Dealing with Uncertainty
in Decision Making Models

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A product mix problem

Choose $x_j \geq 0,\ j = 1, \ldots, 4,$ to maximize:

$$\sum_{j=1}^{4} c_j x_j = 12x_1 + 25x_2 + 21x_3 + 40x_4$$

such that (constraints)

$$t_{c1} + t_{c2} + t_{c3} + t_{c4} \leq d_c \qquad \text{(carpentry)}$$

$$t_{f1} + t_{f2} + t_{f3} + t_{f4} \leq d_f \qquad \text{(finishing)}$$

$d_c(d_f) =$ total time available for carpentry (finishing)

Linear Programming Sol'n

$$\max\langle c,x\rangle \text{ such that } Tx \le d,\ x \in \mathbb{R}^n_+ \qquad \text{with}$$

$$T = \begin{bmatrix} t_{c1} & t_{c2} & t_{c3} & t_{c4} \\ t_{f1} & t_{f2} & t_{f3} & t_{f4} \end{bmatrix} = \begin{bmatrix} 4 & 9 & 7 & 10 \\ 1 & 1 & 3 & 40 \end{bmatrix}, \quad \begin{bmatrix} d_c \\ d_f \end{bmatrix} = \begin{bmatrix} 6000 \\ 4000 \end{bmatrix}$$

Optimal: $x^d = (1{,}333.33,\ 0,\ 0,\ 66.67)$

Linear Programming Sol'n

$\max\langle c,x\rangle$ such that $Tx \le d,\ x \in \mathbb{R}^n_+$ with

$$T = \begin{bmatrix} t_{c1} & t_{c2} & t_{c3} & t_{c4} \\ t_{f1} & t_{f2} & t_{f3} & t_{f4} \end{bmatrix} = \begin{bmatrix} 4 & 9 & 7 & 10 \\ 1 & 1 & 3 & 40 \end{bmatrix}, \quad \begin{bmatrix} d_c \\ d_f \end{bmatrix} = \begin{bmatrix} 6000 \\ 4000 \end{bmatrix}$$

Optimal: $x^d = (1{,}333.33,\ 0,\ 0,\ 66.67)$

but ... "reality": $t_{cj} = t_{cj} + \eta_{cj}, \quad t_{fj} = t_{fj} + \eta_{fj}$

entry	values			
$d_c + \zeta_c$	5,873	5,967	6,033	6,127
$d_f + \zeta_f$	3.936	3.984	4,016	4,064

10 random variables $\Rightarrow L = 1{,}048{,}576$ possible pairs $(T^l,\ d^l)$

Taking recourse into account

What if $\sum_{j=1}^{4}(t_{cj}+\eta_{cj})x_j > d_{cj}+\varsigma_{cj}$??

Taking recourse into account

What if $\sum_{j=1}^{4}(t_{cj} + \eta_{cj})x_j > d_{cj} + \varsigma_{cj}$?? $\Rightarrow$ overtime

With $\xi = (\eta_{\{\cdot,\cdot\}}, \zeta_{\{\cdot\}})$, recourse : $(y_c(\xi), y_f(\xi))$@cost (q_c, q_f)

$$
\begin{array}{llllllll}
\max & \langle c,x\rangle & -p_1\langle q,y^1\rangle & -p_2\langle q,y^2\rangle & \cdots - & p_L\langle q,y^L\rangle & & \\
\text{s.t.} & T^1x & -y^1 & & & & \le & d^1 \\
 & T^2x & & -y^2 & & & \le & d^2 \\
 & \vdots & & & \ddots & & & \vdots \\
 & T^Lx & & & & -y^L & \le & d^L \\
 & x \ge 0, & y^1 \ge 0, & y^2 \ge 0, & \cdots & y^L \ge 0. & &
\end{array}
$$

Structured large scale l.p. $(L \approx 10^6)$

Equivalent Deterministic Program

Define $\Xi = \{\xi = (\eta, \zeta)\}$, $p_\xi = [\xi = \xi]$

$$Q(\xi, x) = \max\left\{\langle -q, y\rangle \middle| T_\xi x - y \geq d_\xi, y \geq 0\right\}$$

EQ concave

$$EQ(x) = E\{Q(\xi, x)\} = \sum_{\xi \in \Xi} p_\xi Q(\xi, x)$$

(DEQ) $\max\langle c, x\rangle + EQ(x)$ such that $x \in \mathbb{R}^n_+$

non-smooth convex optimization problem

Robust Solutions !!!

DEQ Optimal : $x^* = (257,\ 0,\ 665.2,\ 33.8)$

while $x^d = (1{,}333.33,\ 0,\ 0,\ 66.67)$

Expected profit x^*: \$18.051, x^d : \$17,942

x^d not close to optimal (- 6.5%)

x^d isn't pointing in the right direction

x^* robust, considered all 10^6 possibilities.

NewsVendor Problem

$$\max \; -cx+(c+r)y,\; x\geq 0,\quad 0\leq y\leq \min\{x,\xi\}$$

$$\Xi=[0,150]$$

$$\text{c} = 10,\; r=15$$

Pick $\xi^1,\ldots,\xi^L$ (*scenarios*), and find:

$$(x^l,y^l)\in \operatorname*{argmin}_{x\geq 0,y\geq 0}\left\{-cx+(c+r)y \,\middle|\, y\leq \min\left[\xi^l,x\right]\right\}$$

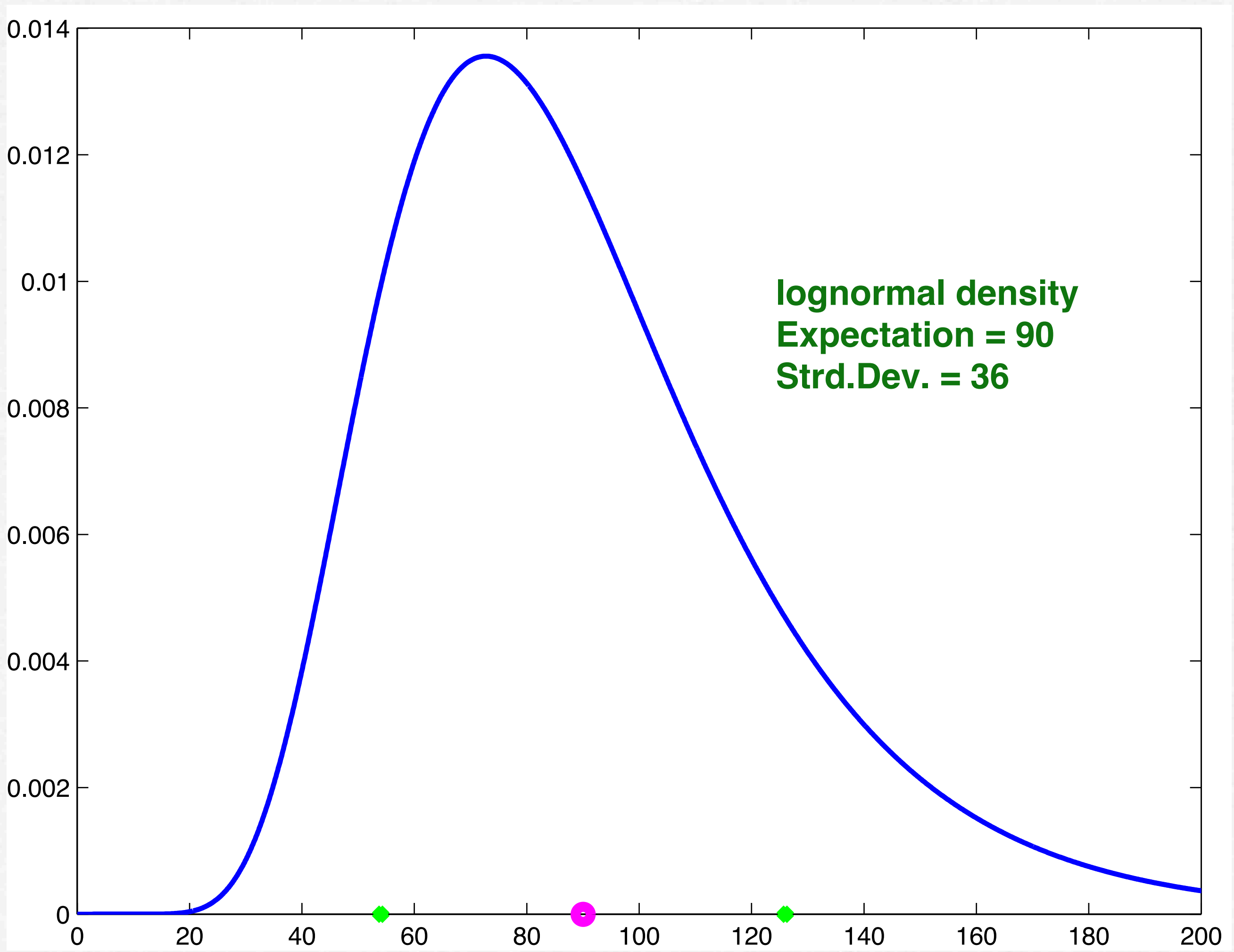
Wait-and-see sol'ns: $x^l=\xi^l$. "Reconciliation"

no help in choosing x^* optimal!

ξ : Estimated Density *h*

$$\xi \;\; \text{log-normal: } h(z) = \left(z\tau\sqrt{2\pi}\right)^{-1} e^{-\frac{(\ln z - \theta)^2}{2\tau^2}}$$

$$\theta = 4.43, \;\; \tau = 0.38; \;\; H(z) = \int_0^z h(s)\,ds$$



Maximize expected return

$$\max \; -cx + E\{(c+r)y_\xi\}$$

$$\text{such that } \; x \geq 0,\; 0 \leq y_\xi \leq \min[\xi, x]$$

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$$\text{such that } \ x \geq 0, \ 0 \leq y_\xi \leq \min[\xi, x]$$

$$\text{DEQ: } \max \ -cx + EQ(x) =$$

$$-cx + \left((c+r) \int_0^x \xi H(d\xi) + \int_x^\infty x H(d\xi) \right)$$

$$\text{sol'n: } x^* = H^{-1}\left(\frac{r}{c+r} \right) = H^{-1}(0.6) = 99.2; \quad c = 10, r = 15$$

Maximize expected return

$$\max\ -cx + E\{(c+r)y_\xi\}$$

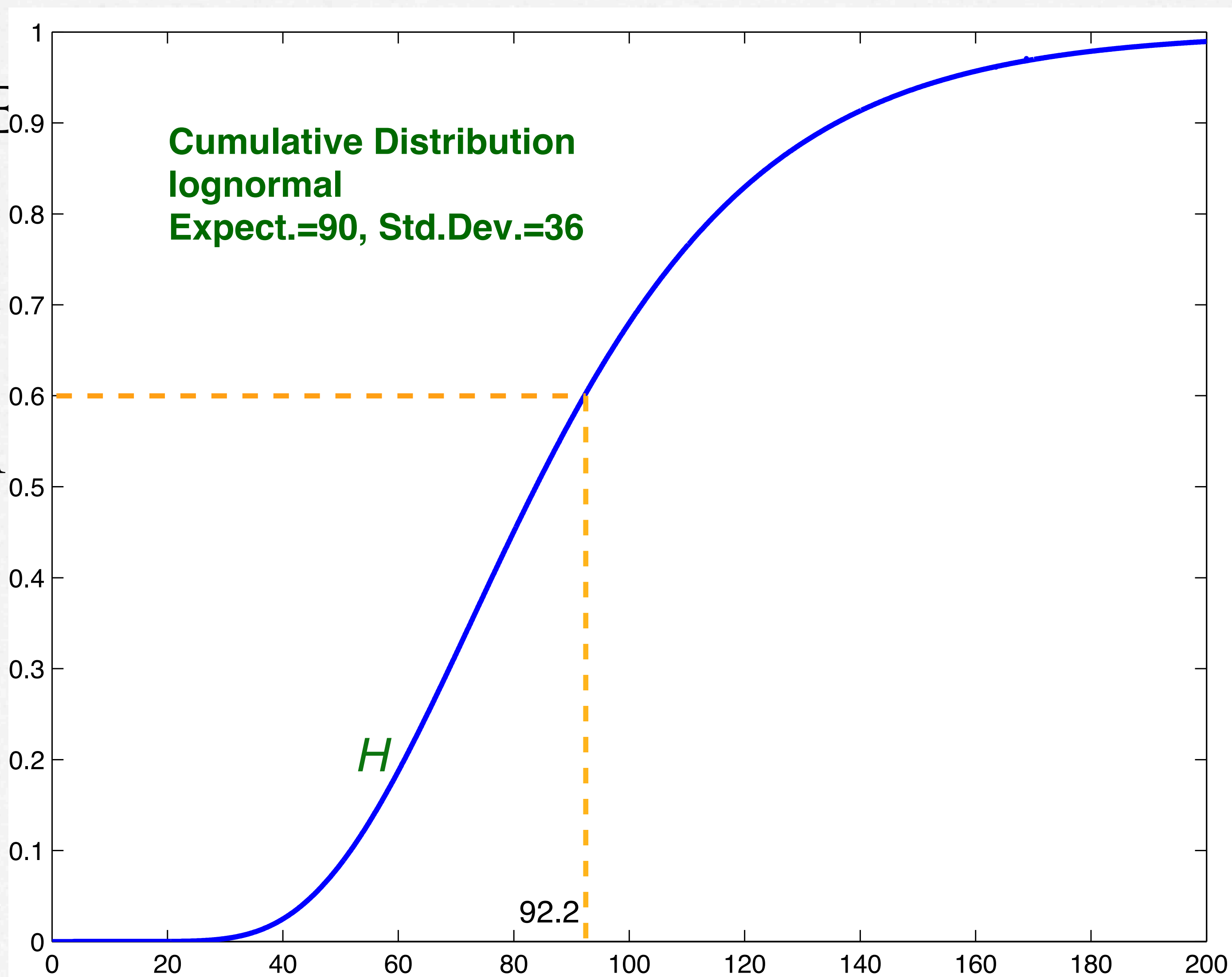
$$\text{such that}\ \ x \geq 0,\ 0 \leq y_\xi \leq \min[\xi, x]$$

DE

$\xi)\Big)$

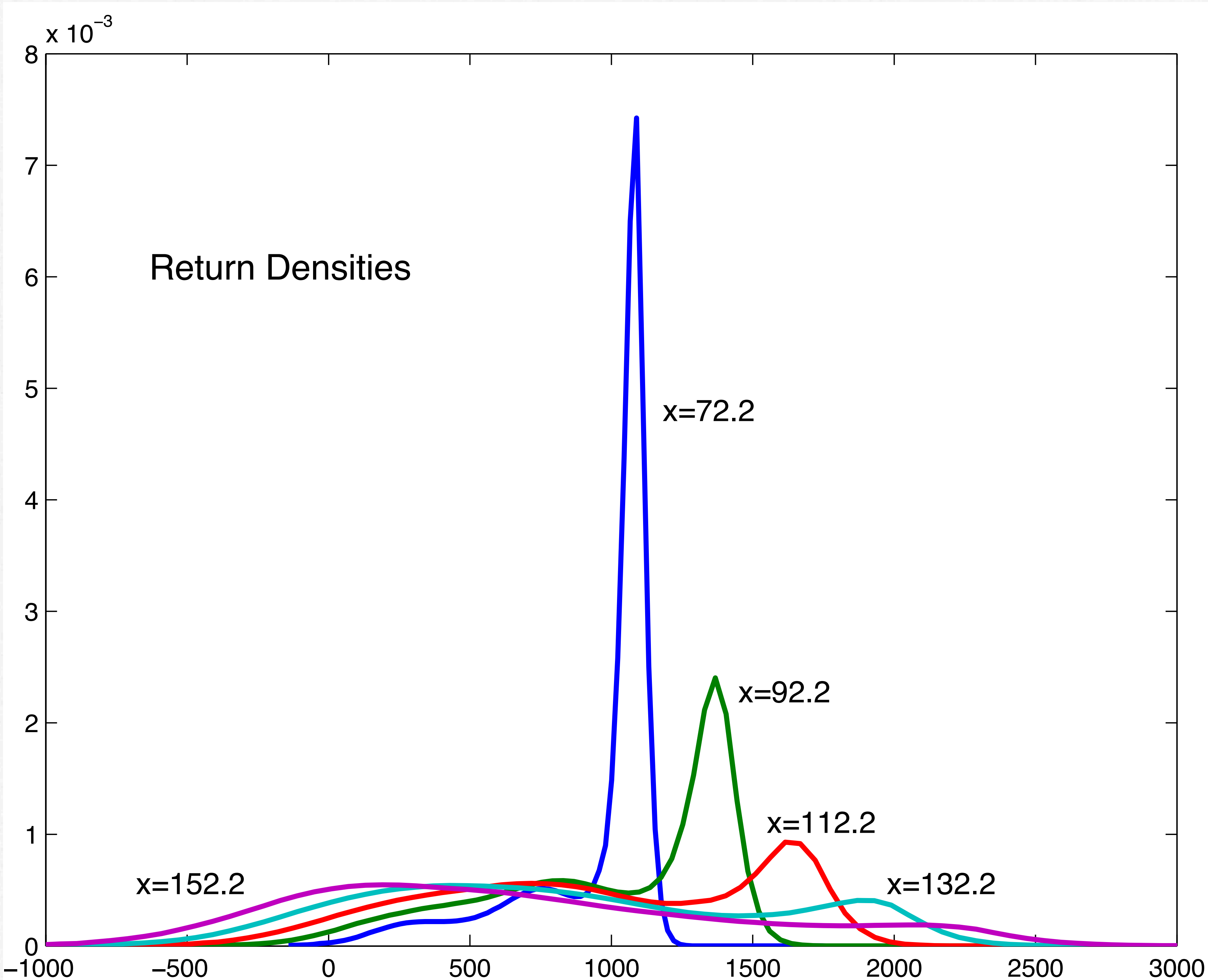
sol

$10, r = 15$

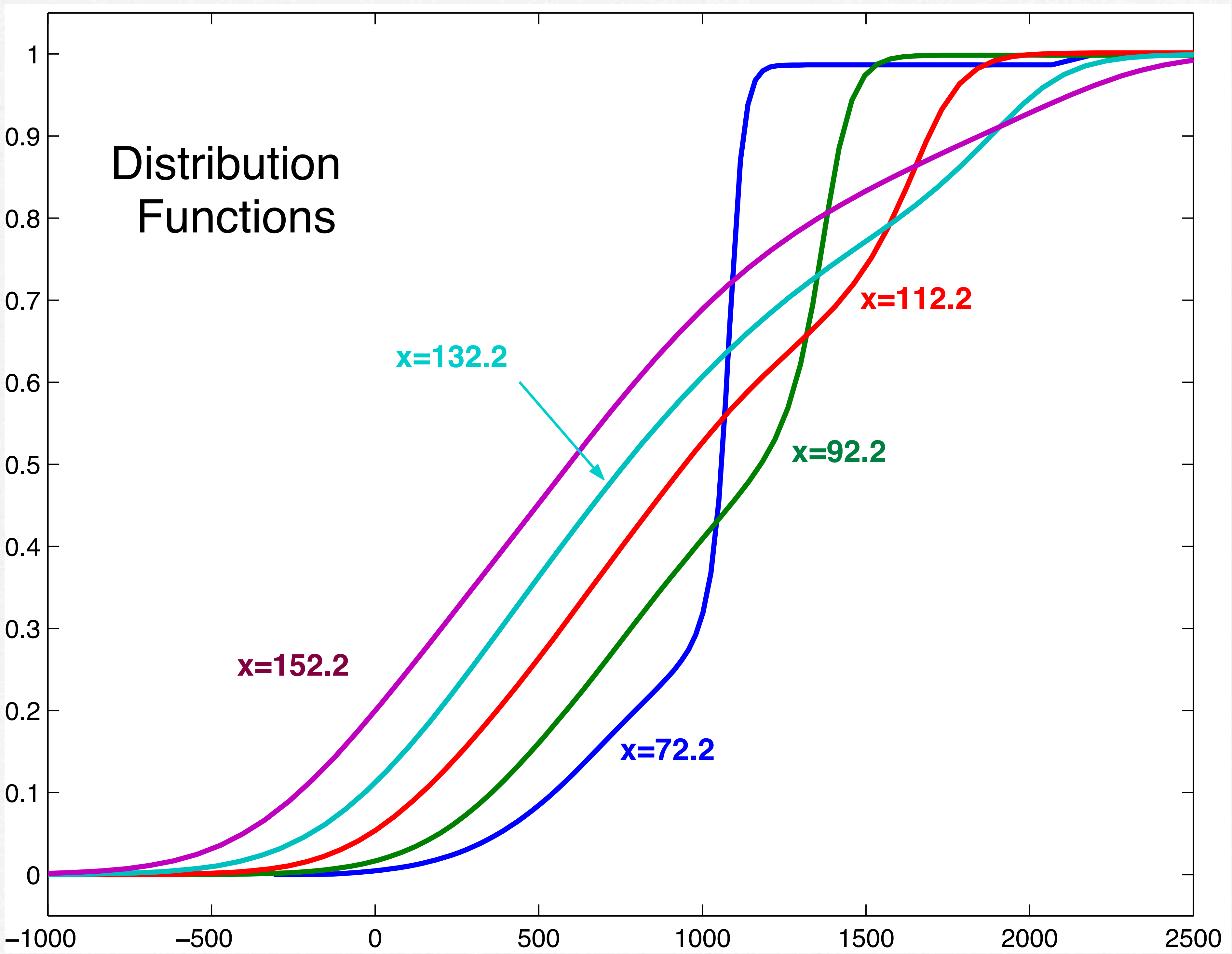


… but is maximum expected return the "real" objective?

The “returns” densities



Choosing: “returns” distribution



Decision criteria:

from a distribution $\Longrightarrow$ a number

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from a distribution $\Longrightarrow$ a number

- maximize expected return (scaled?)
- max. E{return} & minimize customers lost
- minimize Value-at-Risk (V@A, CV@R)
- minimize probability of any loss
- minimize a “safeguarding” measure, ...

$$\max E\{f(\xi,x)\} \Rightarrow \max E\{u(f(\xi,x))\}$$

V@E: Value-at-Risk

$$F(v;x) = \text{prob}\left[-cx + Q(\xi, x) \leq v\right]$$

Value-at-Risk(V@R) for $\alpha \in (0,1)$:

$$\text{V@R}(\alpha;x) = F^{-1}(\alpha;x) \quad \left(= \sup\left\{v \middle| F^{-1}(\alpha;x)\right\}\right)$$

Objective: find x that maximizes $\text{V@R}(\alpha;x)$ given α

Challenge: $x \mapsto \text{V@R}(\alpha;x)$ isn't concave!

Heuristic: F is $\mathcal{N}\left(\mu(x), \sigma(x)^2\right)$ and

$$\boxed{\text{V@R}(\alpha;x) = \mathcal{N}\left(\alpha; \mu(x), \sigma(x)^2\right)}$$

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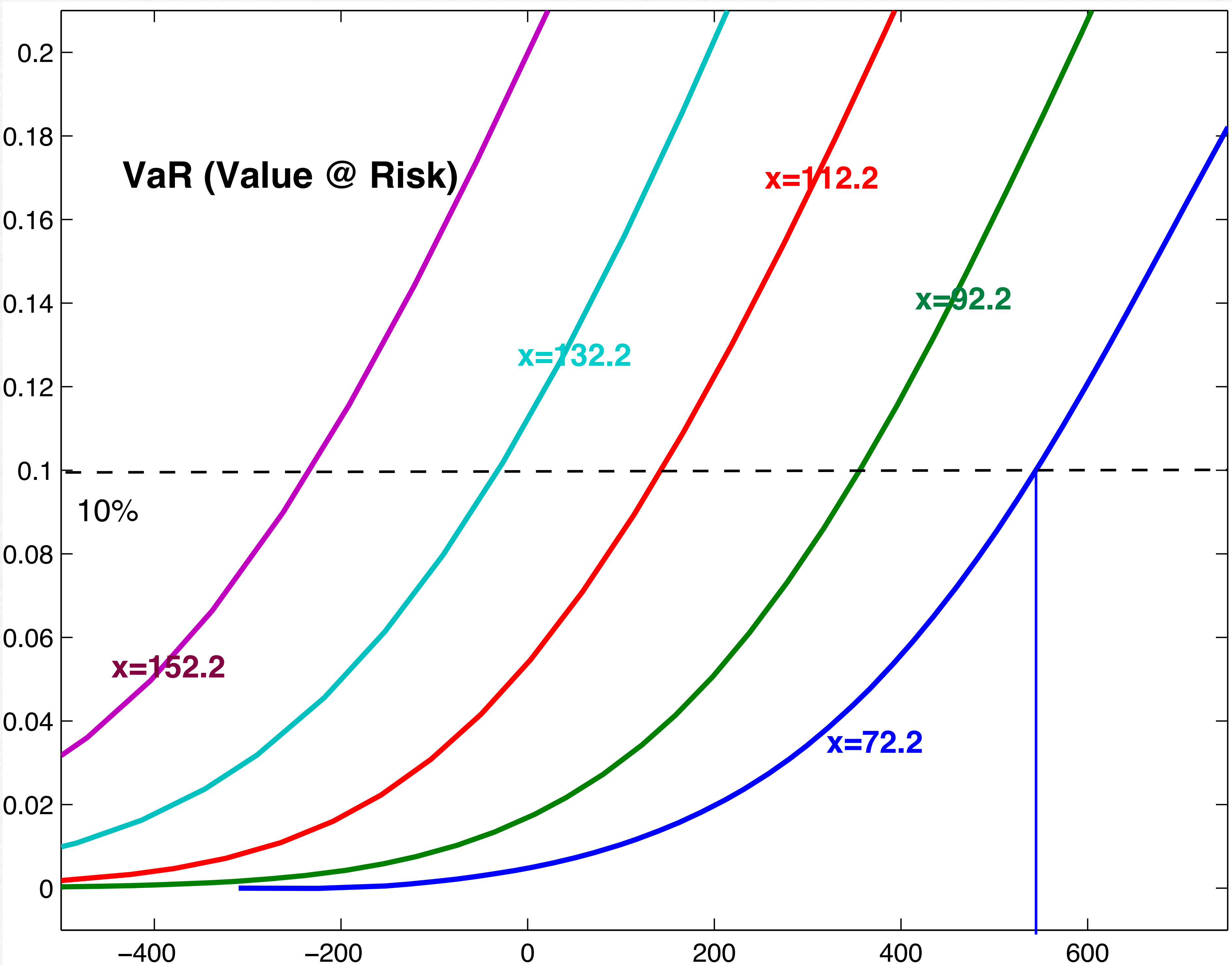
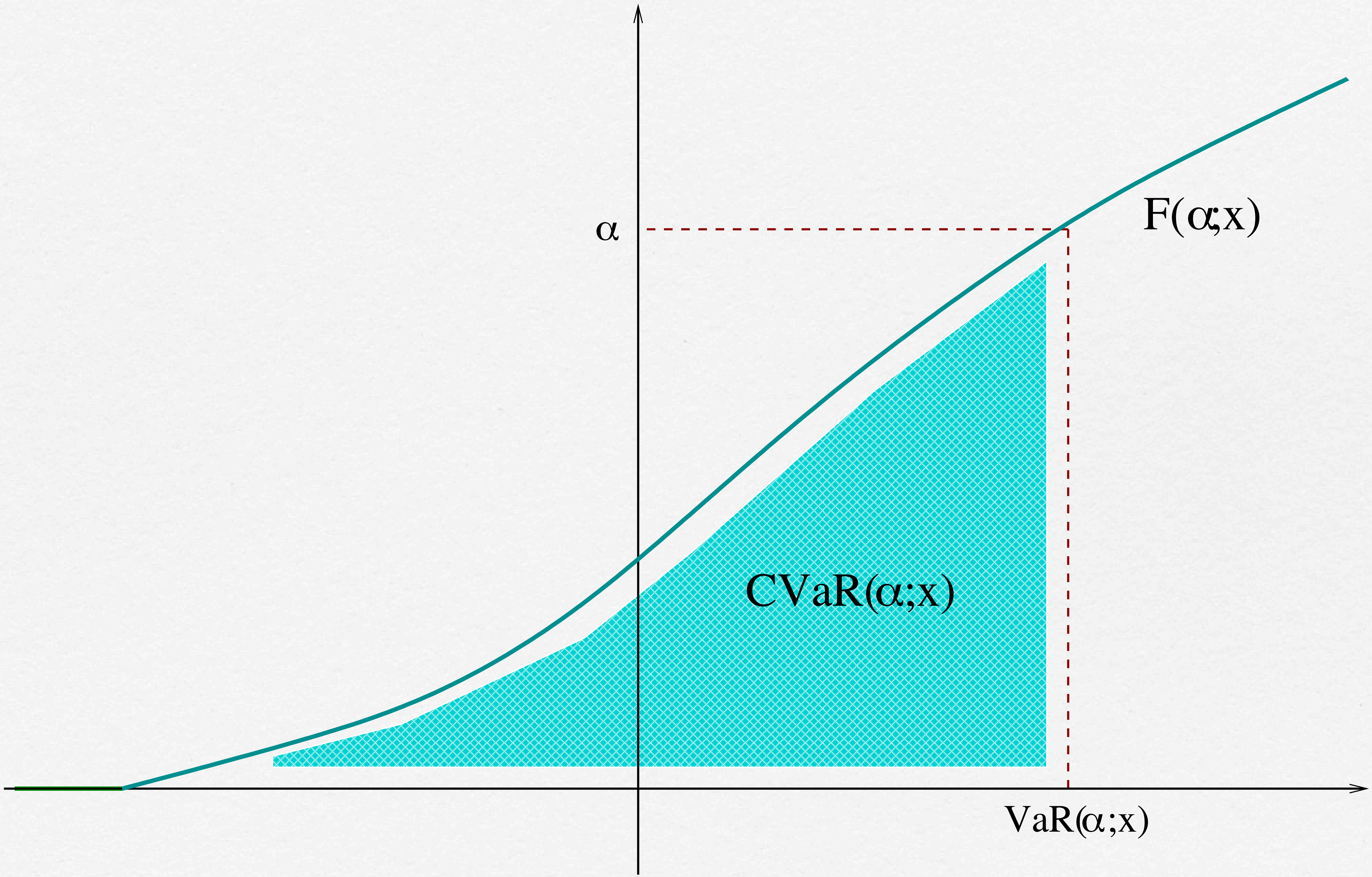
V@E”: the NewsVendor



CV@R: Conditional Value-at-Risk



CV@R: Conditional Value-at-Risk

$$G(v;x) = E\left\{-cx + Q(\xi,x) \middle| -cx + Q(\xi,x) \le v\right\}$$

Conditional Value-at-Risk(CV@R) for $\alpha \in (0,1)$:

$$CV@R(\alpha;x) = G^{-1}(\alpha;x) \quad \left(= \sup\left\{v \middle| G^{-1}(\alpha;x)\right\}\right)$$

$$= \min_r r + (1-\alpha)^{-1} E\left\{\left[-cx + Q(\xi,x) - r\right]_+\right\}$$

Objective: find x that maximizes $CV@R(\alpha;x)$ given α

$x \mapsto CV@R(\alpha;x)$ is concave (convenient u)

Stochastic Programs with Recourse

.. with Simple Recourse

decision: $x \rightsquigarrow$ observation: $\xi \rightsquigarrow$ recourse cost evaluation.

cost evaluation 'simple' $\Rightarrow$ simple recourse, i.e.,

$$\min_{x \in S \subset \mathbb{R}^n} f_0(x) + \mathbb{E}\{Q(\xi, x)\} \quad Q \text{ 'simple'}$$

Product mix problem. With $\xi = (T, d)$,

$$f_0(x) = \langle c, x \rangle, \;\; S = \mathbb{R}^4_+, \;\; Q(\xi, x) = \sum_{i=c,f} \max\left[0, \gamma_i(\langle T_i, x\rangle - d_i)\right]$$

NewsVendor: cost: γ, sale price δ,

$\boldsymbol{\xi}$, demand distribution P, order x,

expected "loss": $\gamma x + \mathbb{E}\{Q(\boldsymbol{\xi}, x)\}$

$Q(\xi, x) = -\delta \cdot \min\{x, \xi\}$

$\Rightarrow$ explicit sol'n

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Extensive formulation

EXF

$$
\begin{array}{llllllll}
\min & \langle -c, x\rangle & +p_1\langle q, y^1\rangle & +p_2\langle q, y^2\rangle & \cdots + & p_L\langle q, y^L\rangle & & \\
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\end{array}
$$

Deterministic Equivalent Problem

$$Q(\xi, x) = \min \left\{ \langle q, y\rangle \,\middle|\, T_\xi x + y \geq d_\xi,\ y \geq 0 \right\}$$

$$EQ(x) = \mathbb{E}\{Q(\boldsymbol{\xi}, x)\} = \sum_{\xi\in\Xi} p_\xi Q(\xi, x)$$

the equivalent deterministic program:

DEP

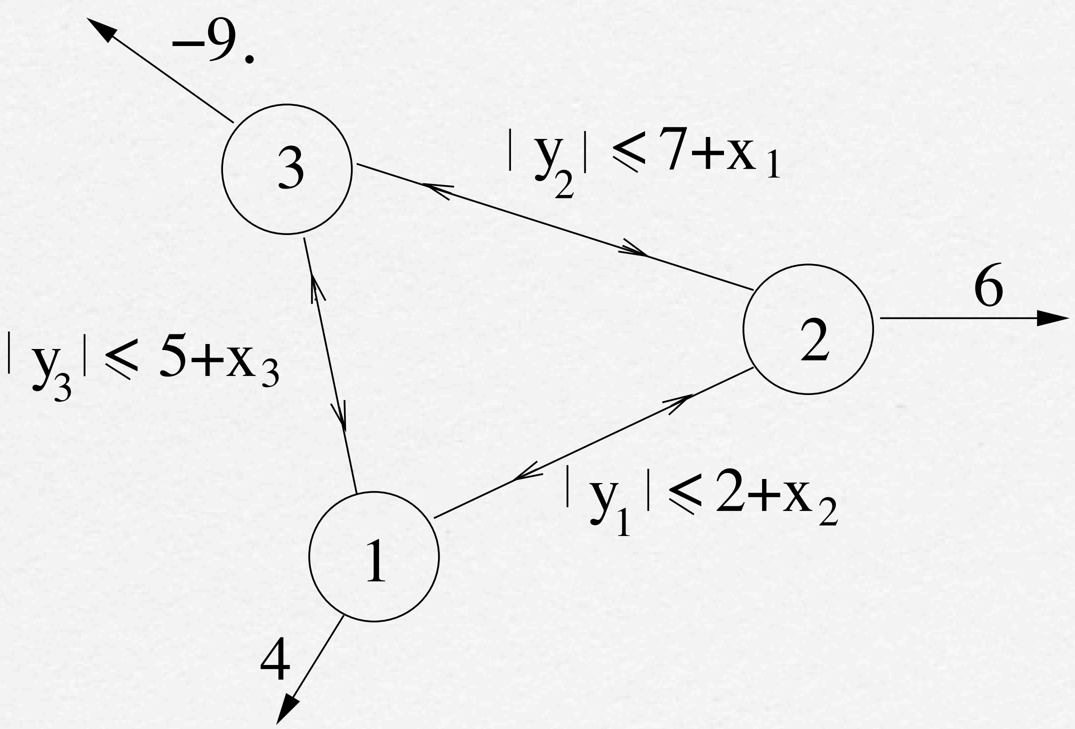
$$\min\langle -c, x\rangle + EQ(x) \quad \text{such that } x \in \mathbb{R}^n_+$$

product mix problem

Network capacity expansion

Deterministic Version:

$$\min \sum_{j=1}^n \psi_j(x_j), \text{ such that } 0 \le x_j \le \upsilon_j, \qquad j = 1, \dots, n$$
$$|y_j| \le \gamma_j + x_j, \ \ j = 1, \dots, n, \ \ \sum_{j \in \odot(i)} y_j \ge e_i, \ \imath = 1, \dots, m$$



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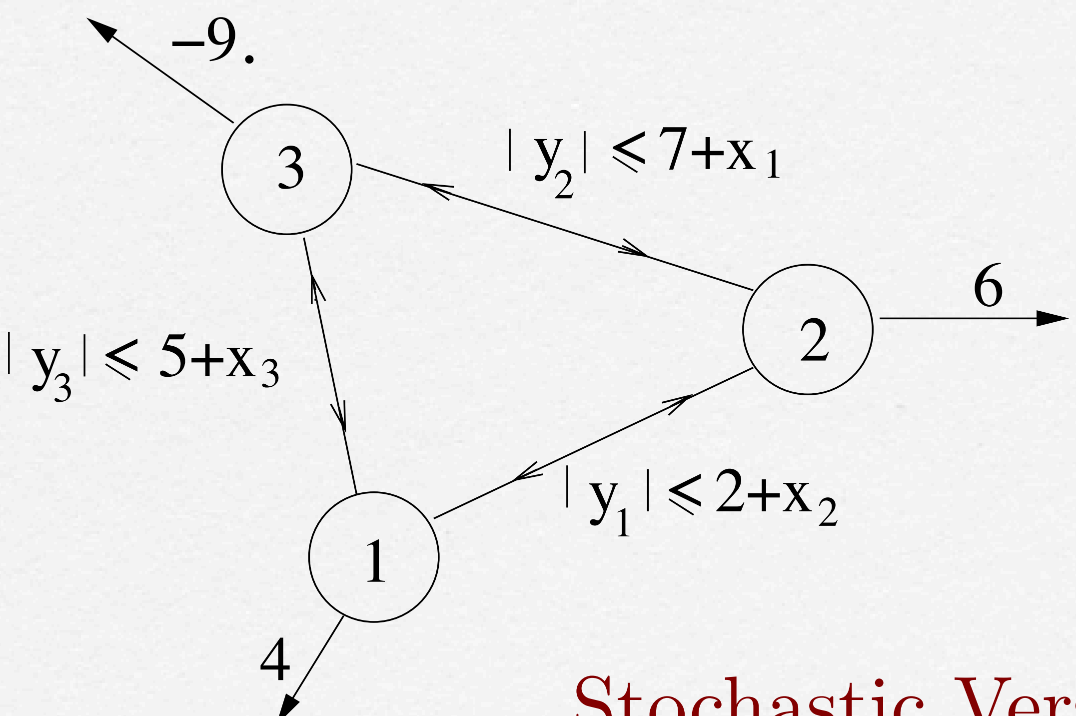

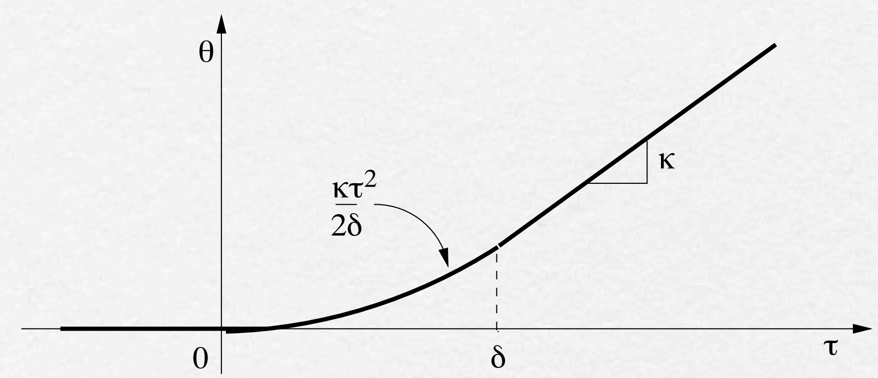

monitoring function

Stochastic Version: $\boldsymbol{\xi}^l, l = 1, \dots, L,$

$$\min_{x, y^l} \sum_{l=1}^{L} \Big[p_l \sum_{i=1}^{m} \theta_i \big(\xi_i^l - \sum_{j \in \odot(i)} \boldsymbol{y}_j^l \big) \Big]$$
$$\text{s.t. } \sum_{j=1}^{n} \psi_j(x_j) \le \beta, \quad 0 \le x_j \le \upsilon_j, \;\; j = 1, \dots, n,$$
$$|\boldsymbol{y}_j^l| - x_j \le \gamma_j, \;\; j = 1, \dots, n, \; l = 1, \dots, L$$

Aggregation Principle in Stochastic Optimization

Here-&-Now vs. Wait-&-See

- Basic Process: decision --> observation --> decision

$$x^1 \rightarrow \xi \rightarrow x^2_\xi$$

- Here-&-now problem! x^1

  not all contingencies available at time 0

  cannot depend on ξ!

- Wait-&-see problem

  implicitly all contingencies available at time 0

  choose (x^1_ξ, x^2_ξ) after observing ξ

- incomplete information to anticipative information ?

Stochastic Optimization: Fundamental Theorem

Stochastic Optimization: Fundamental Theorem

A here-and-now problem can be “reduced” to a wait-and-see problem by introducing the

appropriate ‘information’ costs

(price of non-anticipativity)

Price of Nonanticipativity

Here-&-now

$$\min\ \mathbb{E}\left\{f(\boldsymbol{\xi}, x^1, x^2_{\xi})\right\}$$

$$x^1 \in C^1 \subset \mathbb{R}^n,$$

$$x^2_{\xi} \in C^2(\xi, x^1),\ \forall \xi.$$

Price of Nonanticipativity

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Explicit non-anticipativity

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$w_\xi \perp$ subspace of constant fcns

multipliers

$$\Rightarrow \mathbb{E}\left\{ w_\xi \right\} = 0$$

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$$\Rightarrow \mathbb{E}\left\{w_\xi\right\} = 0$$

$$\min\ \mathbb{E}\left\{f(\boldsymbol{\xi},x^1_\xi,x^2_\xi) - \langle w_\xi, x^1_\xi\rangle + \langle w_\xi, \mathbb{E}\{x^1_\xi\}\rangle\right\}$$

such that $x^1_\xi \in C_1,\quad x^2_\xi \in C_2(\boldsymbol{\xi},x^1_\xi)$

Price of Nonanticipativity

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$w_\xi \perp$ subspace of constant fcns

multipliers

$$\Rightarrow\mathbb{E}\left\{w_\xi\right\}=0$$

$$\min\ \mathbb{E}\left\{f(\boldsymbol{\xi},x^1_\xi,x^2_\xi)-\langle w_\xi,x^1_\xi\rangle+\langle w_\xi,\mathbb{E}\{x^1_\xi\}\rangle\right\}$$

such that $x^1_\xi\in C_1,\quad x^2_\xi\in C_2(\boldsymbol{\xi},x^1_\xi)$

Adjusted Here-&-Now

$\min \; \mathbb{E}\left\{ f(\boldsymbol{\xi}, x^1, x^2_\xi) \right\}$ such that $x^1 \in C^1 \subset \mathbb{R}^n, \; x^2_\xi \in C^2(\xi, x^1), \forall \xi$

x^1 must be $\mathcal{G}$-measurable, $\mathcal{G} = \sigma\text{-}\{\emptyset, \Xi\}$

x^2 is $\mathcal{A}$-measurable, $\mathcal{A} \supset \mathcal{G}$,

in general, interchange $\mathbb{E}$ & ∂ is not valid

required: $\forall\, \xi, x^1 \in C^1, C^2(\xi, x^1) \neq \emptyset$ $\mathcal{G}$-measurability of constraints

Now, suppose w_ξ are the (optimal) non-anticipativity multipliers (prices)

$\min \; \mathbb{E}\left\{ f(\boldsymbol{\xi}, x^1_\xi, x^2_\xi) - \langle w_\xi, x^1_\xi \rangle + \langle w_\xi, \mathbb{E}\{x^1_\xi\} \rangle \right\}$

such that $x^1_\xi \in C^1 \subset \mathbb{R}^n, \; x^2_\xi \in C^2(\xi, x^1_\xi), \forall \xi$

Interchange is now O.K. , $\mathbb{E}\left\{ \langle w_\xi, \mathbb{E}\{x^1_\xi\} \rangle \right\} = \langle \mathbb{E}\{w_\xi\}, \mathbb{E}\{x^1_\xi\} \rangle = 0$, yields

$\forall\, \xi$, solve: $\min \; f(\boldsymbol{\xi}, x^1, x^2) - \langle w_\xi, x^1 \rangle$ s.t. $x^1 \in C^1, \; x^2 \in C^2(\xi, x^1)$

a collection of deterministic optimization problems in $\mathbb{R}^{n_1 + n_2}$

Progressive Hedging Algorithm

0. w_ξ^0 such that $\mathbb{E}\left\{w_\xi^0\right\} = 0,\ \nu = 0.$ Pick $\rho > 0$

1. for all ξ :

$$(x_\xi^{1,\nu}, x_\xi^{2,\nu}) \in \arg\min f(\xi; x^1, x^2) - \langle w_\xi^\nu, x^1 \rangle$$

$$x^1 \in C^1 \subset \mathbb{R}^{n_1},\ x^2 \in C^2(\xi, x^1) \subset \mathbb{R}^{n_2}$$

2. $\bar{x}^{1,\nu} = \mathbb{E}\left\{x_\xi^{1,\nu}\right\}.$ Stop if $\left|x_\xi^{1,\nu} - \bar{x}^{1,\nu}\right| = 0$ (approx.)

otherwise $w_\xi^{\nu+1} = w_\xi^\nu + \rho\left[x_\xi^{1,\nu} - \bar{x}^{1,\nu}\right]$, return to 1. with $\nu = \nu + 1$

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otherwise $w_\xi^{\nu+1} = w_\xi^\nu + \rho\left[x_\xi^{1,\nu} - \bar{x}^{1,\nu}\right]$, return to 1. with $\nu = \nu + 1$

Convergence: add a proximal term

$$f(\xi; x^1, x^2) - \langle w_\xi^\nu, x^1 \rangle - \frac{\rho}{2}\left|x^1 - \bar{x}^{1,\nu}\right|^2$$

linear rate in $(x^{1,\nu}, w^\nu)$ ... eminently parallelizable

Nonanticipativity

Recall $\min Ef(x) = \mathbb{E}\{f\big(\boldsymbol{\xi}, x(\boldsymbol{\xi})\big)\}$ such that $x(\xi) = \mathbb{E}\{x(\boldsymbol{\xi})\}$ P–$a.s.$

Nonanticipativity <u>constraints</u>:

$\mathcal{N}_a = \big\{x : \Xi \to \mathbb{R}^n\big\} \subset$ linear subspace of constant fcns

$\implies \exists\, w : \Xi \to \mathbb{R}$ "multipliers" $\perp \mathcal{N}_a$ ($\Rightarrow \mathbb{E}\{w(\boldsymbol{\xi})\} = 0$) such that

$$x^* \in \operatorname{argmin} Ef \quad \implies \quad x^* \in \operatorname{argmin} \big\{\mathbb{E}\{f\big(\boldsymbol{\xi}, x(\boldsymbol{\xi})\big) + \langle w(\boldsymbol{\xi}), (x(\boldsymbol{\xi}) - \mathbb{E}\{x(\boldsymbol{\xi})\})\rangle\big\}$$

$$\implies \quad x^* \in \operatorname{argmin} \big\{\mathbb{E}\{f\big(\boldsymbol{\xi}, x(\boldsymbol{\xi})\big) + \langle w(\boldsymbol{\xi}), x(\boldsymbol{\xi})\rangle\}\big\}$$

$$P\text{–}a.s. \quad \implies \quad x^* \in \operatorname*{argmin}_{x \in E} \{f\big(\xi, x\big) + \langle w(\xi), x\rangle\}\big\}, \;\; \xi \in \Xi$$

$w(.)$: contingencies equilibrium prices, $\sim$ 'insurance' prices

PH: Implementation issues

implementation: choice of ρ ... scenario ($\times$), i*th*-decision (i) dependent

(heuristic) extension to problems with integer variables

non-convexities: e.g. ground-water remediation with non-linear PDE recourse

asynchronous

partitioning (= different information feeds)

$$\min \mathbb{E}\{f(\xi,x)\}\ , \quad f(\xi,x) = f_0(x) + \iota_{C(\xi,x)}(x)$$

$S = \{\Xi_1, \Xi_2, \ldots, \Xi_K\}$ a partitioning of Ξ, $p_k = P(\Xi_k)$

Bundling

$\mathbb{E}\{f(\xi,x)\} = \sum_n p_n \mathbb{E}\{f(\xi,x) \mid \Xi_n\}$ (Bundling)

defining $g(k,x) = \mathbb{E}\{f_0(\xi,x) \mid \Xi_n\}$ if $x \in C_k = \bigcap_{\xi \in \Xi_k} C_\xi$

solve the problem as: $\min \sum_{n=1}^{N} p_k g(k,x)$

Multistage Stochastic Programs

$\min_{x \in \mathcal{N}^a} \mathbb{E}\{f(\boldsymbol{\xi}, x(\boldsymbol{\xi}))\}, \quad x(\xi) = \left(x^1(\xi), \ldots, x^T(\xi)\right)$

filtration : $\mathcal{A}_0 \subset \mathcal{A}_1 \subset \cdots \subset \mathcal{A}_T = \mathcal{A}, \ \mathcal{A}_0$ trivial

$x \in \mathcal{N}^a$ if x^t $\mathcal{A}_{t-1}$-measurable $\approx \sigma$-field($\overset{\rightarrow \nu-1}{\boldsymbol{\xi}}$)

(here $\boldsymbol{\xi}^0$ deterministic, $x^1(\boldsymbol{\xi}) \equiv x^1$)

under usual $\mathbb{C}.\mathbb{Q}.$ (convex case): $\bar{x} \in \mathcal{X}$ optimal if

$\exists\, \bar{w} \perp \mathcal{N}^a, \bar{w} \in \mathcal{X}^*$ such that $\bar{x} \in \arg\min_{x \in \mathcal{X}} Ef(x) - \mathbb{E}\{\langle \bar{\boldsymbol{w}}, x\rangle\}$

$\bar{w} \perp \mathcal{N}^a \Leftrightarrow \mathbb{E}\{\bar{w}(\boldsymbol{\xi}) \,|\, \mathcal{A}_{t-1}\} = 0, \forall t = 1, \ldots, T$

$\bar{w}$ non-anticipativity prices

at which to buy the right to adjust decision (after observation)

can be viewed as insurance premiums, ...

Monday, October 28, 13

just a bit of “math”

Expectation Functionals

Expectation of $\overline{\mathbb{R}}$-valued functions (Fatou, monotone convergence, ...):

$$E\{f(\boldsymbol{\xi})\} = \int_\Xi f(\xi)P(d\xi) = \begin{cases} \infty & \text{if } P([f(\boldsymbol{\xi}) = \infty]) > 0 \\ \int_\Xi f(\xi)P(d\xi) & \text{otherwise,} \end{cases}$$

or $E\{f(\boldsymbol{\xi})\} = E\{\max[f(\boldsymbol{\xi}), 0]\} - E\{\max[-f(\boldsymbol{\xi}), 0]\}$, $\infty - \infty = \infty$ (convention).

$f : \Xi \times \mathbb{R}^n \to \overline{\mathbb{R}}$, $\quad Ef : \mathbb{R}^n \to \mathbb{R} \cup \{\infty\}$, assume $Ef \not\equiv \infty$

- **Convexity**. $x \mapsto f(\xi, x)$ is convex (resp. affine, sublinear), then so is Ef.
- **Lower semicontinous**. $x \mapsto f(\xi, x)$ lsc & convex or summably bounded below $\Rightarrow Ef$ lsc.
- **Subdifferentials**. Ef finite near x, for all $\xi \in \Xi$, $f(\xi, \cdot)$ convex, then

$$\partial Ef(x) = \mathbb{E}\{\partial f(\boldsymbol{\xi}, x)\} = \left\{ \int_\Xi v(\xi)\, P(d\xi) \;\middle|\; v \text{ integrable}, v(\xi) \in \partial f(\xi, x) \right\}.$$

.

Characterization of minimizers

Theorem. Ef an expectation functional with $f(\xi,\cdot)$ convex.
Then, $x^0 \in \operatorname{argmin} Ef \iff \exists\, v : \Xi \to \mathbb{R}$, $\mathbb{E}\{v(\boldsymbol{\xi})\} = 0$, $v(\xi) \in \partial f(\xi, x^0)$, i.e.,

$$x^0 \in \operatorname*{argmin}_{x \in \mathbb{R}} \left\{ f(\xi,x) - v(\xi)x \right\} \quad \forall \xi \in \Xi$$

Proof. If $v(\cdot)$ exists, then $0 \in \partial Ef(x^0)$, i.e., $x^0 \in \operatorname{argmin} Ef$.

On the other hand, if $0 \in \partial Ef(x^0)$, $\exists\, v$ such that $\mathbb{E}\{v(\boldsymbol{\xi})\} = 0$ and $v(\xi) \in \partial f(\xi, x^0)$ is guaranteed by 'Subdufferential property'. The equivalence

$$v(\xi) \in \partial f(\xi, x^0) \;\&\; x^0 \in \operatorname{argmin}_x \{f(\xi,x) - v(\xi)x\}$$

is validated by Fermat's rule. □

Characterization of minimizers

Theorem. Ef an expectation functional with $f(\xi, \cdot)$ convex.
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$$v(\xi) \in \partial f(\xi, x^0) \ \&\ x^0 \in \operatorname{argmin}_x \{ f(\xi, x) - v(\xi)x \}$$

is validated by Fermat's rule. $\square$

Knowing v allows the interchange of minimization and expectation

Unit Commitment SCUC (PH with binary variables)

Transmission Network

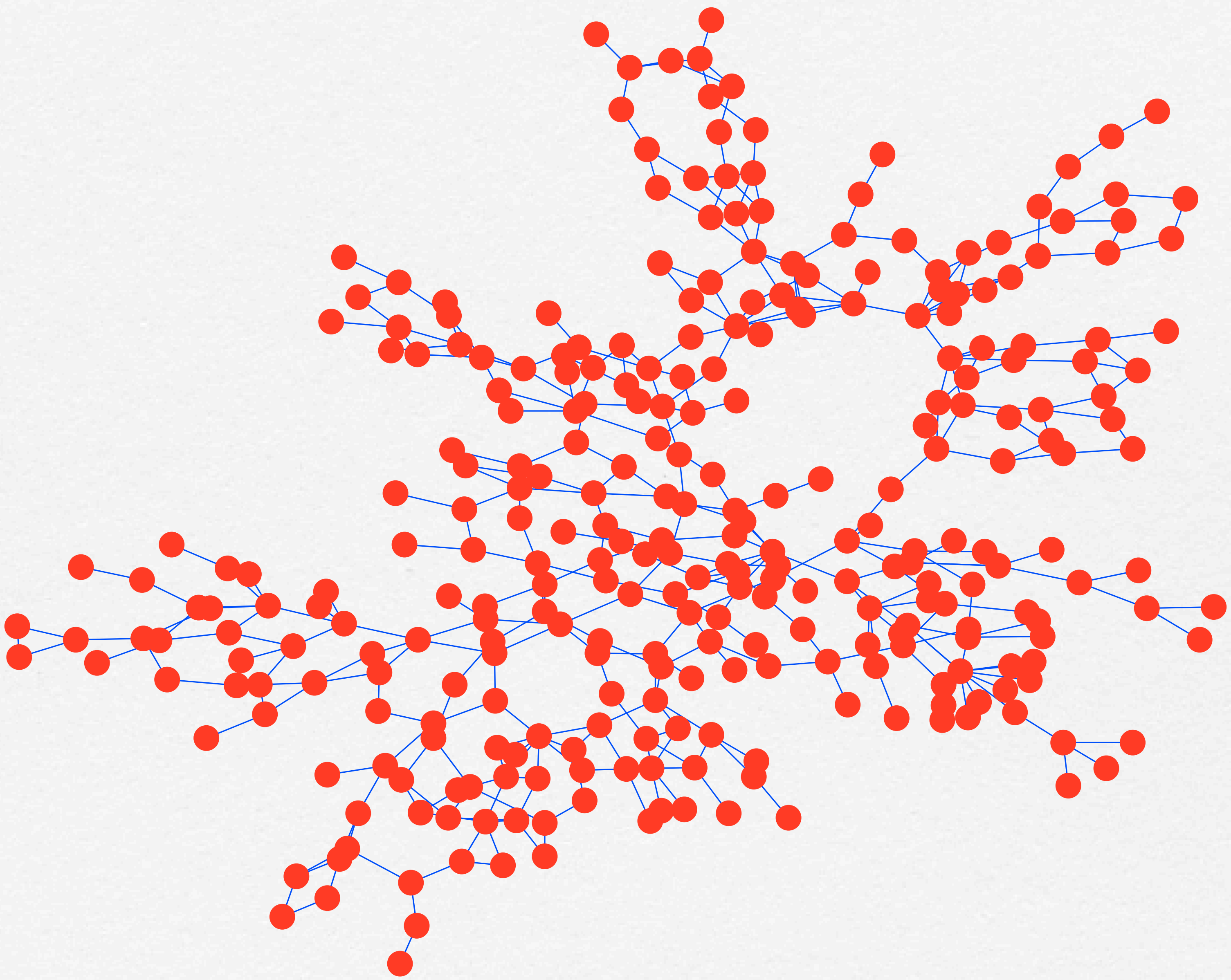
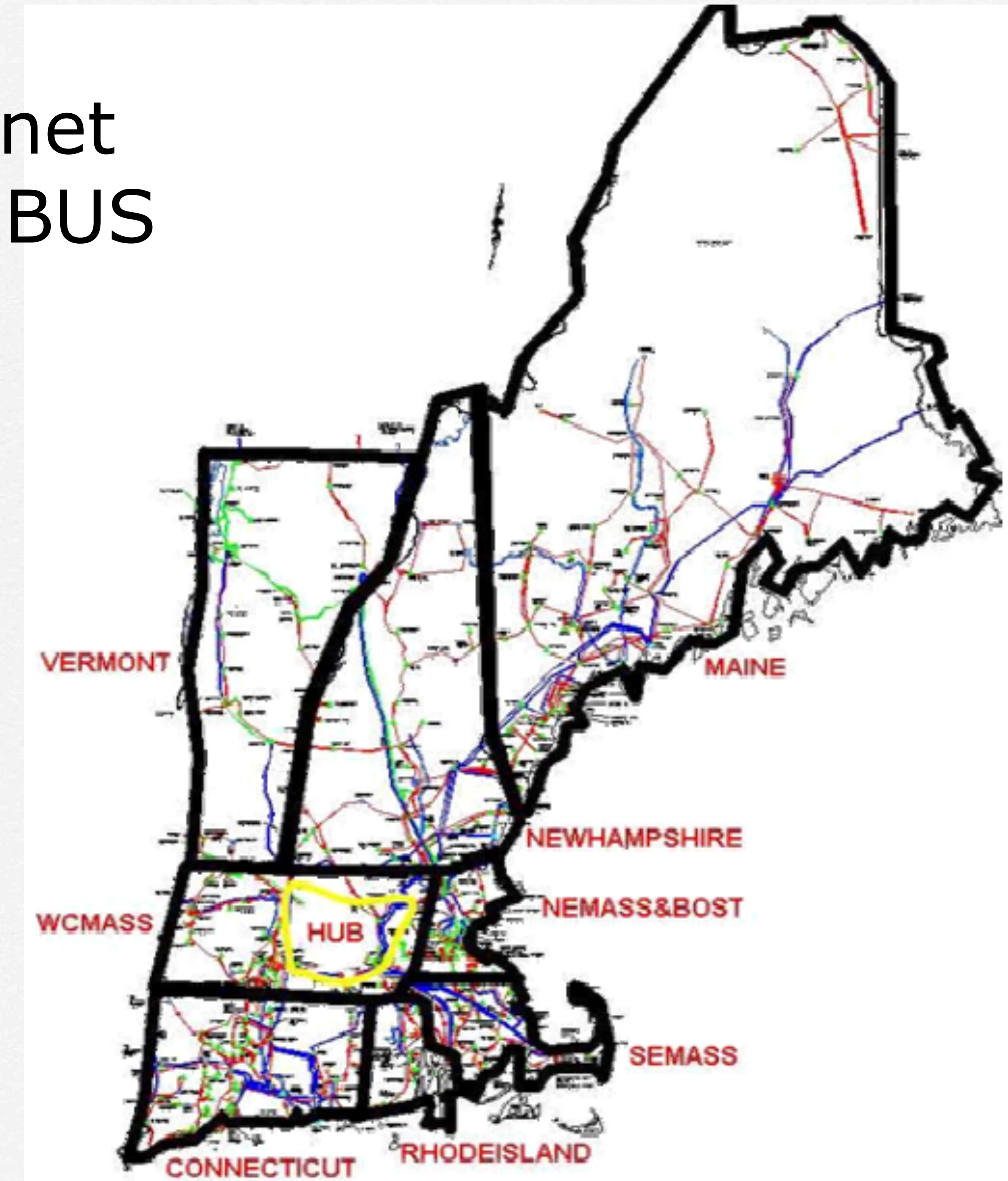


Figure 1. Topology of the IEEE 300 node system

Transmission Network

NE-ISO net
~30,000 BUS



FERC

Federal Energy Regulatory Commission

RTO

In the US is an organization that is responsible for moving electricity over large interstate areas; coordinates, controls and monitors an electricity transmission grid that is larger with much higher voltages than the typical power company's distribution grid.

ISO

*Is an organization formed at the direction or recommendation of the **FERC,** in the areas where an ISO is established, it coordinates, controls and monitors the operation of the electrical power system, usually within a single US State, but sometimes encompassing multiple states.*

ISO New England Inc. (*ISO-NE*) *is an independent, non-profit RTO, serving Connecticut, Maine, Massachusetts, New Hampshire, Rhode Island and Vermont. Its Board of Directors and its over 400 employees* have no financial interest or ties to any company doing business in the region's wholesale electricity marketplace.

Energy Sources

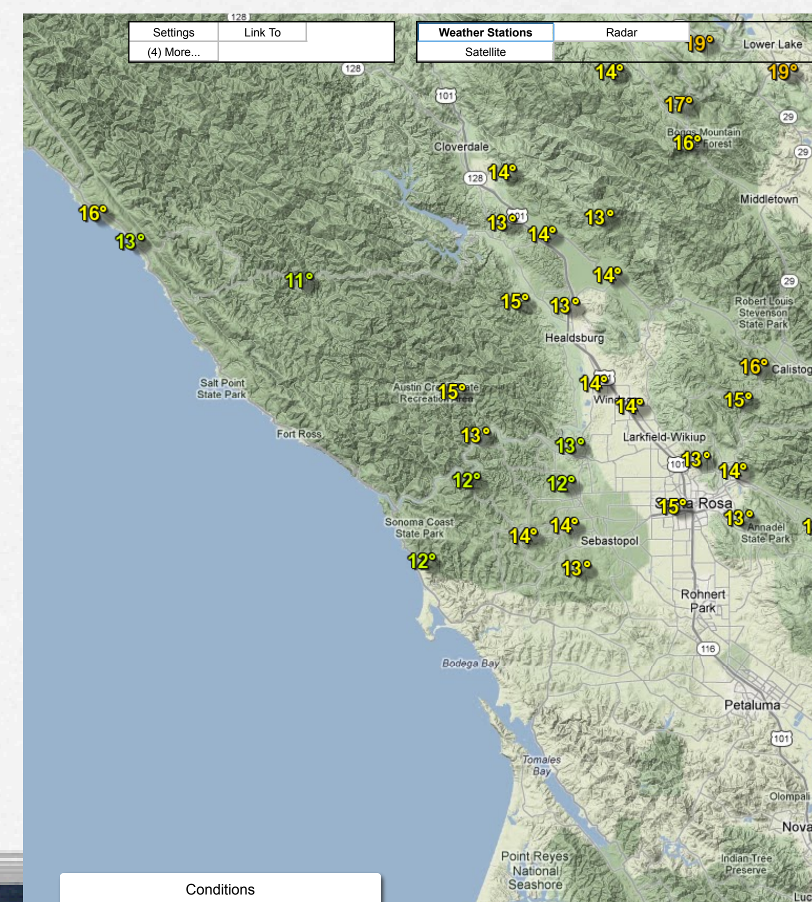
- nuclear energy
- hydro-power
- thermal plants (coal, oil, shale oil, bio, rubish, . . .)
- gas turbines (natural gas, from ”cracking’)
- renewables (wind, solar, ..., ocean waves)

different characteristics

Uncertainties

- WEATHER: demand & supply (especially renewables)
- industrial-commercial environment (demand)
- seasonal, day of the week, time of the day
- contingencies: transmission lines, generators

El Cerrito, California (94530) Conditions & Forecast http://www.wunderground.com/cgi-bin/findweather/getForecast...



Uncertainties

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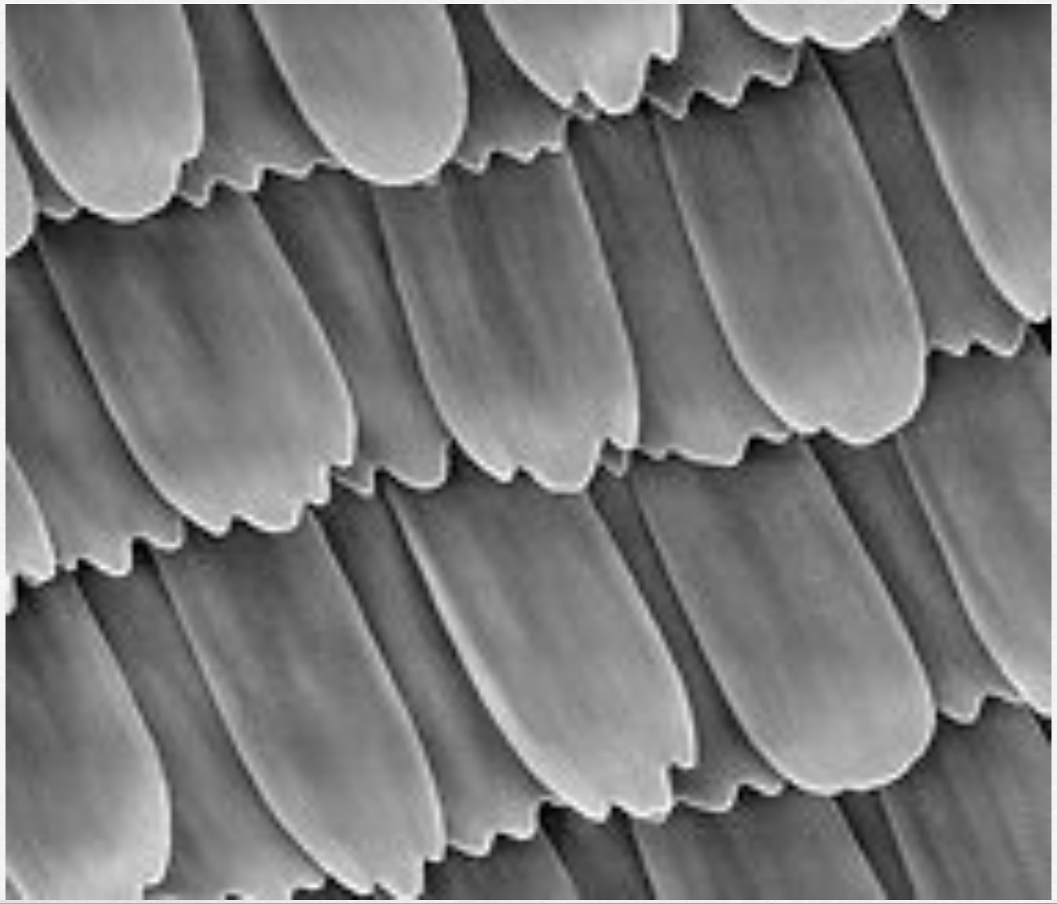

Uncertainties

- WEATHER: demand & supply (especially renewables)
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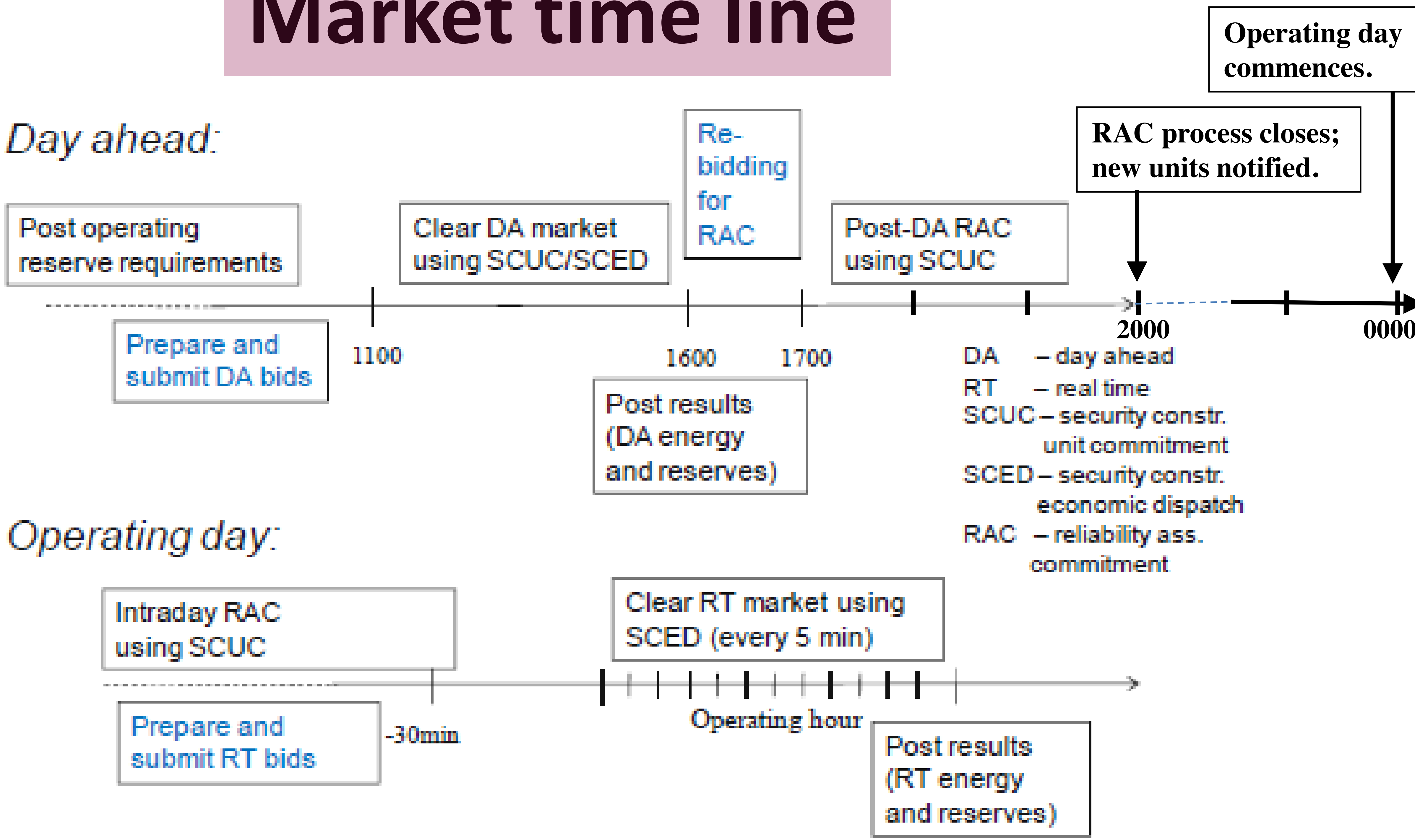
- contingencies: transmission lines, generators

Market time line



	MISO	NYISO	PJM	ERCOT	CAISO
Market timeline	DA offers due: 11am DA results: 4pm Re-bidding due: 5pm RT offers due: OH -30 min	DA offers due: 5 am DA results: 11 am RT offers due: OH -75 min	DA offers due: noon DA results: 4pm RT offers due: 6pm DA	DA bids due (reserves): 1pm/4pm DA results (reserves): 1.30pm/6pm RT offers due: OH -60 min	DA offers: 10am DA results: 1pm RT offers: OH - 75 min

Ref: A. Botterud, J. Wang, C. Monteiro, and V. Miranda “Wind Power Forecasting and Electricity Market Operations,” available at www.usaee.org/usaee2009/submissions/OnlineProceedings/Botterud_etal_paper.pdf

Short history of ISO-management techniques

- *RT: deterministic optimization with LMP (dual variables associated with demand(s) constraints).*
- *SCUC/SCED: Lagrangian relaxation with conservative reliability constraints*
- *SCUC/SCED: deterministic MIP with conservative RUT*
- *ARPA-"E (project): "take into account uncertainty"*

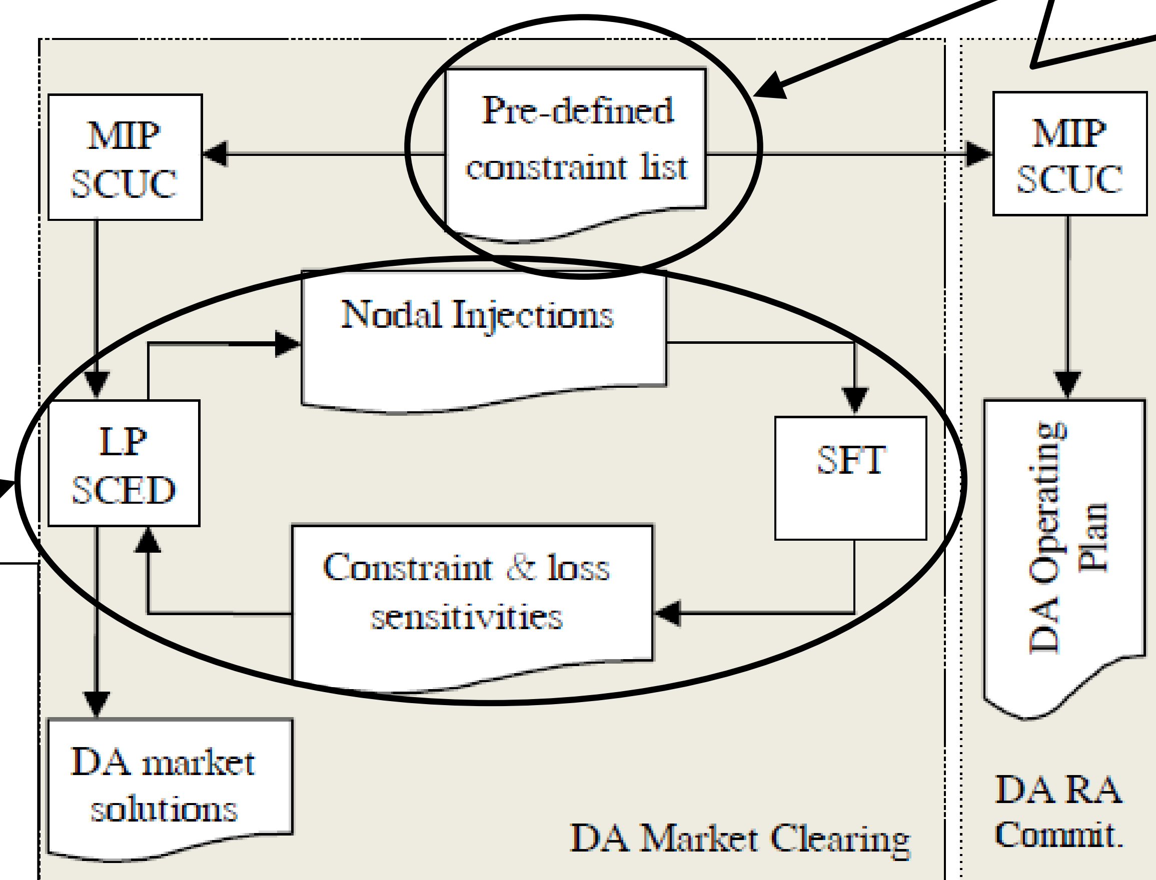
A collection of stochastic-programs

- DA-SCUC/SCED unit commitment *binaries*
- DA-RAC rebidding assessment bidding *(binaries)*
- DA-RUT - reliability commitments (spinning, N-1)
- RT - 3 min (real time adjustments) LMP's
- SCED2 - 3 or 4 hours schedule to foresee ramp ups/down, etc.

DA = day ahead

Day-Ahead Market

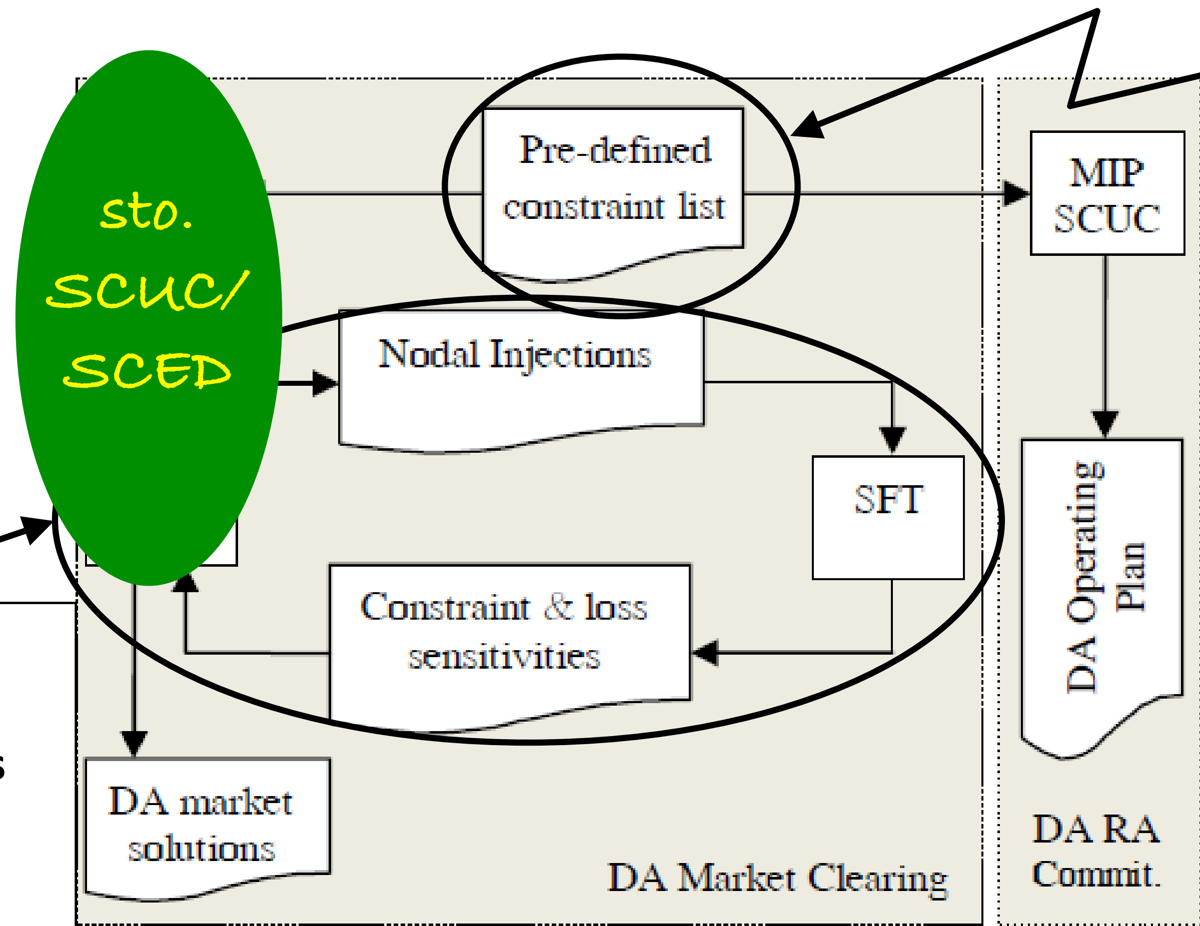
SCUC enforces limited number of transmission constraints on the commitment solution.



Each hourly SCED performs SFT, which tests all contingencies in a list and for violations, imposes appropriate constraints in SCED and resolves it.

Ref: Xingwang Ma, Haili Song, Mingguo Hong, Jie Wan, Yonghong Chen, Eugene Zak, “The Security-constrained Commitment and Dispatch For Midwest ISO Day-ahead Co-optimized Energy and Ancillary Service Market,” Proc. of the 2009 IEEE PES General Meeting.

Day-Ahead Market



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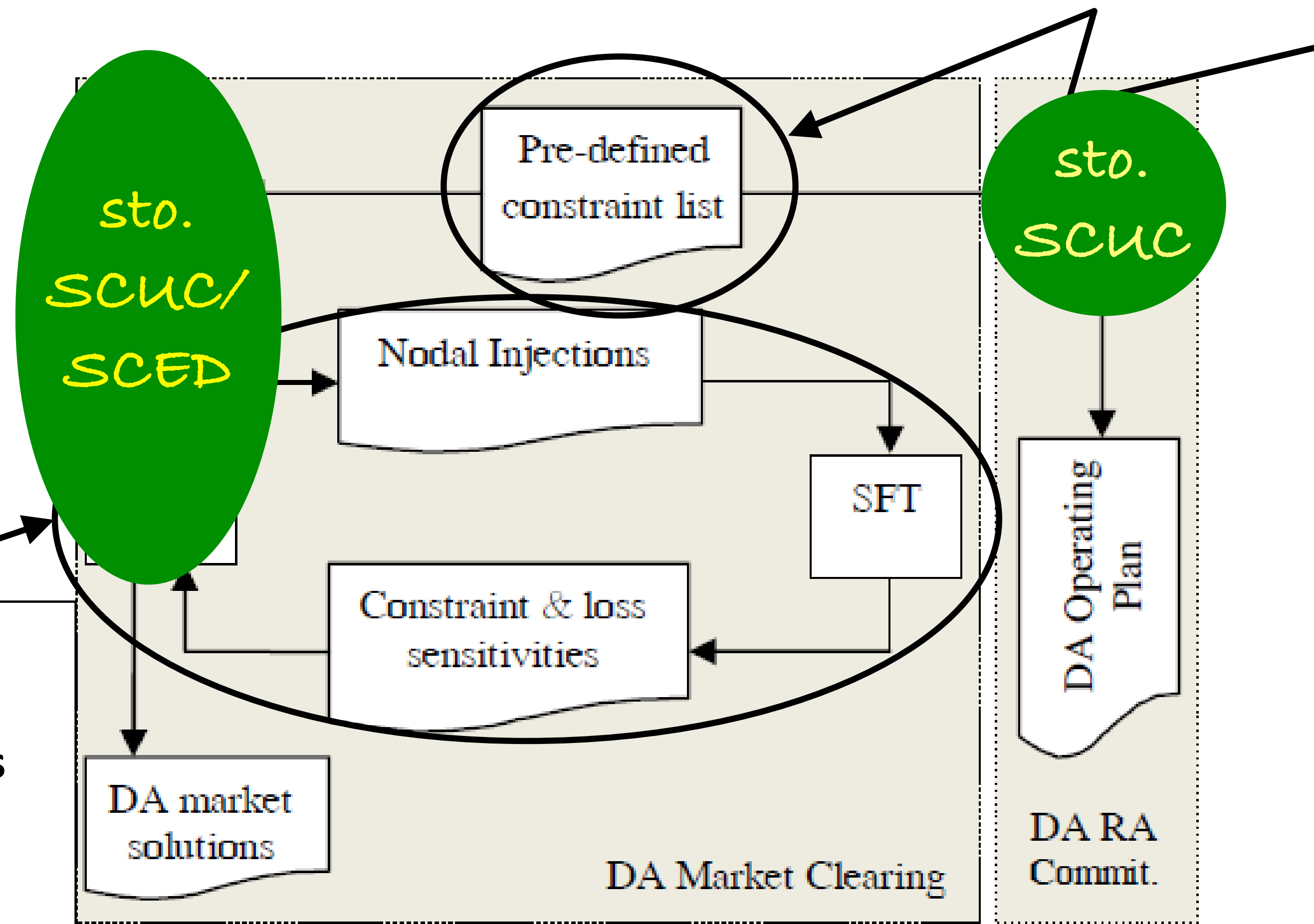
Day-Ahead Market



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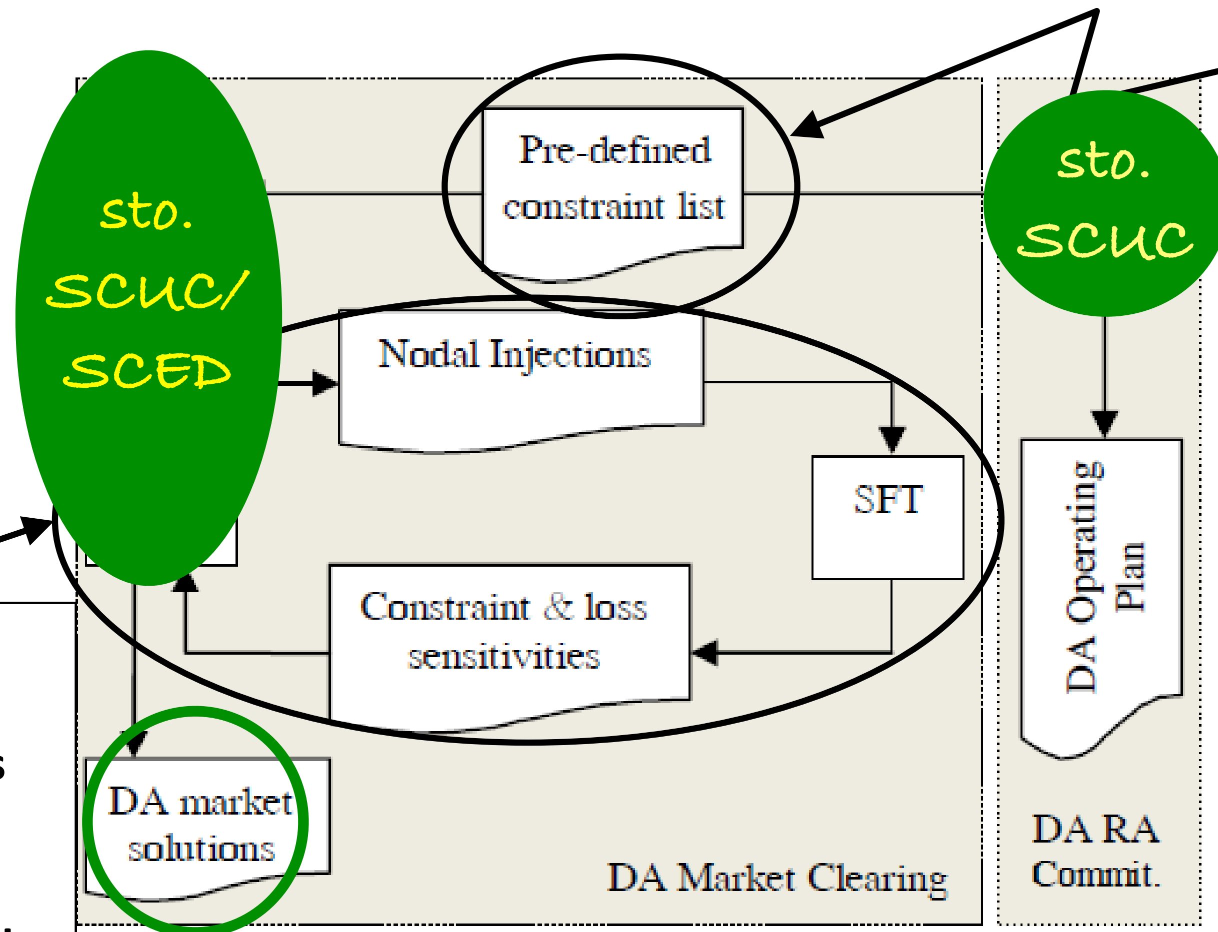
Day-Ahead Market



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Abstract Unit Commitment

$$\text{Minimize} \sum_{k\in K}\sum_{j\in J} c_j^P(k) + c_j^u(k) + c_j^d(k) \qquad \text{with}$$

$$\sum_{j\in J} p_j(k) = D(k), \quad \forall k \in K$$

$$\sum_{j\in J} \bar{p}_j(k) \geq D(k) + R(k), \quad \forall k \in K$$

$$p_j(k), \overline{p}_j(k) \in \Pi, \quad \forall j \in J, \ \forall k \in K$$

Π region of feasible production, all generating units, all time periods. The specific nature of Π is model-dependent.

Abstract Unit Commitment

$$\text{Minimize} \sum_{k \in K} \sum_{j \in J} c_j^P(k) + c_j^u(k) + c_j^d(k) \qquad \text{with}$$

J generating units

$$\sum_{j \in J} p_j(k) = D(k), \quad \forall k \in K$$

$$\sum_{j \in J} \bar{p}_j(k) \geq D(k) + R(k), \quad \forall k \in K$$

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Abstract Unit Commitment

$$\text{Minimize} \sum_{k \in K} \sum_{j \in J} c_j^P(k) + c_j^u(k) + c_j^d(k) \qquad \text{with}$$

K time periods *J generating units*

$$\sum_{j \in J} p_j(k) = D(k), \quad \forall k \in K$$

$$\sum_{j \in J} \bar{p}_j(k) \geq D(k) + R(k), \quad \forall k \in K$$

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Π region of feasible production, all generating units, all time periods. The specific nature of Π is model-dependent.

Abstract Unit Commitment

production cost

$$\text{Minimize} \sum_{k \in K} \sum_{j \in J} c_j^P(k) + c_j^u(k) + c_j^d(k) \qquad \text{with}$$

K time periods *J generating units*

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Π region of feasible production, all generating units, all time periods. The specific nature of Π is model-dependent.

Abstract Unit Commitment

production cost   *startup cost*

$$\text{Minimize} \sum_{k \in K} \sum_{j \in J} c_j^P(k) + c_j^u(k) + c_j^d(k) \qquad \text{with}$$

K time periods   *J generating units*

$$\sum_{j \in J} p_j(k) = D(k), \quad \forall k \in K$$

$$\sum_{j \in J} \bar{p}_j(k) \geq D(k) + R(k), \quad \forall k \in K$$

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Π region of feasible production, all generating units, all time periods. The specific nature of Π is model-dependent.

Abstract Unit Commitment

production cost   startup cost   shutdown cost

$$\text{Minimize} \sum_{k \in K} \sum_{j \in J} c_j^P(k) + c_j^u(k) + c_j^d(k) \qquad \text{with}$$

K time periods   J generating units

$$\sum_{j \in J} p_j(k) = D(k), \quad \forall k \in K$$

$$\sum_{j \in J} \bar{p}_j(k) \geq D(k) + R(k), \quad \forall k \in K$$

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Π region of feasible production, all generating units, all time periods. The specific nature of Π is model-dependent.

Abstract Unit Commitment

production cost *startup cost* *shutdown cost*

$$\text{Minimize} \sum_{k \in K} \sum_{j \in J} c_j^P(k) + c_j^u(k) + c_j^d(k) \quad \text{with}$$

K time periods *J generating units*

demand

$$\sum_{j \in J} p_j(k) = D(k), \quad \forall k \in K$$

$$\sum_{j \in J} \bar{p}_j(k) \geq D(k) + R(k), \quad \forall k \in K$$

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Π region of feasible production, all generating units, all time periods. The specific nature of Π is model-dependent.

Abstract Unit Commitment

production cost startup cost shutdown cost

$$\text{Minimize} \sum_{k\in K}\sum_{j\in J} c_j^P(k) + c_j^u(k) + c_j^d(k) \quad \text{with}$$

K time periods J generating units

power output demand

$$\sum_{j\in J} p_j(k) = D(k), \quad \forall k \in K$$

$$\sum_{j\in J} \bar{p}_j(k) \geq D(k) + R(k), \quad \forall k \in K$$

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Abstract Unit Commitment

production cost   startup cost   shutdown cost

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K time periods   J generating units

demand

power output

$$\sum_{j \in J} p_j(k) = D(k), \quad \forall k \in K$$

max power output

$$\sum_{j \in J} \bar{p}_j(k) \geq D(k) + R(k), \quad \forall k \in K$$

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Π region of feasible production, all generating units, all time periods. The specific nature of Π is model-dependent.

Abstract Unit Commitment

production cost *startup cost* *shutdown cost*

$$\text{Minimize} \sum_{k \in K} \sum_{j \in J} c_j^P(k) + c_j^u(k) + c_j^d(k) \qquad \text{with}$$

K time periods *J generating units*

power output *demand*

$$\sum_{j \in J} p_j(k) = D(k), \quad \forall k \in K$$

max power output *spinning reserve*

$$\sum_{j \in J} \bar{p}_j(k) \geq D(k) + R(k), \quad \forall k \in K$$

$$p_j(k), \bar{p}_j(k) \in \Pi, \quad \forall j \in J, \ \forall k \in K$$

Π region of feasible production, all generating units, all time periods. The specific nature of Π is model-dependent.

Abstract Unit Commitment

production cost *startup cost* *shutdown cost*

$$\text{Minimize} \sum_{k \in K} \sum_{j \in J} c_j^P(k) + c_j^u(k) + c_j^d(k) \quad \text{with}$$

K time periods *J generating units*

demand

power output $$\sum_{j \in J} p_j(k) = D(k), \quad \forall k \in K$$

spinning reserve

max power output $$\sum_{j \in J} \bar{p}_j(k) \geq D(k) + R(k), \quad \forall k \in K$$

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Π region of feasible production, all generating units, all time periods. The specific nature of Π is model-dependent.

Abstract Unit Commitment

production cost   startup cost   shutdown cost

$$\text{Minimize} \sum_{k \in K} \sum_{j \in J} c_j^P(k) + c_j^u(k) + c_j^d(k) \quad \text{with}$$

K time periods   J generating units

power output   demand

$$\sum_{j \in J} p_j(k) = D(k), \quad \forall k \in K$$

max power output   spinning reserve

$$\sum_{j \in J} \bar{p}_j(k) \geq D(k) + R(k), \quad \forall k \in K$$

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Π region of feasible production, all generating units, all time periods.
The specific nature of Π is model-dependent.

"Stochastic Version"

Monday, October 28, 13

Abstract Unit Commitment

min. expectation (actually: risk measure) with penalties

production cost · startup cost · shutdown cost

$$\text{Minimize} \sum_{k \in K} \sum_{j \in J} c_j^P(k) + c_j^u(k) + c_j^d(k) \quad \text{with}$$

K time periods · J generating units

power output · demand

$$\sum_{j \in J} p_j(k) = D(k), \quad \forall k \in K$$

adjust node balance eq'ns

max power output · spinning reserve

$$\sum_{j \in J} \bar{p}_j(k) \geq D(k) + R(k), \quad \forall k \in K$$

$$p_j(k), \bar{p}_j(k) \in \Pi, \quad \forall j \in J, \ \forall k \in K$$

Π region of feasible production, all generating units, all time periods. The specific nature of Π is model-dependent.

"Stochastic Version"

between a rock and a hard place

CPLEX-MIP: can handle a few scenarios

PH : not designed for binary vairables

Progressive Hedging Algorithm

Review

0. w_ξ^0 such that $\mathbb{E}\{w_\xi^0\} = 0$, $\nu = 0$. Pick $\rho > 0$

1. for all ξ :

$$(x_\xi^{1,\nu}, x_\xi^{2,\nu}) \in \arg\min f(\xi; x^1, x^2) + \langle w_\xi^\nu, x^1 \rangle + \frac{\rho}{2}\left|x^1 - \bar{x}^{1,\nu-1}\right|^2$$

$$x^1 \in C^1 \subset \mathbb{R}^{n_1},\; x^2 \in C^2(\xi, x^1) \subset \mathbb{R}^{n_2}$$

2. $\bar{x}^{1,\nu} = \mathbb{E}\{x_\xi^{1,\nu}\}$. Stop if $\left|x_\xi^{1,\nu} - \bar{x}^{1,\nu}\right| = 0$ (approx.)

   otherwise $w_\xi^{\nu+1} = w_\xi^\nu + \rho\left[x_\xi^{1,\nu} - \bar{x}^{1,\nu}\right]$, return to 1. with $\nu = \nu + 1$

Implementation: bundling, $\rho \to \rho_s$, ...
Watson & Woodruff (Hart, Siirola, ...)
Chile: Sistemas Complejos de Ingeneria (L.F. Solari, ...)
& Centro de Modelamiento Matematico
Carl Laird (Texas A& M), Ryan Sarah (Iowa), ...

PH: binary variables

$\min\langle c, x\rangle + \sum_{\xi\in\Xi} p_\xi \langle q_\xi, y_\xi\rangle$ such that

$x \in C_1,\ \ y_\xi \in C_2(\xi, x)\ \forall\, \xi \in \Xi$

binary (integer) variables: some x's, some y_ξ's.

PH: binary variables

$\min\langle c, x\rangle + \sum_{\xi\in\Xi} p_\xi \langle q_\xi, y_\xi\rangle$ such that

$x \in C_1,\ \ y_\xi \in C_2(\xi, x)\ \forall\, \xi \in \Xi$

binary (integer) variables: some x's, some y_ξ's.

Choice of $\rho \to \rho_j$ depending on $c_j, |x_j|, \ldots$

Variable Fixing, in particular binaries, $x_j(s) =$ constant (k iterations)
Variable Slamming: aggressive variable fixing $x_j(s) \approx$ constant (& $c_j x_j(s)$)
"Sufficient" variable convergence $\sim$ for small values of $c_j x_j(s)$

Termination criterion: variable slamming when $x_j^\nu(\xi) - x_j^{\nu+1}(\xi)$ small

Detecting cycling behavior: (simple) hashing scheme

PH: binary variables

$\min\langle c, x\rangle + \sum_{\xi\in\Xi} p_\xi \langle q_\xi, y_\xi\rangle$ such that

$x \in C_1, \;\; y_\xi \in C_2(\xi, x) \; \forall\, \xi \in \Xi$

binary (integer) variables: some x's, some y_ξ's.

Choice of $\rho \to \rho_j$ depending on $c_j, |x_j|, \ldots$

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Termination criterion: variable slamming when $x_j^\nu(\xi) - x_j^{\nu+1}(\xi)$ small

Detecting cycling behavior: (simple) hashing scheme

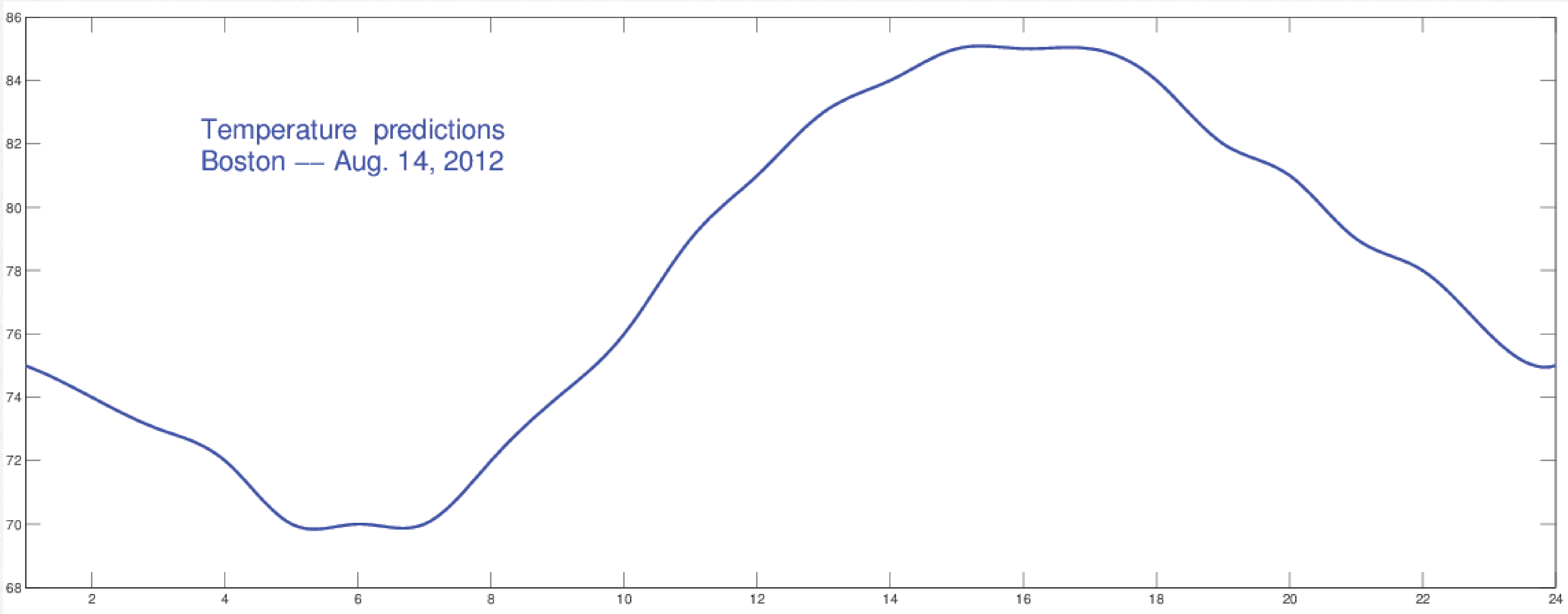
Enough variables fixed ⇒ clean up with CPLEX-MIP

Generating Scenarios

Roger J-B Wets
David Woodruff
Kai Spürkel & Ignacio Rios
@ UC Davis
Sarah Ryan & Yonghan Feng
@ Iowa State U.

Robust decisions in a stochastic environment demand a <u>robust</u> model of the uncertainty.

from predictions on day D-1 to load forecasts on day D



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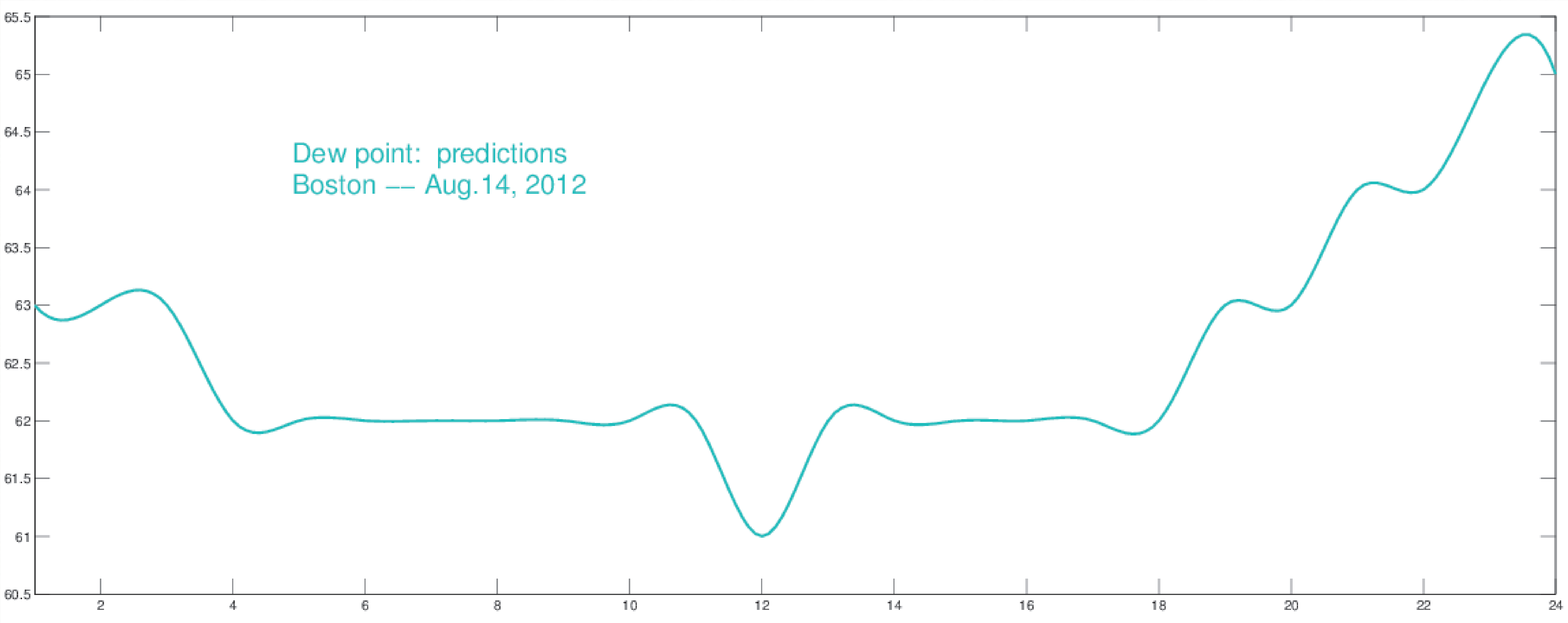

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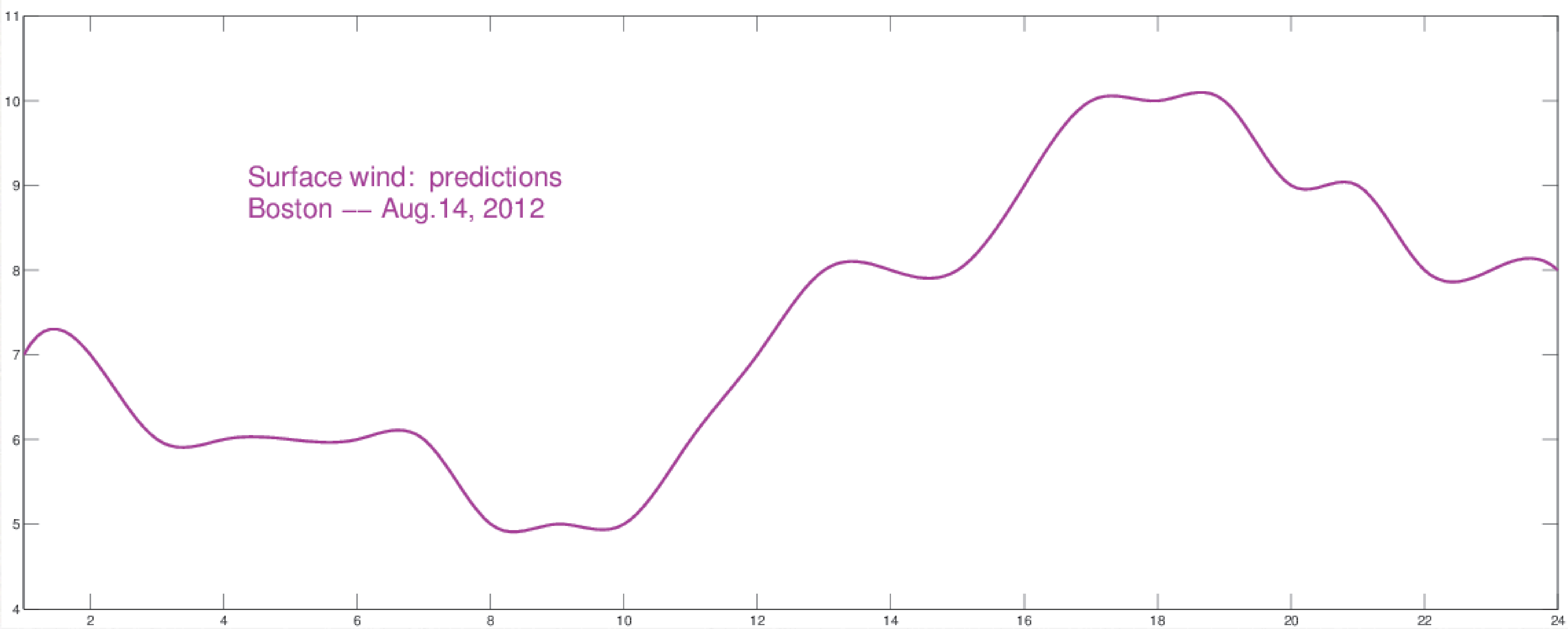
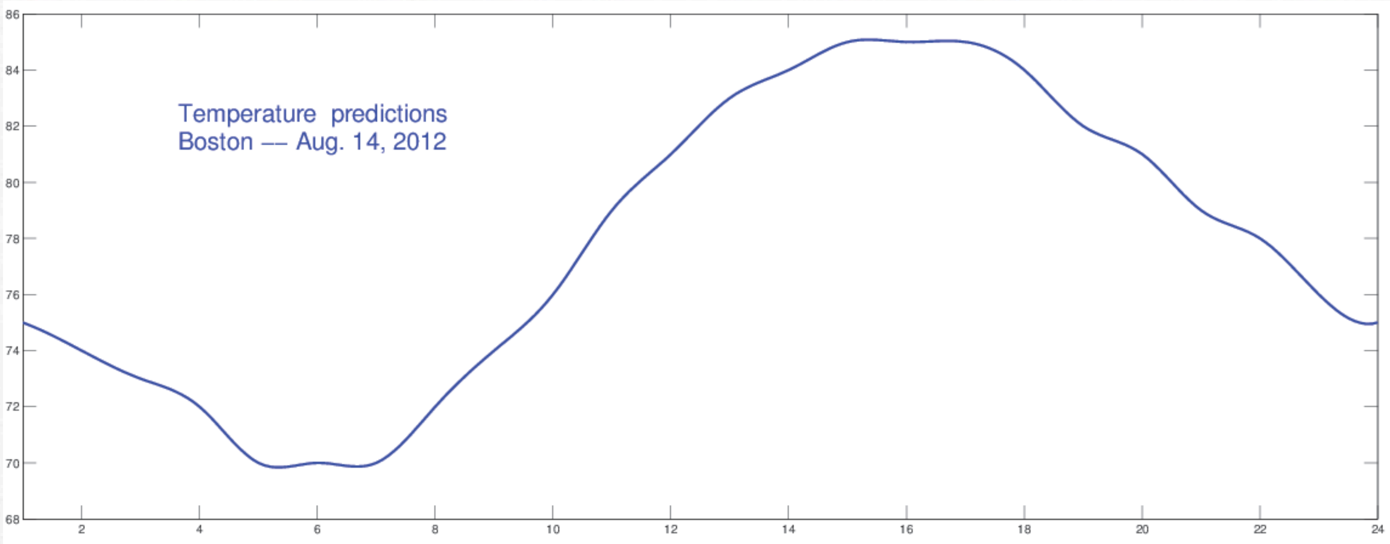
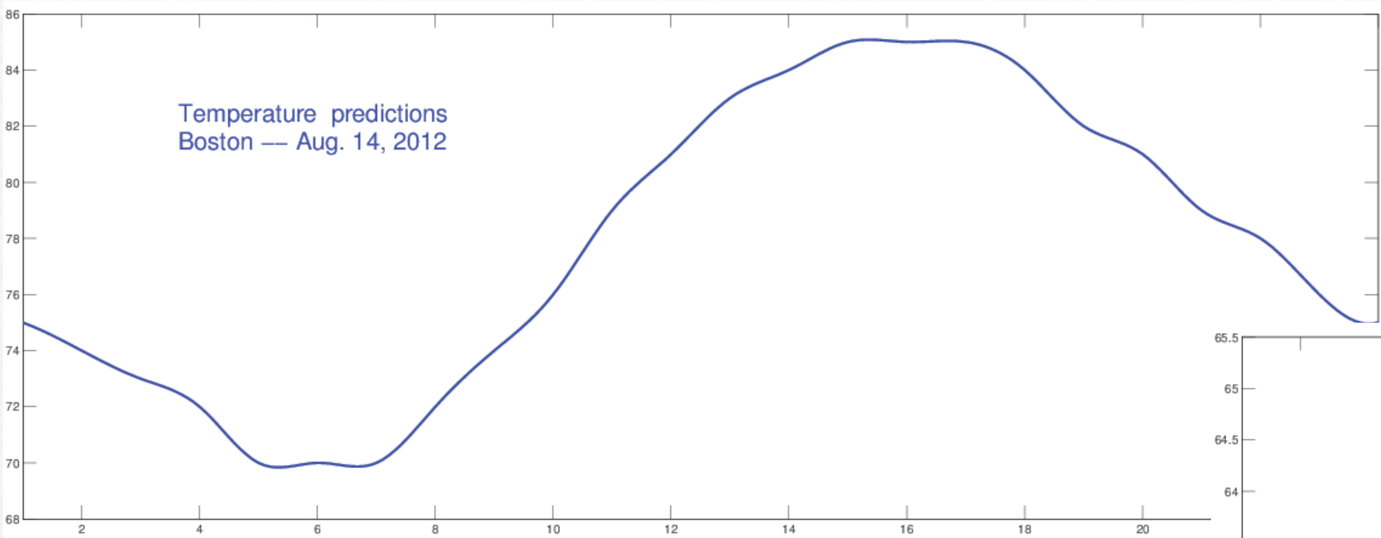


THE DATA


Temperature predictions
Boston — Aug. 14, 2012
86
84
82
80
78
76
74
72
70
68
2
4
6
8
10
12
14
16
18
20
22
24

THE DATA


Temperature predictions
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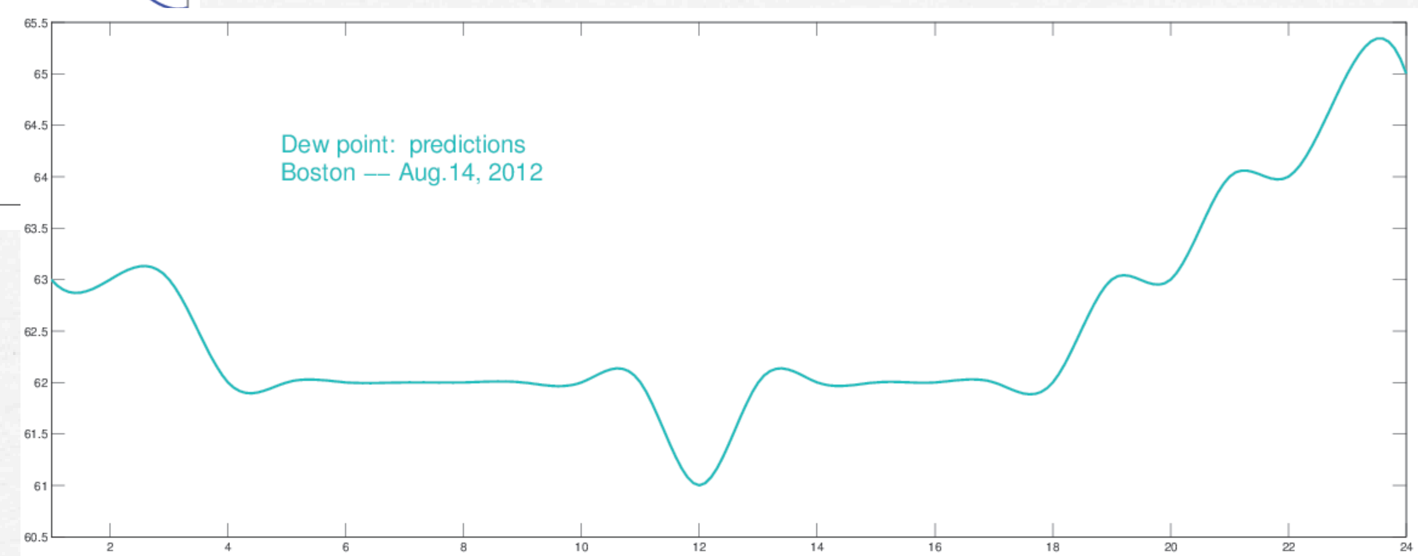
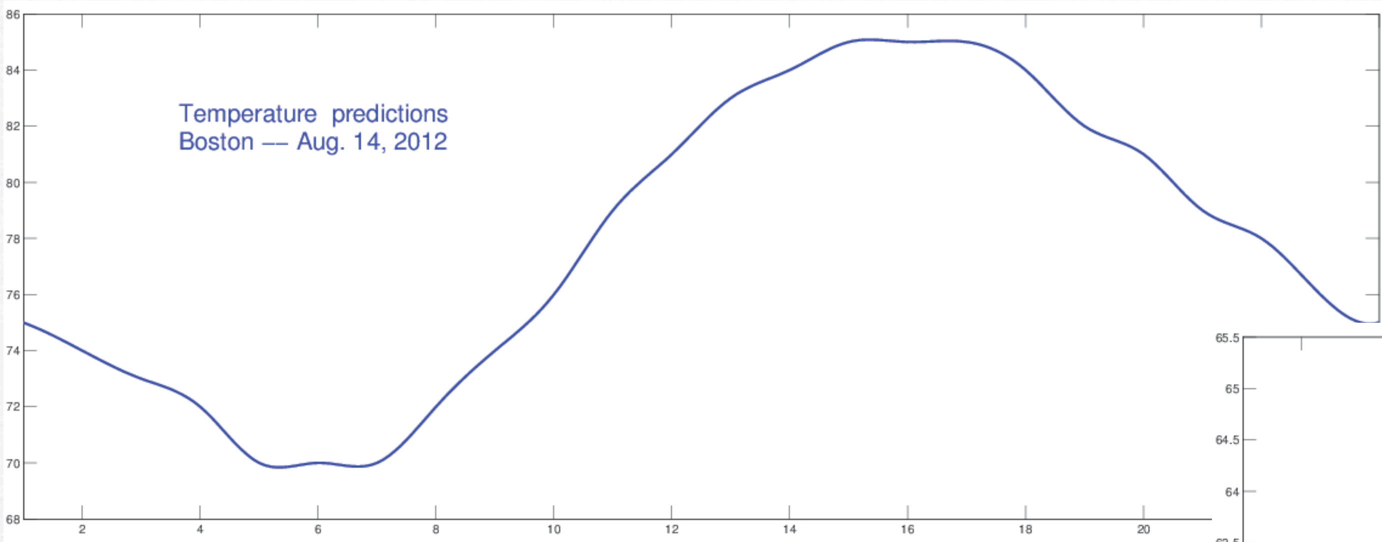

Dew point: predictions
Boston — Aug.14, 2012

THE DATA


Temperature predictions
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86
84
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8
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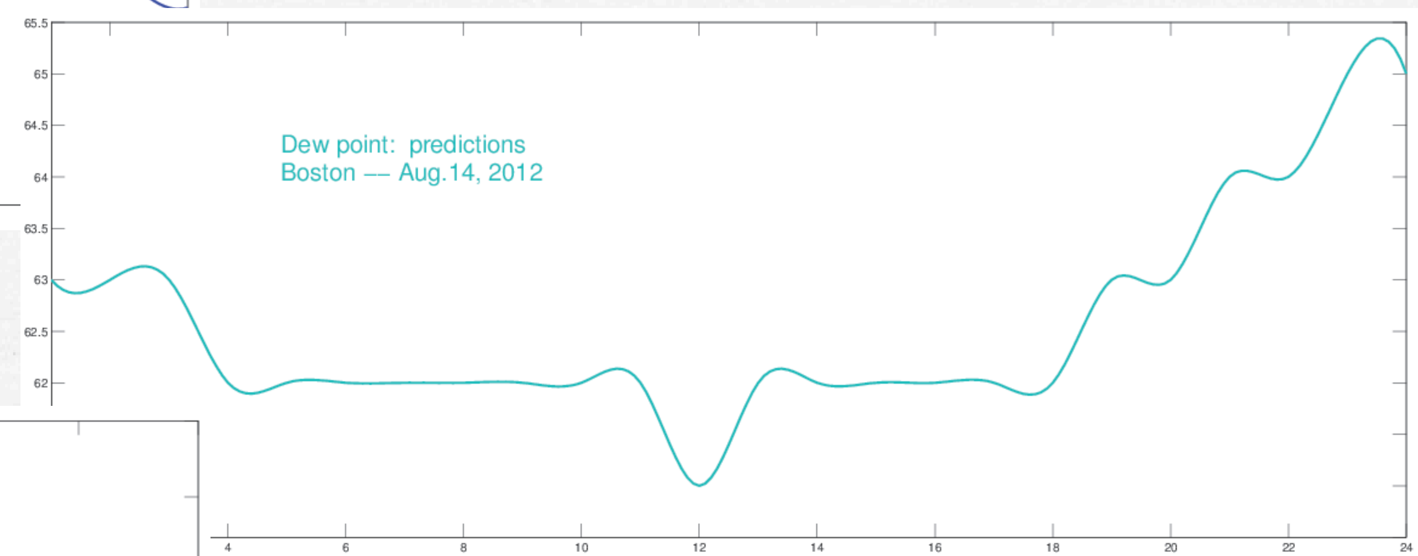
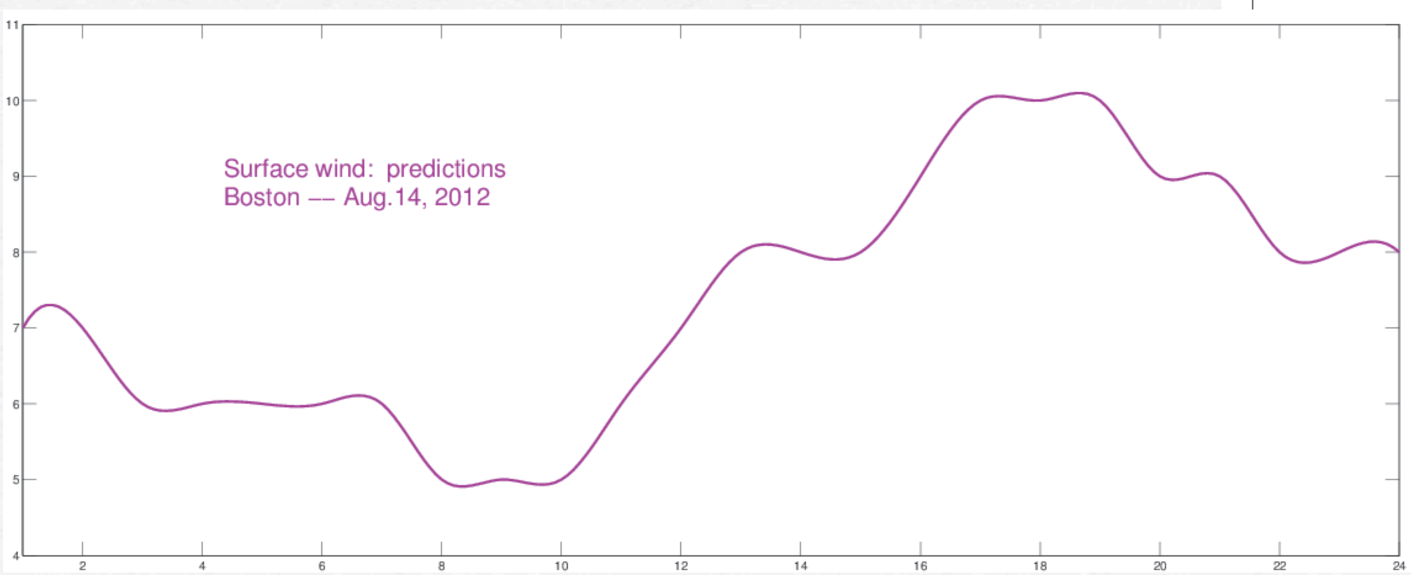
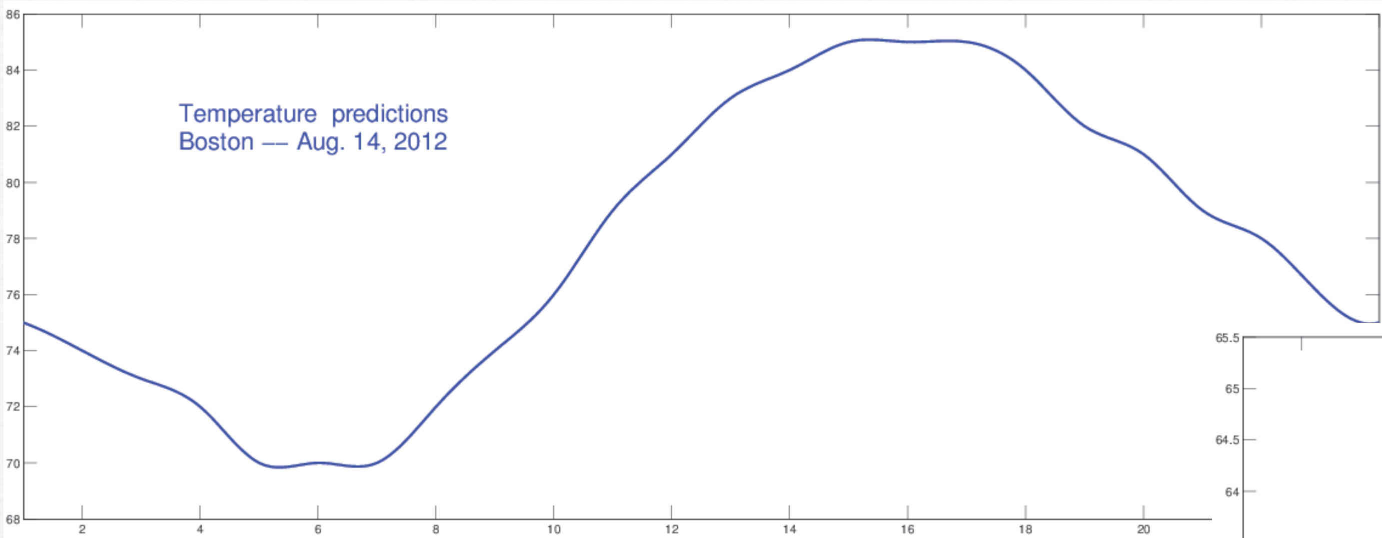
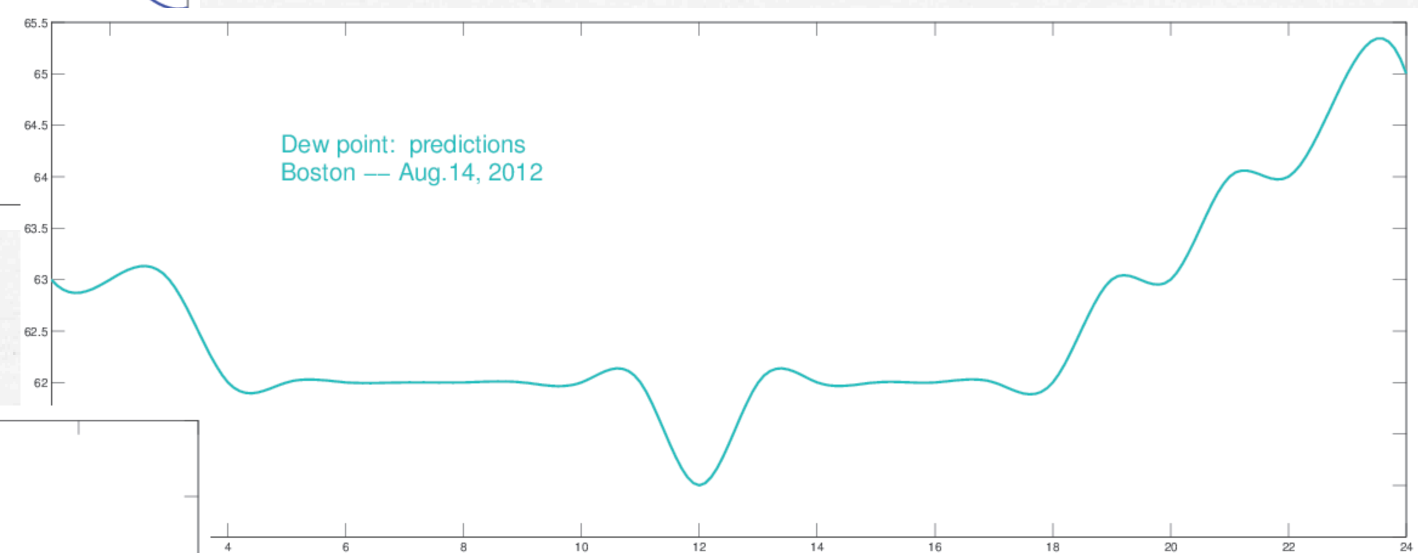
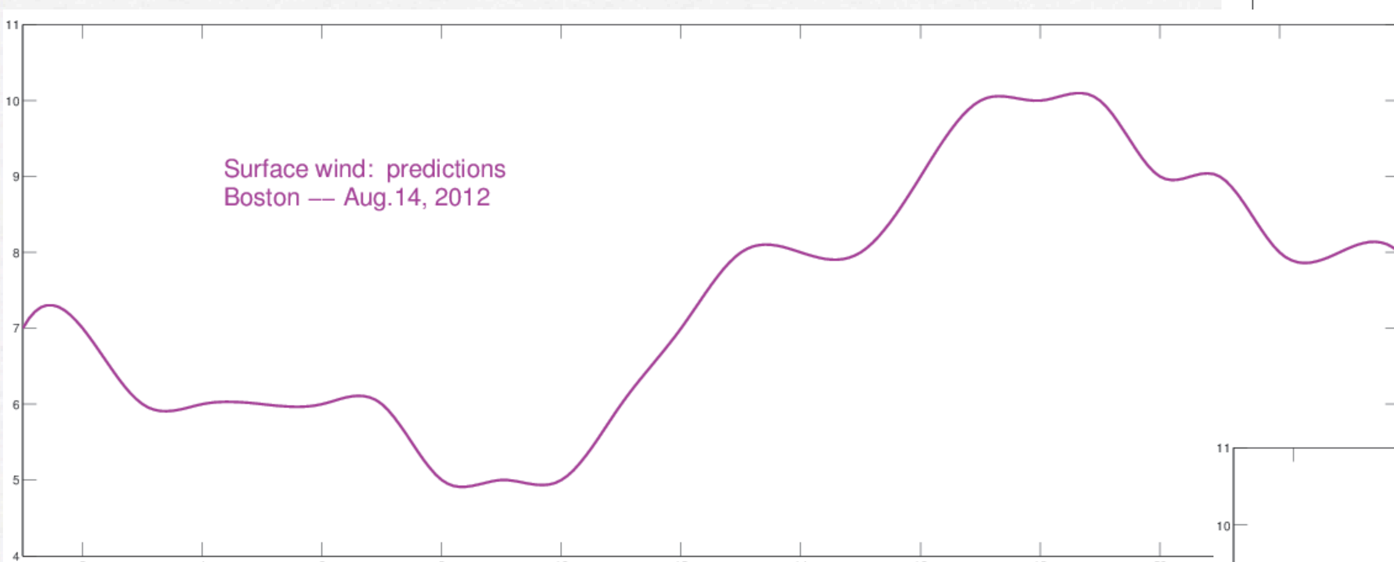

Dew point: predictions
Boston — Aug.14, 2012
65.5
65
64.5
64
63.5
63
62.5
62
4
6
8
10
12
14
16
18
20
22
24

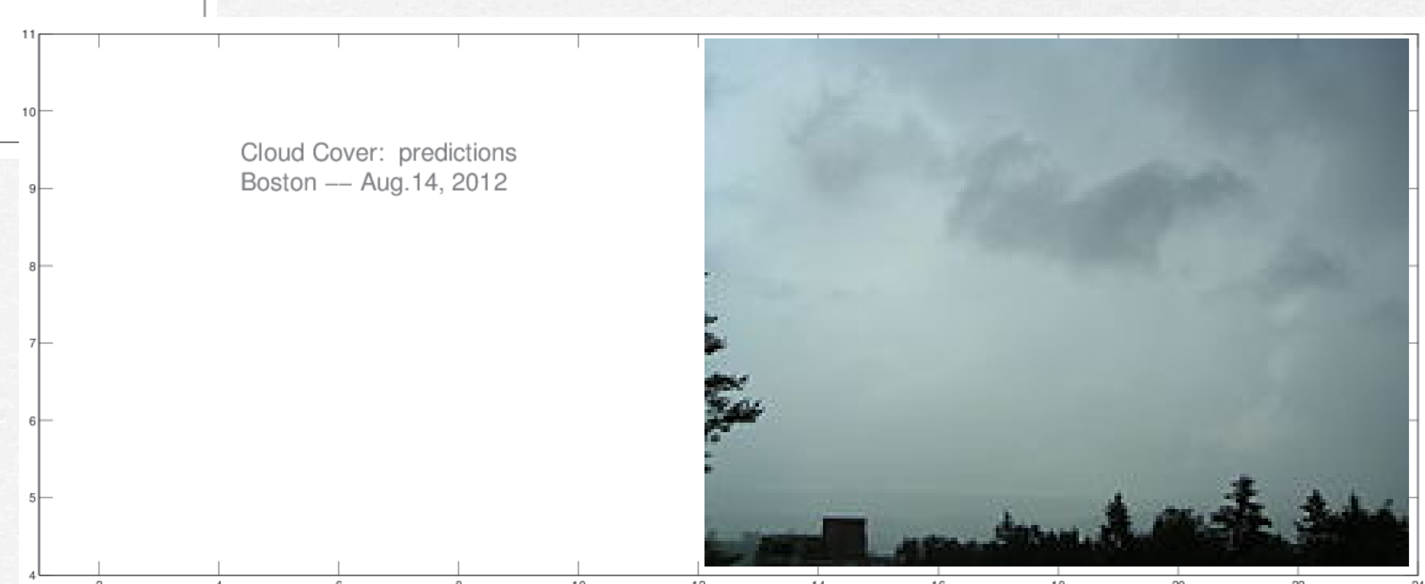

Surface wind: predictions
Boston — Aug.14, 2012
11
10
9
8
7
6
5
4
2
4
6
8
10
12
14
16
18
20
22
24

THE DATA







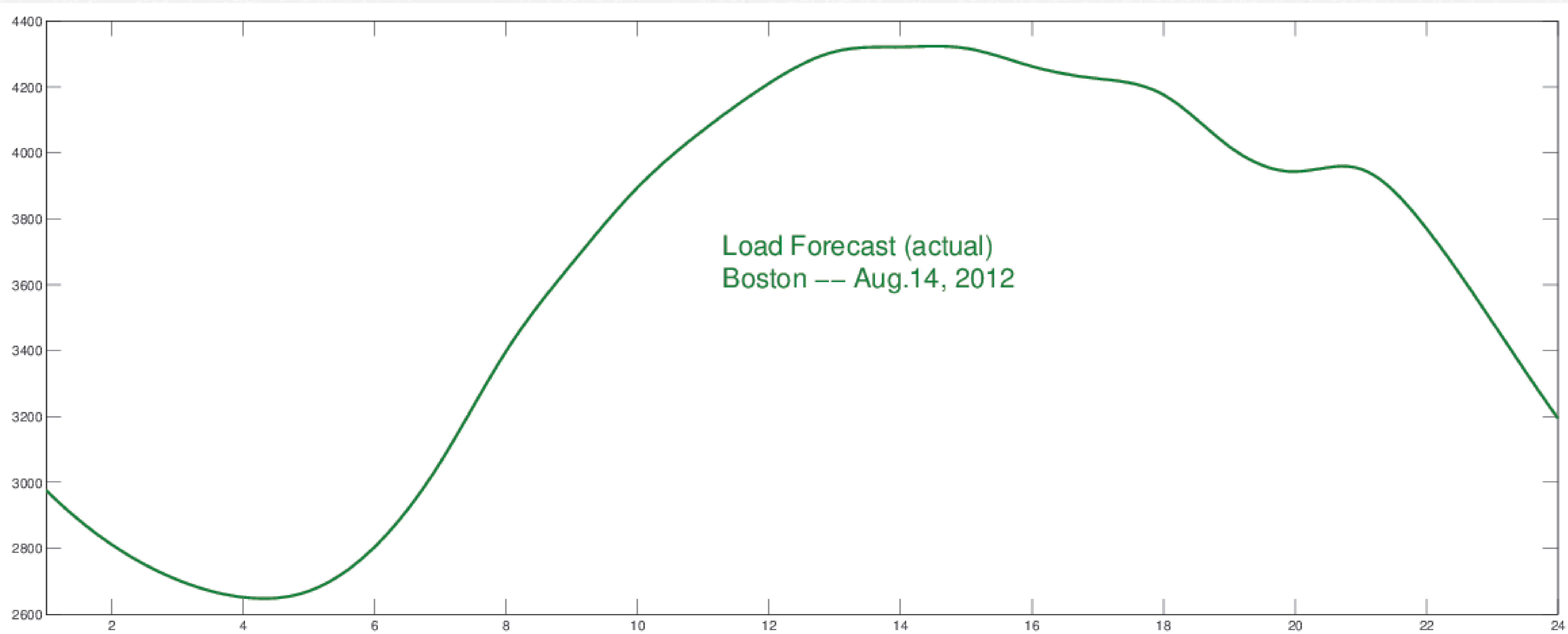


NOAA: ACTUALS

... to load on day D

to be delivered-load: $l(t)$

$$= \text{fcn}\big(\text{temp}(\tau \le \text{t}), \text{dewpt}(\tau \le \text{t}), \text{clcover}(\tau \le \text{t}), \text{wind}(\tau \le \text{t})\big), \ t \le 24$$



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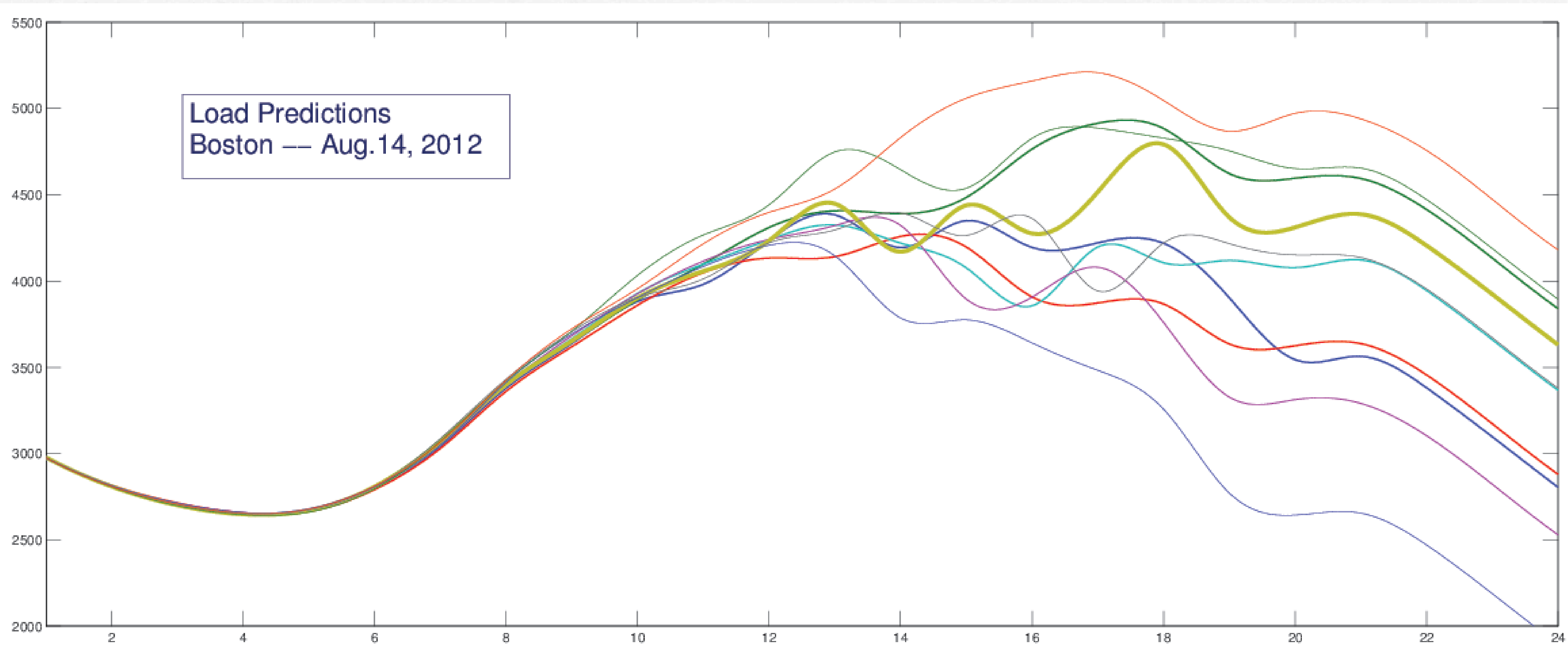
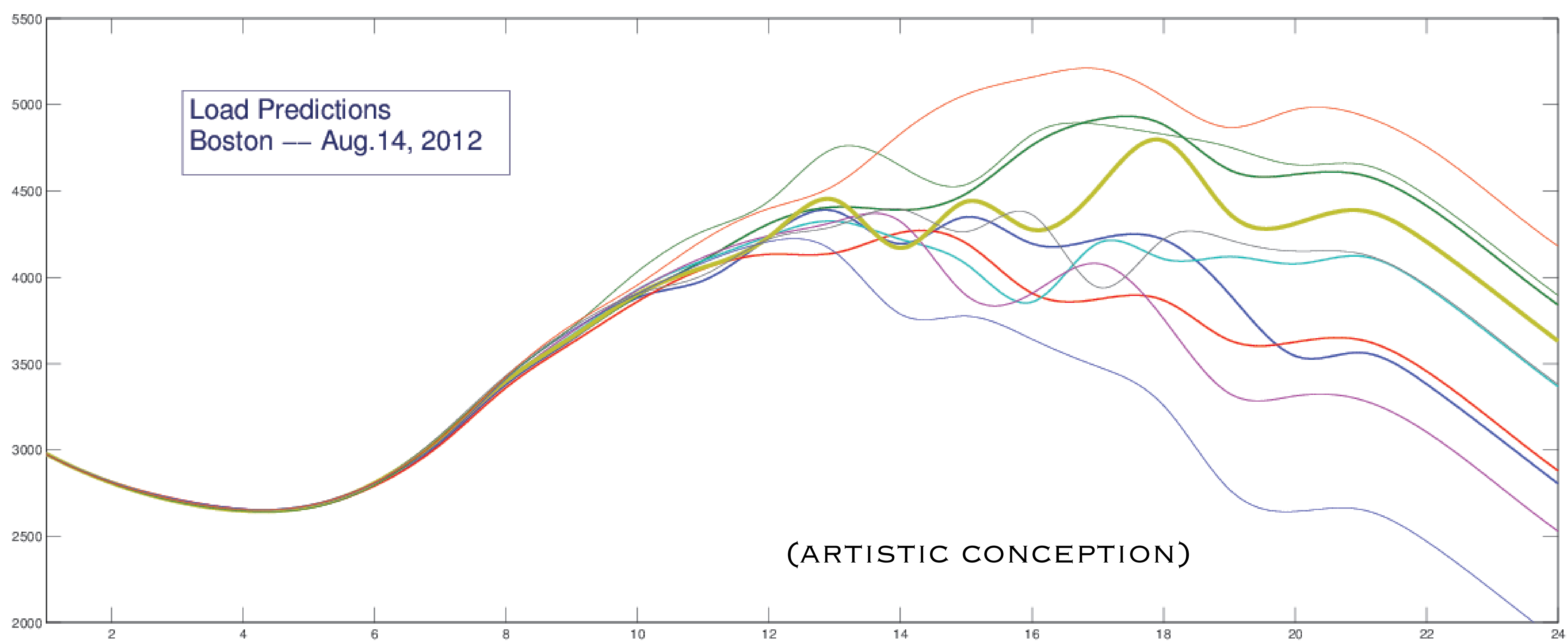
But that wouldn't capture the uncertainty!

one would expect:

“Realistic” Forecasts



“Realistic” Forecasts



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- weather prediction @ 11 a.m.
  better @ 11 p.m. ... but too late!

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- weather prediction @ 11 a.m.
  better @ 11 p.m. ... but too late!
- surface wind =>? power wind
- cloud cover (no historical prediction data) -- only actuals are available
- model to be used for the stochastic load predictions model: SDE, time series, ???
  all inappropriate

Stochastic Load Process ⟻ Scenarios

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a) segmentation: season + day characteristics

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b) functional regression for given segment

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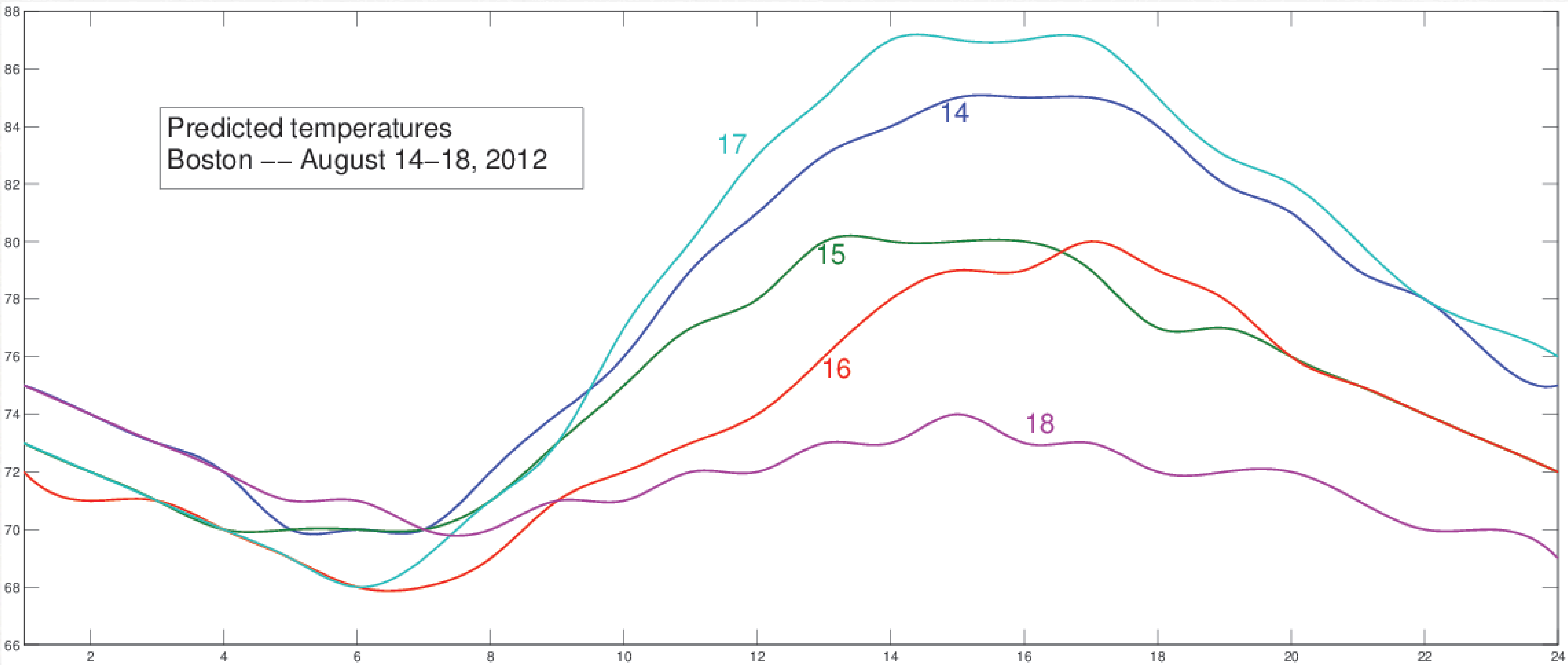
Stochastic Load Process ⟻ Scenarios

a) segmentation: season + day characteristics
b) functional regression for given segment
c) hourly distribution of errors per segment
HOW THIS IS CARRIED OUT (this p.m.)
d) conditional distribution of errors => process
e) discretization of the process => scenarios

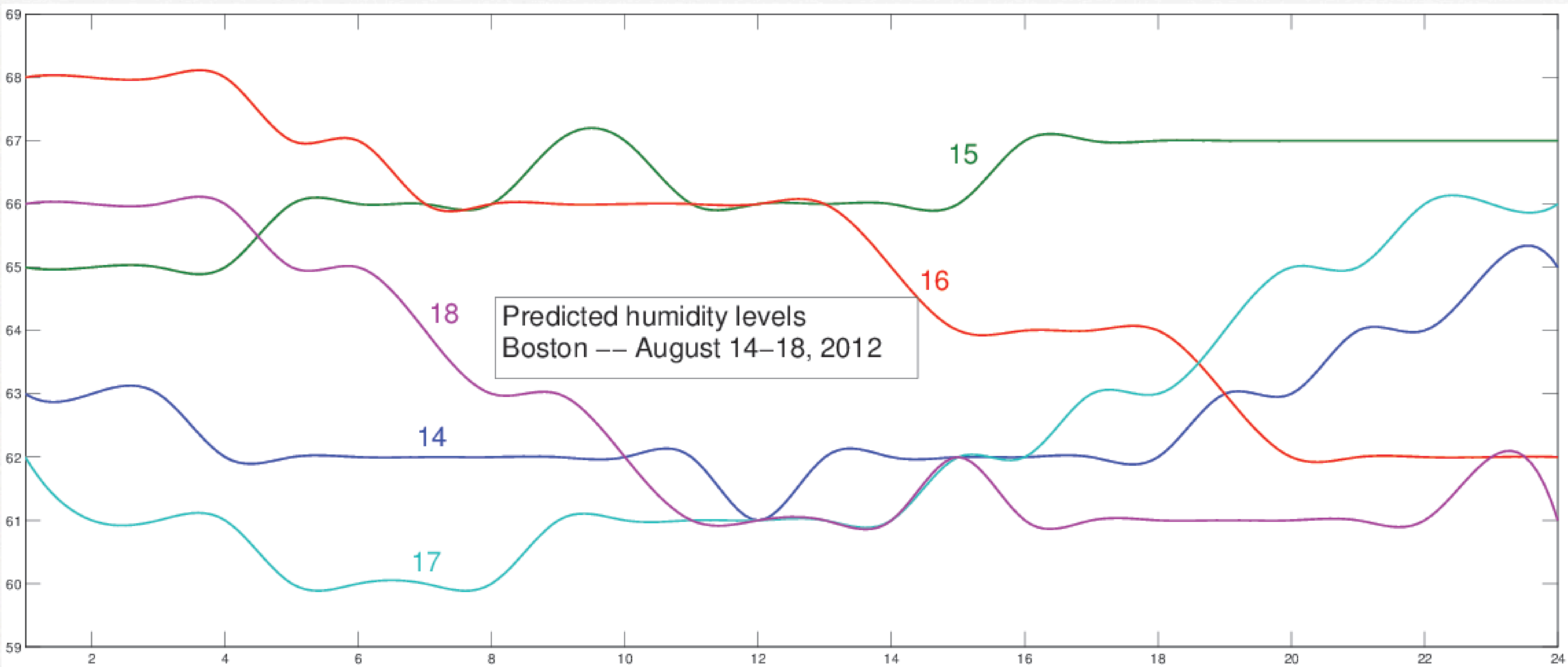
Segmentation

- ~ similars, analogs (± standard)
  to enrich data: Wednesday rule, zone rule?
- seasons: (factor analysis, 'heuristics')
  - ± spring & fall : temperature
  - winter: temperature & cloud cover
  - summer: temperature & dew point
- wind power (at present): handled independently
  based on 3TIER analogs
  total load ≈ load scenario - wind power scenario

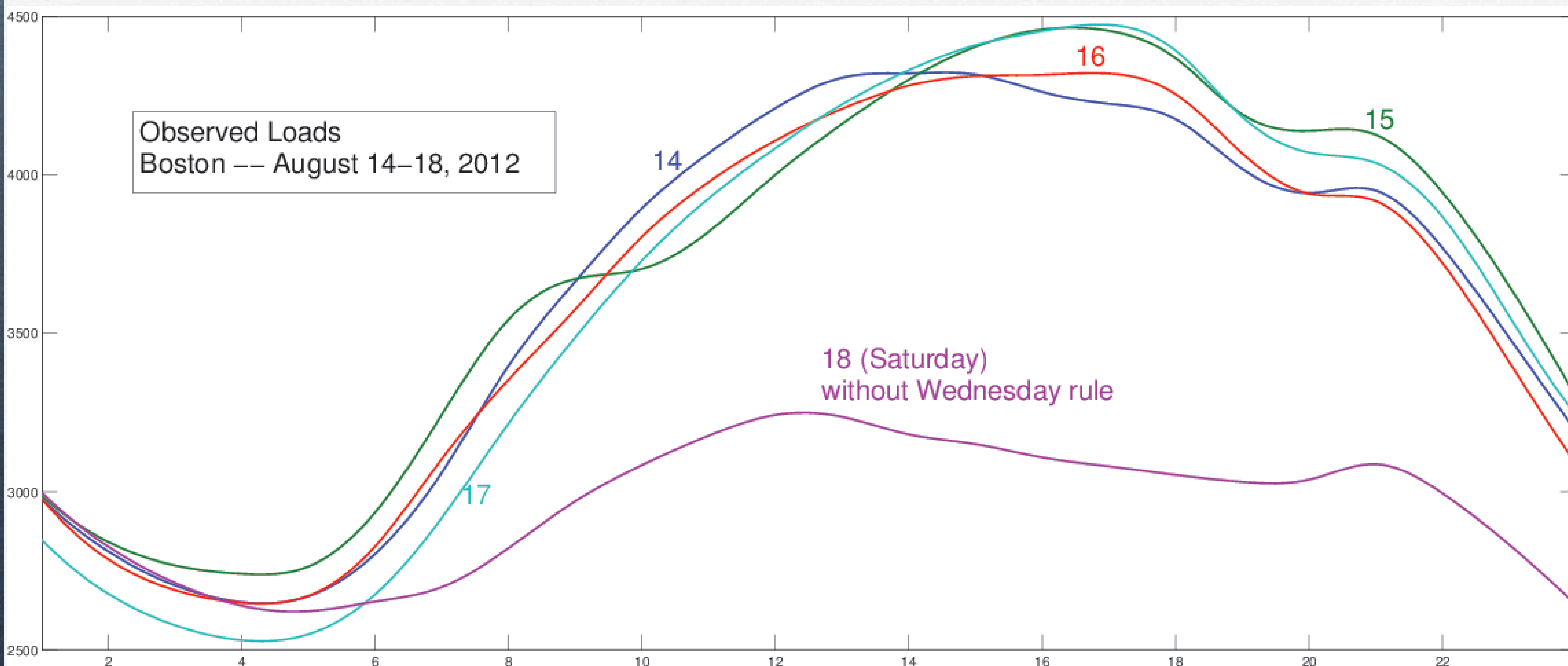
Summer segment "#1"

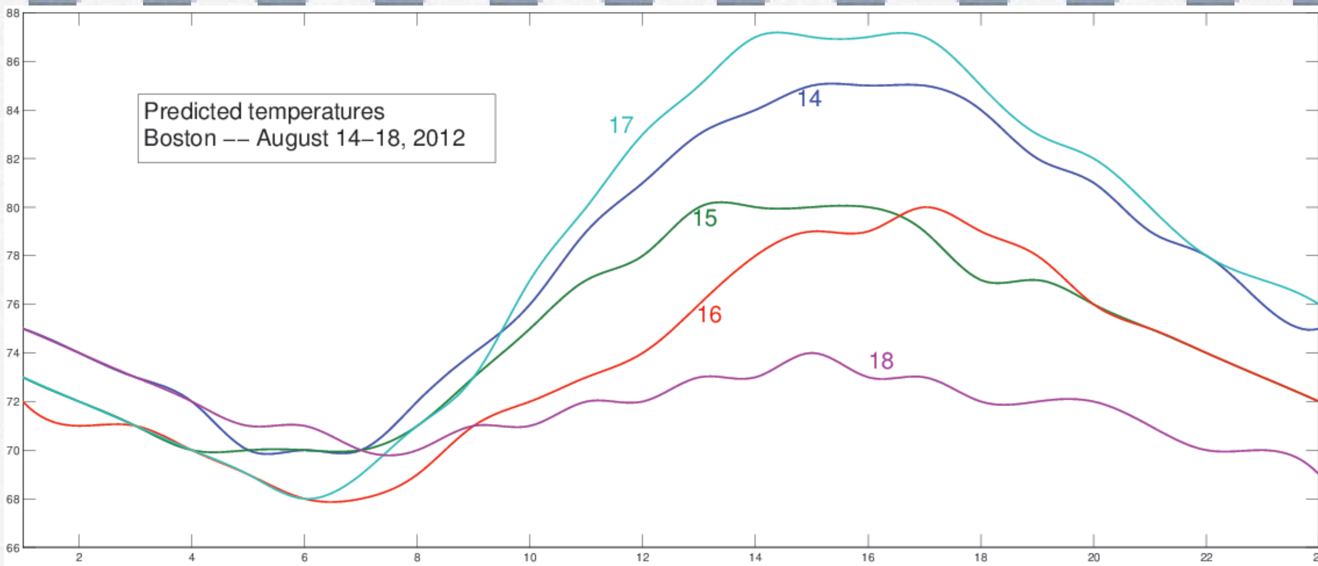


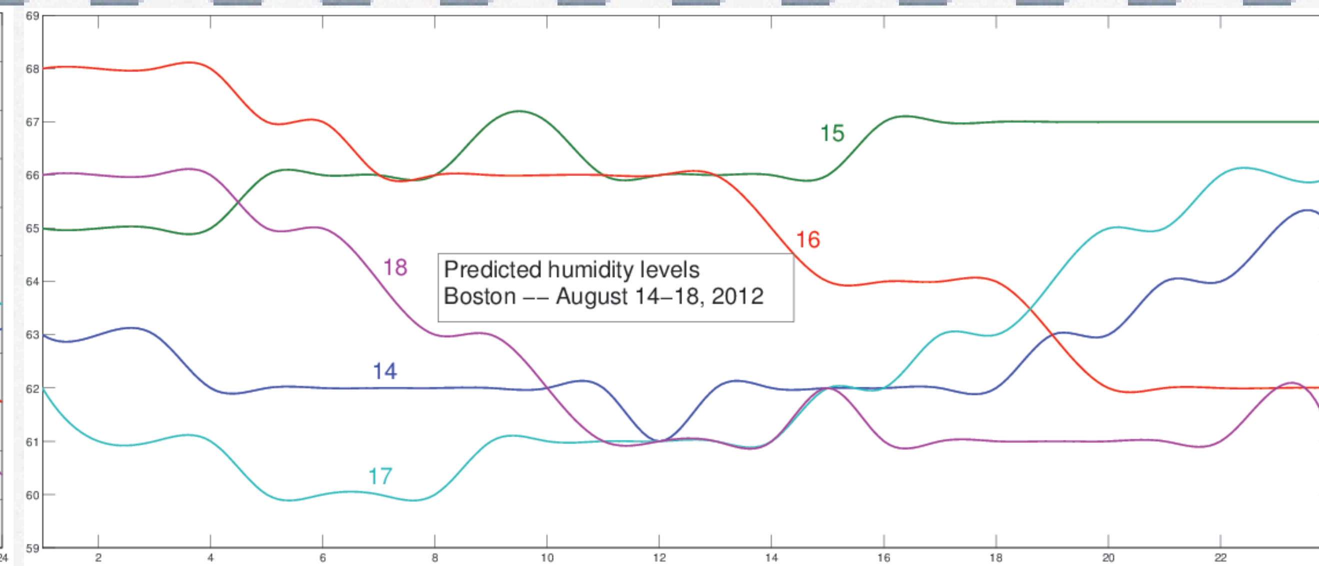
Summer segment "#1"



Summer segment “#1”

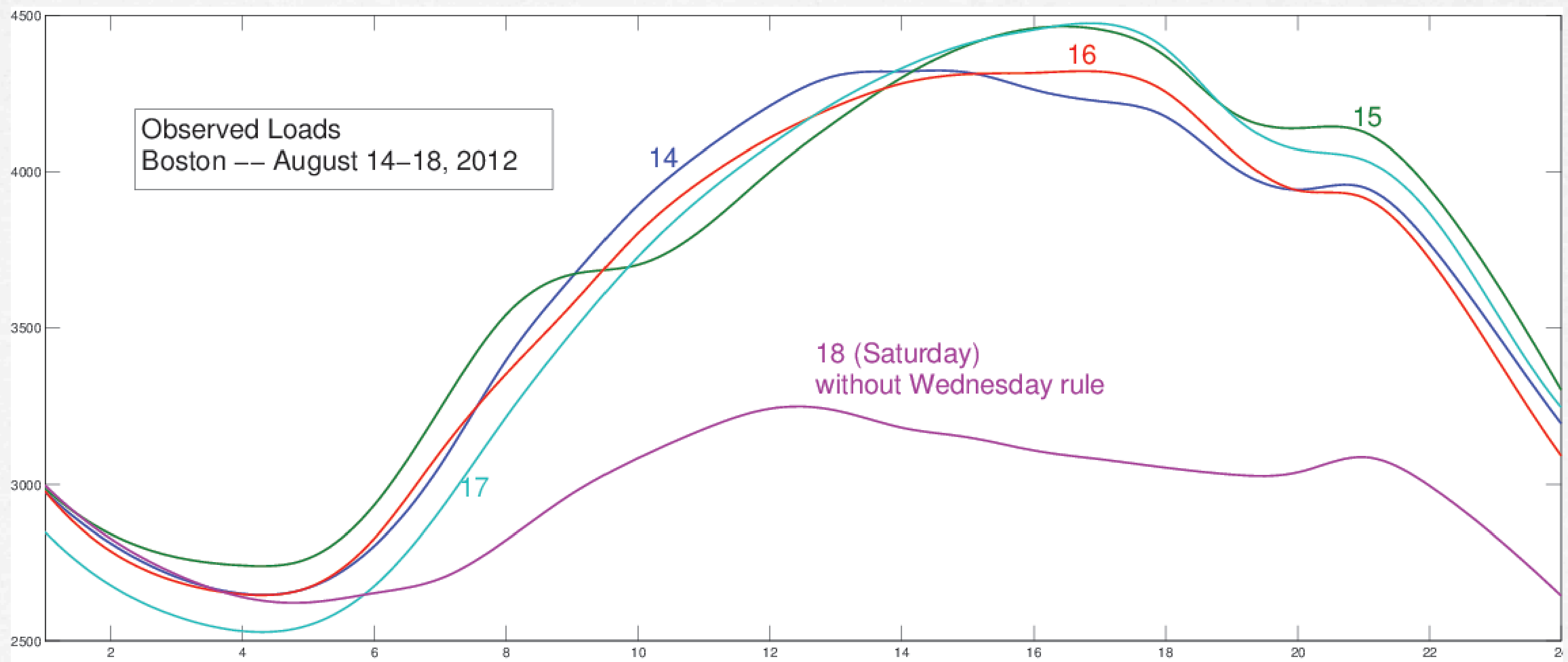






from day d-1 $\Rightarrow$ possible load on day d $d = 14, ...,18$

1. regression(temp. curve, humid. curve) $\Rightarrow$ 'expected' load curve
2. get distribution of the errors (hourly, ... at any time)



The Regression Problem

find a function r that minimizes errors (with respect to $\|\square\|$)

$$\sum_{\text{days d in segment}} \sum_{\text{hours h in day}} \left\| r\left((tmp_{d,h}, hum_{d,h})\right) - \text{load}_{d,h} \right\|$$

an infinite dimensional problem!

Our approach: rely on 2-dimensional epi-splines ("innovation")

- epi-splines approximate with arbitrary accuracy 'any' function
- epi-splines are completely determined by a finite # of parameters
- allows (via constraints) to include 'soft' (non-data) information

The Errors Distributions

Given segment # and associated r, for fixed hour h

$$e_{d,h} = \text{load}_{d,h} - r\big((tmp_{d,h}, hum_{d,h})\big),\ d \in \text{segment \#}$$

$\Rightarrow$ estimate the density f_h of the errors (at h in segment #)

yields an overall estimate of the 'volatility' (in fact, more)

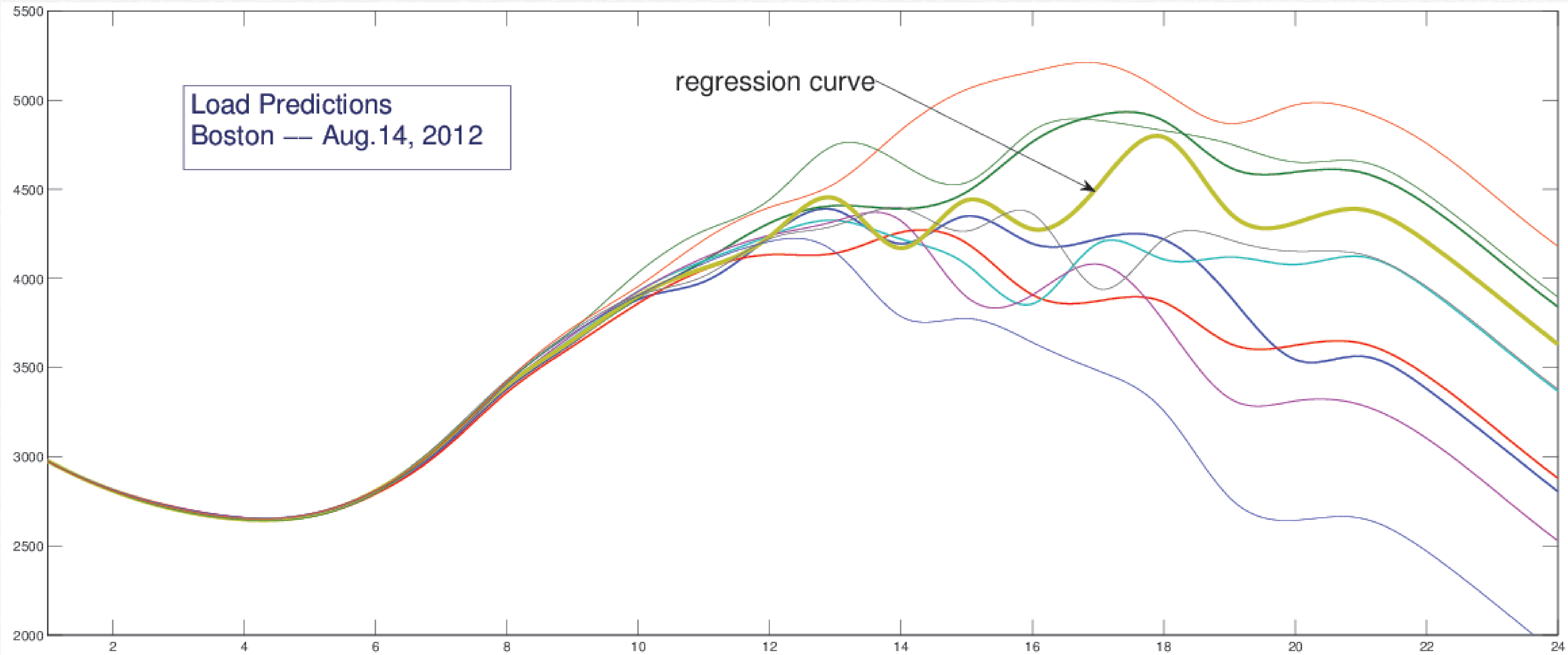
another infinite dimensional problem & data might be scarce

Our approach: estimation via exponential epi-spline (novel):

- $f_h = \exp(-s_h)$, s_h an epi-spline $\quad (\Rightarrow f_h \geq 0)$
- same properties as epi-spline, could include unimodality restriction

... et voilà!

regression curve & sampling from errors distribution



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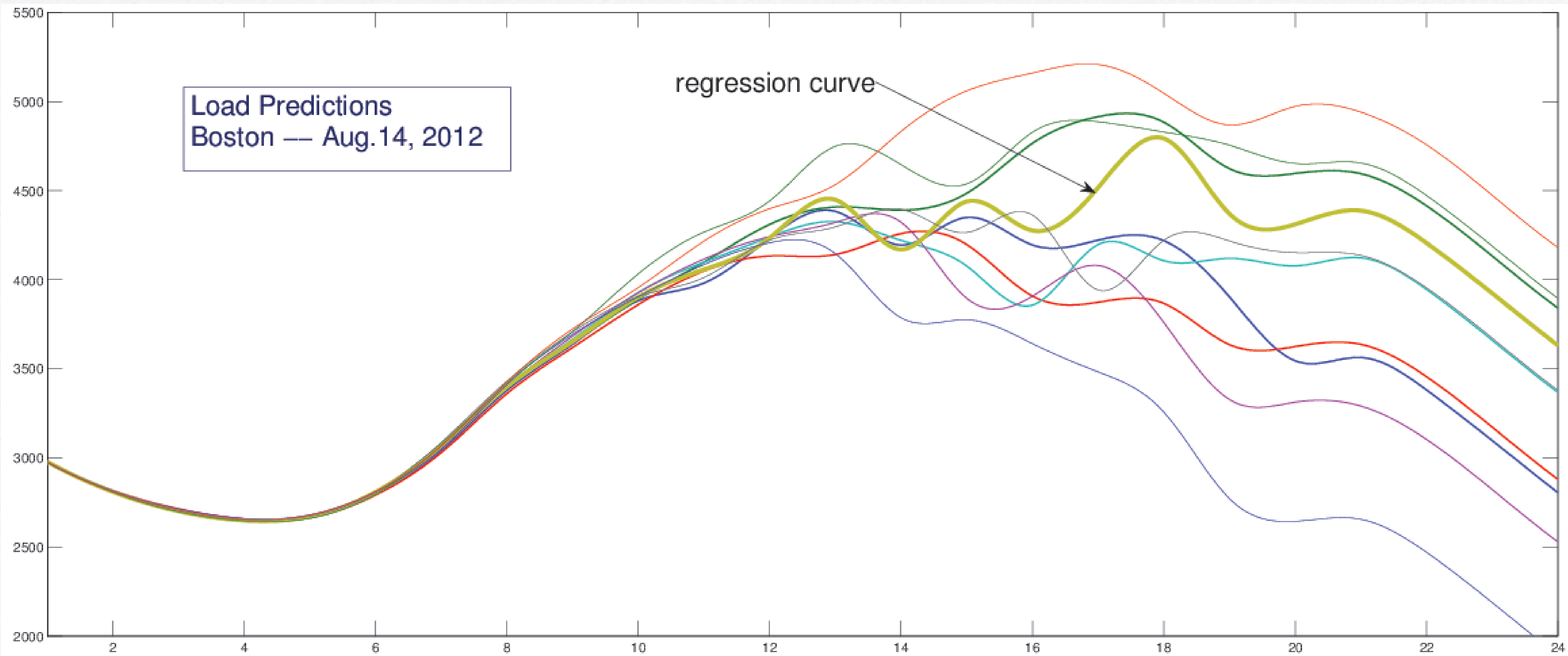
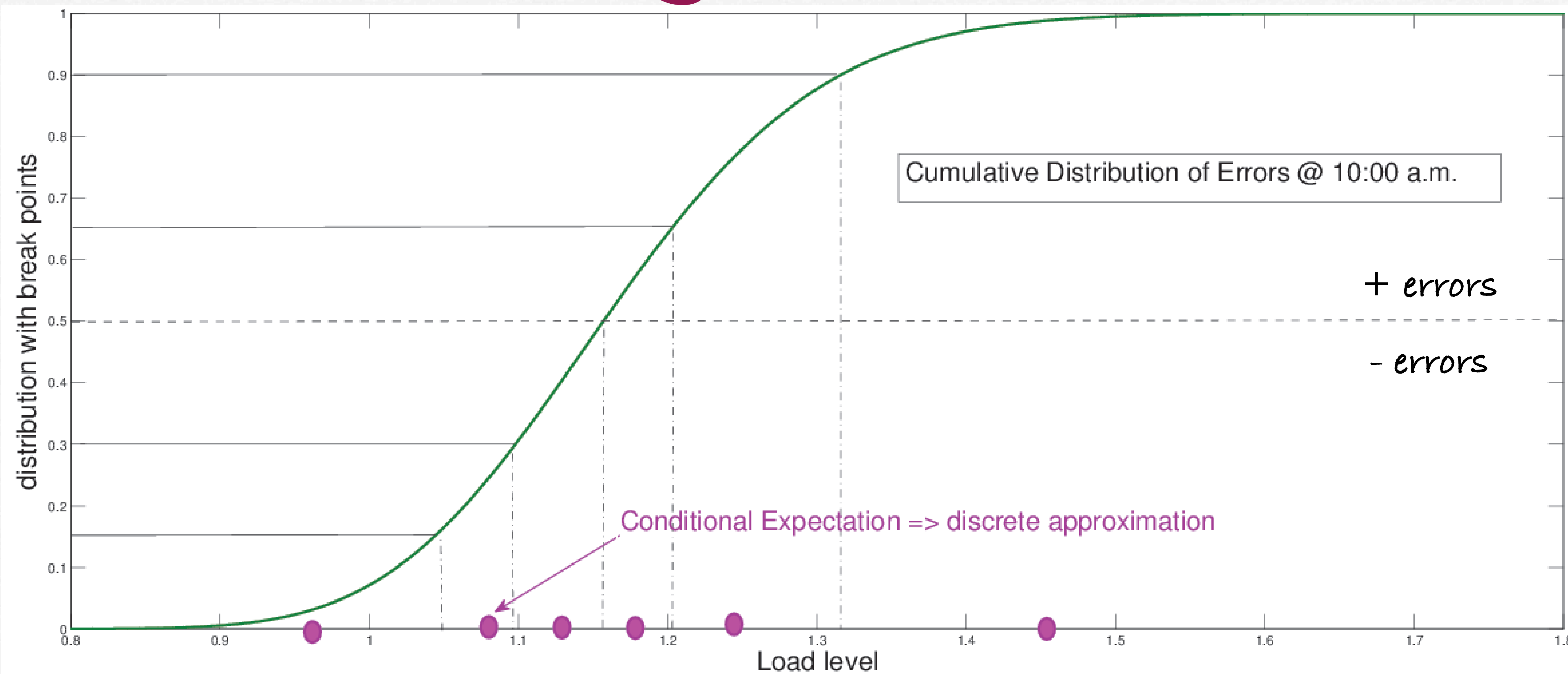


$a.$ how many samples? $10^3, 10^5, \ldots$?

$b.$ conditioning: @10 o'clock above or below the regression curve

Continuation: actually Building Scenario Trees

Conditioning & Discretization



a. identify all observed load curves in each sub-segment

b. for each sub-segment: re-calculate regression and errors distribution

c. repeat for each sub-segment @ (say, 1 p.m.) $\Rightarrow$ sub-sub-segment