Annotated Bibliography for my BanffIRS Lecture: May 2005
Equilibrium in a Stochastic Environment
a 'constructive' approach *

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Warning. This bibliography is not comprehensive, even biased. It refers mostly to our publications. For a more complete-balanced bibliography, please consult the references lists of the articles/books cited.

1 Stochastic Programming

1.1 Overview

These couple of articles provide an overview of the properties of two- and multi-stage stochastic programs with recourse:

1. R.J-B Wets. Stochastic programs with fixed recourse: the equivalent deterministic problem. *SIAM Review*, 16:309–339, 1974.

2. R.J-B Wets. Stochastic programming. In G. Nemhauser, A. Rinnooy Kan, and M. Todd, editors, *Handbook for Operations Research and Management Sciences, Vol 1*, pages 573–629. Elsevier Science Publishers B.V. (North Holland), 1989.

1.2 Non-anticipativity as a constraint

This collection of papers introduce and explore the implications of dealing with non-anticipativity as an explicit constraint. Non-anticipativity of the decision process is an inherent component of stochastic optimization problems and distinguishes, in a fundamental way, stochastic from deterministic optimization problems. The second paper gives an interpretation of the 'multipliers' associated with these non-anticipativity constraints as *equilibrium prices*. The third paper introduces the notion of a *non-anticipative constraint mapping* for discrete-time models that will be key when dealing with stochastic dynamic equilibrium models. The next paper is more of an expository nature but makes the tie between the quite common approach via scenario analysis and stochastic programming models. Finally, the last paper, gives a (simplified) proof of justification for

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the interchange of minimization and expectation and, consequently, relates the here-and-now and the wait-and-see models; for earlier and more comprehensive proofs one should consult the papers by Rockafellar-Wets mentioned in the references.

1. R.J-B Wets. On the relation between stochastic and deterministic optimization. In A. Bensoussan and J.L. Lions, editors, *Control Theory, Numerical Methods and Computer Systems Modelling*, Lecture Notes in Economics and Mathematical Systems, 107, pages 350–361. Springer, 1975.

2. R.T. Rockafellar and R.J-B Wets. Stochastic convex programming: Kuhn-tucker conditions. *J. of Mathematical Economics*, 2:349–370, 1975.

3. R.T. Rockafellar and R.J-B Wets. Nonanticipativity and $\updownarrow^1$-martingales in stochastic optimization problems. *Mathematical Programming Study*, 6:170–187, 1976.

4. R.J-B Wets. The aggregation principle in scenario analysis and stochastic optimization. In S. Wallace, editor, *Algorithms and Model Formulations in Mathematical Programming*, NATO ASI Vol.51, pages 91–113. Springer, NATO ASI Vol.51,, 1989.

5. R.J-B Wets. Stochastic programming models: Wait-and-see versus here-and-now. In François Auzerais, R. Burridge, C. Greengard, and A. Ruszczynski, editors, *Decision Making under Uncertainty: Energy and Environmental Models*. Springer, 2001.

1.3 Solution procedures

Solution procedures for stochastic programs are usually based on decomposition schemes such as described in the first paper. The two following papers lay out procedures based on 'progressively' imposing the non-anticipativity constraint, or equivalently, hedging against uncertainty, i.e., finding the prices associated with the non-anticipativity constraints,

1. R.J-B Wets. Large-scale linear programming techniques in stochastic programming. In Y. Ermoliev and R. Wets, editors, *Numerical Techniques for Stochastic Optimization*, pages 61–89. Springer, 1988.

2. R.T. Rockafellar and R.J-B Wets. Scenarios and policy aggregation in optimization under uncertainty. *Mathematics of Operations Research*, 16:119–147, 1991.

3. R.T. Rockafellar and R.J-B Wets. A dual strategy for the implementation of the aggregation principle in decision making under uncertainty. *Applied Stochastic Models and Data Analysis*, 8:245–255, 1992.

2 Equilibrium

2.1 Existence, Ky Fan's Inequality

For classical results about the existence of equilibrium prices one can refer to Debreu's monograph or the more recent book by Balasko, for example. However, our approach will rely on relating equilibrium points to the max-inf points of a bivariate function, called the *Walrasian*. The existence of max-inf points relies on a fundamental results due to Ky Fan; a more comprehensive treatment can be found in Chapter 6 of the book by of Aubin and Ekeland listed below.

1. G. Debreu. *Theory of Value*. J. Wiley, 1959.

2. Y. Balasko. *Foundations of the theory of General Equilibrium*. Academic Press, 1988.

3. K. Fan. A minimax inequality and applications. In A. Shisha, editor, *Inequalities, 3*, pages 103–113. Academic Press, 1972.

4. J.-P. Aubin and I. Ekeland. *Applied Nonlinear Analysis*. J. Wiley Intersciences, 1984.

2.2 Stability

For the state-of-the-art (80's) can be found in the review article by Hahn. The second article relies on the study of the stability of the max-inf points of bivariate functions from a variational viewpoint, and marks a departure from the 'classical' approach.

1. F.Hahn. Stability. In K. Arrow and M. Intriligator, editors, *Chapter 16, Handbook of Mathematical Economics*, pages 745–793. North-Holland, 1981.

2. A. Jofré and R.J-B Wets. Continuity properties of Walras equilibrium points. *Annals of Operations Research*, 114:229–243, 2002.

3 Variational Convergence

3.1 Epi-convergence (or hypo-convergence)

Chapter 7 of *Variational Analysis* provides a comprehensive, but not exhaustive, treatment of approximation theory for optimization problems. The convergence notion is *epi-convergence* that 'guarantees' the convergence of the optimal solutions and optimal values. The last part of that chapter shows that epi-convergence defines a metric-topology for the space of lower semicontinuous functions and reviews a number of useful quantitative results.

1. R.T. Rockafellar and R.J-B Wets. *Variational Analysis (2nd Edition)*. Springer, 2004.

3.2 Hypo/epi-convergence

Hypo/epi-convergence is the convergence notion for bivariate function that yields the convergence of their saddle points. In particular, one relies on this notion to obtain the continuity of the non-anticipativity multipliers (prices) as a function of the goods-price system.

1. H. Attouch and R.J-B Wets. A convergence theory for saddle functions. *Transactions of the American Mathematical Society*, 280:1–41, 1983.

2. H. Attouch and R.J-B Wets. A convergence for bivariate functions aimed at the convergence of saddle values. In J.P. Cecconi and T. Zolezzi, editors, *Mathematical Theories of Optimizations*, pages 1–42. Springer, 1983.

3. H. Attouch, D. Azé, and R.J-B Wets. Convergence of convex-concave saddle functions: continuity properties of the Legendre-Fenchel transform with applications to convex programming and mechanics. *Annales de l'Institut H. Poincaré: Analyse Nonlinéaire*, 5:537–572, 1988.

4. H. Attouch, D. Azé, and R.J-B Wets. On continuity properties of the partial Legendre-Fenchel transform: convergence of sequences of augmented Lagrangian functions, Moreau-Yosida approximates and subdifferential operators. In J.-B. Hiriart-Urruty, editor, *Fermat-Days 85: Mathematics for Optimization*, pages 1–42. North Holland, 1986.

3.3 Lopsided convergence

Lopsided convergence was introduced as a convergence notion for bivariate functions aimed at the convergence of their max-inf points. The first paper in the list gave a definition that was too restrictive, and has now been adjusted, cf. the subsequent papers, to handle a much larger class of bivariate functions that includes those arising in equilibrium issues, fixed point problems, non-cooperative games and their Nash equilibrium points, etc.

1. H. Attouch and R.J-B Wets. Convergence des points min/sup et de points fixes. *Comptes Rendus de l'Académie des Sciences de Paris*, 296:657–660, 1983.
2. A. Jofré and R.J-B Wets. Variational convergence of bivariate function: Theoretical foundations manuscript, September 2004.
3. A. Jofré and R.J-B Wets. Variational convergence of bivariate function: Motivating applications I manuscript, February 2005.
4. A. Jofré and R.J-B Wets. Variational convergence of bivariate function: Motivating applications II manuscript, 2005.

4 Variational Inequalities & Equilibrium

The first book in the list can be consulted for general results and solution procedures for variational inequalities in finite dimensions. The three following papers deal with the relationship between variational inequalities and economic equilibrium. The last one of these paper introduces a 'functional' variational inequality that doesn't quite fit the classical variational inequalities formulation.

1. F. Facchinei and J.-S. Pang. *Finite-Dimensional Variational Inequalities and Complementarity Problems*. Springer, 2003.
2. S. Dafermos. Exchange price equilibria and variational inequalities. *Mathematical Programming*, 46:391–402, 1990.
3. A. Jofré, R.T. Rockafellar, and R.J-B Wets. A variational inequality scheme for determining an economic equilibrium of classical or extended type. In F. Giannessi and A. Maugeri, editors, *Variational Analysis and Applications*, pages 553–578. Springer, 2005.
4. A. Jofré, R.T. Rockafellar, and R.J-B Wets. Variational inequalities and economic equilibrium. manuscript, February 2005.

5 Extensions

When the initial endowments do not (necessarily) belong to the survival sets of the agents, one is lead to consider transfer functions. We studied this problem and have preliminary results. A major reference in this area:

1. R. Guesnerie. *A Contribution to the Pure Theory of Taxation*. Cambridge University Press, 1999.