A Tribute to David Blackwell

George G. Roussas, Coordinating Editor

On July 8, 2010, Professor David Blackwell of the University of California, Berkeley, passed away at the age of ninety-one.

In appreciation of this great mathematical scientist, the editor of *Notices of the American Mathematical Society*, Steven Krantz, decided to compile a rather extensive article consisting primarily of short contributions by a number of selected invitees. Krantz, being a mathematician, felt the need for a bridge toward the community of statisticians and probabilists. I had the good fortune to be invited to fill this role and to be given the opportunity to offer a token of my affection and immense respect for Professor Blackwell.

A limited number of invitations were sent to a selected segment of statisticians/probabilists. The response was prompt and enthusiastic. At the final stage, twenty contributions were collected of average length of about one and a half pages. The contributors were selected to represent four groups of people: former students of Professor Blackwell; former students at the University of California, Berkeley, but not Professor Blackwell's students; faculty of the University of California, Berkeley; faculty from other institutions.

The heart of the present article is the set of contributions referred to above.

Deep gratitude is due to the twenty contributors to this article for their response to the extended invitations; for their sharing with the mathematical sciences community at large their experiences with Professor Blackwell; their reminiscences about him; and their expressed appreciation of Professor Blackwell's work.

—*George Roussas*

George G. Roussas is Distinguished Professor of Statistics at the University of California, Davis. His email address is `roussas@wald.ucdavis.edu`.

Manish Bhattacharjee

In David Blackwell, we have that extraordinary combination of exceptional scholarship and superb teaching that all academicians aspire to but rarely achieve. The teaching aspect was manifest at all levels, ranging from a basic introductory course to cutting-edge research. Anyone who has been at Berkeley for any length of time is familiar with the fact that his section of "Stat-2" class routinely had more students than the combined enrollment of all other sections of the same course, which were typically taught by others. My own introduction to his teaching style was also through an undergraduate course in dynamic programming that I took in my second or third semester as a graduate student. His engaging style of explaining a problem and bringing it to life in this class I believe, on reflection, was a decisive influence on my choosing stochastic dynamic programming and its applications to probability as the primary area for my dissertation research. One of my fond memories of how he brought a problem to life in class concerns a story of how, over a period of time, he and Samuel Karlin wrestled with the problem of proving the optimality of the so-called "bold play" strategy in a subfair "red-and-black" game that is an idealized version of roulette.

The *New York Times* obituary of July 17, 2010, describes him as "a free-ranging problem solver", which of course he truly was. His was a mind constantly in search of new challenges, breaking new ground in different areas every few years with astounding regularity. As Thomas Ferguson told *UC Berkeley News* in an interview recently, Blackwell "went from one area to another, and he'd write a fundamental paper in each." His seminal

Manish Bhattacharjee is professor of mathematical sciences at the New Jersey Institute of Technology. His email address is `manish.bhattacharjee@njit.edu`.

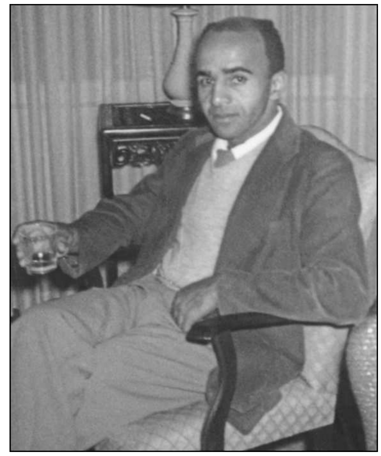
contributions to the areas of statistical inference, games and decisions, information theory, and stochastic dynamic programming are well known and widely acknowledged. He laid the abstract foundations of a theory of stochastic dynamic programming that, among other things, led Ralph Strauch, in his 1965 doctoral dissertation under Blackwell's guidance, to provide the first proof of Richard Bellman's *principle of optimality*, which had remained until then a paradigm and just a principle without a proof!

Each of us who have been fortunate enough to count ourselves to be among his students will no doubt have our personal recollections of him that we will fondly cherish. A recurring theme among such recollections and the lasting impressions they have left on us individually, I believe, would be his mentoring philosophy and style. He encouraged his students to be independent and did not at all mind even if you did not see him for extended periods as a doctoral student under his charge. He trusted that you were trying your level best to work things out yourself and waited until you were ready to ask for his counsel and opinion. A consequence of this, exceptions notwithstanding, was perhaps that his students took a little longer than average to complete their dissertations (although I have no hard data on this), but it made them more likely to learn how to think for themselves.

Richard Lockhart

I was David Blackwell's Ph.D. student from mid-1977 to early 1979, having asked him to supervise after hearing the clearest lectures I had, and have, ever heard. I was Ph.D. student number sixty-two of sixty-five, according to the Mathematics Genealogy Project, where it will be seen that David had four students finish in 1978, two in 1979, and two more in 1981. I remember the chair outside his office where I would wait for my weekly half hour. I remember that he suggested a problem to me by giving me a paper and saying he didn't think he had done everything there that he could have. A few months later I gave up and asked to work on something he was doing currently—programmable sets.

David gave a talk at Stanford in the Berkeley-Stanford colloquium series in which he described these objects. I emerged from the talk amazed by how clear and easy it all was. Trying to tell others about it, I saw quickly that the lecture was a very clear presentation of something not so easy at all and that David had the capacity to organize arguments far, far better than I. The idea is simple as he described it. Imagine a computer represented as a countably infinite sequence of lights. A computer program is a function f that turns off some lights. Which lights are extinguished is a function of the current pattern of on and off lights. You start with some initial pattern of lights, x_0 say, and let the program run by computing $x_1 = f(x_0)$, $x_2 = f(x_1)$, and so on inductively, generating the sequence $x_n; n = 0, 1, 2, \ldots$. The sequence x_n decreases so it has a limit, which we call x_ω. But for a general function f the program might not be finished—x_ω need not be a fixed point of f. So continue applying f inductively, once for each countable ordinal α arriving at x_{ω_1}, the limit at ω_1, the first uncountable ordinal. The limit must actually be reached at some countable ordinal, which may depend on x_0.



David Blackwell early in his career.

Photo from Paul R. Halmos Photograph Collection, di_07305, Dolph Briscoe Center for American History, U. of Texas at Austin.

The iterates x_α are, of course, iterates of the function f applied to x_0, so that we may think of the functions f_α converging (pointwise) to the limit function f_{ω_1}, and we may wonder what sorts of functions g have the form of such limits if the basic function f is Borel? David looked at the output program f_{ω_1} as a step in defining general functions g between two polish spaces U and V, which were defined by mapping U into the computer's state space by a Borel function, running the program, and then mapping the result into V in a Borel way. Any function admitting such a factorization is Borel programmable (BP), and sets are BP if their indicator functions are. David's 1978 paper, establishing the basic properties of the sets and functions and showing that they had potential as useful objects in probability theory, is typical of his work—very clear, very concise, questions not answered set forth, and not quite four pages long.

In my thesis I solved one or two minor problems connected with these ideas. I was trying to clarify the relation between BP-sets, R-sets studied by Kolmogorov and others, and C-sets. I wanted to know, for instance, if letting the program, encoding, and decoding functions be BP, rather than just Borel, gave you a larger class of sets than the BP class. (John Burgess did the problem properly in *Fundamenta Mathematica* in 1981 using results from logic concerning monotone operators, the lightface and boldface set hierarchies and game quantifiers.) I cannot remember how I came to realize that I needed to think not about sigma fields but about collections of sets like the analytic sets which had fewer closure properties. I now think,

Richard Lockhart is professor in the Department of Statistics and Actuarial Science at Simon Fraser University. His email address is `lockhart@sfu.ca`.

though, that David planted the seed of this idea in my head one day—just by emphasizing the importance of closure properties to me; I rather suspect that my thesis would not have taken him more than a few weeks to put together and that the result would have been much shorter and far clearer.

I wish I could say I knew him well, but my weekly meetings were all I knew—and they were all mathematics. I was young and shy and David was friendly but formal, I think. I wish we could all learn mathematics from people who see it as clearly as David did.

References

[1] D. H. Blackwell, Borel-programmable functions, *Ann. Prob.* **6** (1978), 321–324.

John Rolph

I was a statistics graduate student at Berkeley in the mid-1960s. It was my good fortune that David Blackwell taught the inference course to the first-year students in my cohort. I was so taken with him and his teaching that I subsequently enrolled in virtually every course he taught. They embraced a dizzying array of topics, ranging from information theory to game theory to seminars on dynamic programming, bandit problems, search, and related topics. In his teaching as in his research, his interests and knowledge were broad.

David Blackwell was a teacher without parallel. I was particularly impressed by how he could make truly deep concepts transparent—even to the beginner. Indeed, the depth was such that understanding sometimes blossomed mysteriously and gradually. I recall going over class notes after class and only then beginning to understand how deep and subtle some of the concepts he had presented were. To make sure we understood the ideas, a group that included Steve Stigler, Bruce Hoadley, and me made a practice of assigning two to take notes and one to listen intently to his lectures. We would meet afterward, reconstructing the lectures to make sure we all actually understood the concepts and results he covered.

Unlike most faculty members who specialized in one or perhaps two areas, Blackwell was a man of remarkably diverse interests; thus his students worked on a wide variety of topics. While I was there, he supervised dissertations in information theory, dynamic programming, game theory, Bayesian inference, search theory, probability theory, stochastic processes, and stopping rules. He moved from student to student and hence topic to topic with astonishing flexibility and focus.

To say he was a quick study is a remarkable understatement. When you met with him, he would look over what you had done, ask a few probing questions, suggest an approach or two to your current problem, and send you on your way. These sessions typically ran ten to twenty minutes. Indeed, when I finally brought him a draft of what I hoped was my completed dissertation, he read it, asked a few penetrating questions, listened to my responses, then told me I was finished—all in a half hour! This modus operandi obviously worked splendidly—he supervised sixty-five doctoral students during his career.

Although David Blackwell had strong beliefs and points of view, he was not an avid proselytizer. Indeed, it was only gradually that I came to understand and appreciate that David Blackwell was the lone Bayesian in the Berkeley Statistics Department. Interestingly, the content of the first-year inference course he taught us was very similar to the course Erich Lehmann customarily gave—Bayes estimates only came up as a convenient mathematical concept, not as a philosophy of inference. That Blackwell was not a vociferous proponent of his belief in the superiority of the Bayesian approach was typical of his nonconfrontational way of interacting with his colleagues.

David Blackwell was a man we shall all remember with great respect and affection. He was a man of modest demeanor who would solve seemingly intractable problems with mathematical rigor, elegance, and transparency. The impact of his research was both substantial and broad. And he treated his students as equals, both helping them and challenging them to be successful in their research. He was a person of extraordinary character and ability; it was a privilege to know him and to learn from him.

John Rolph is emeritus professor of statistics at the Marshall School of Business at the University of Southern California. His email address is jrolph@usc.edu.

George Roussas

I joined the Department of Statistics at the University of California, Berkeley (UCB) in the spring semester 1960, after having graduated with a degree in mathematics from the University of Athens, Greece, and having served my two-year military service in the Greek army.

David Blackwell was the chair of the department at that time, and Lucien Le Cam was the graduate advisor. Blackwell would have a brief interview with every incoming graduate student, and it was in this capacity that I first met him. Right after my interview with him, a couple of other, also new, graduate students pointed out to me that we had a black man as chair of the department. In a way,

George Roussas is Distinguished Professor of Statistics at the University of California, Davis. His email address is roussas@wald.ucdavis.edu.

I was taken by surprise, and I responded "come to think of it, we certainly do!" Professor Blackwell's personality was overwhelmingly strong and yet gentle and enchanting, radiating kindness around him. These attributes transcended any trivialities such as noticing the color of his skin. Besides, my cultural background was alien to it.

The UCB campus was an earthly paradise, and the faculty of the department were like Olympian figures to me in the statistics pantheon. I had the utmost respect and admiration for all and each one of them.

Nevertheless, some did stand out, as it were. Thus it was the grand old man Jerzy Neyman from whom, foolishly, I never took a course. He was nice to me, and more than once he recounted his experiences in Greece as a member of a certain commission soon after World War II. It was Michel Loève from whom I took my first course in probability and who inspired me with rigor and deep interest in the subject matter. Later, I also took his year-long course in probability and stochastic analysis. It was the mathematician-philosopher Edward Barankin from whom I learned measure-theoretic probability, and I was also introduced to time series analysis and sufficiency. It was Lucien Le Cam from whom I managed to chip away bits of his vast knowledge of asymptotics in statistics. They were destined to play a formative role in my academic career.

And it was David Blackwell who was destined to be my thesis advisor under some peculiar circumstances.

If I remember well, it was in the early 1960s that Lester Dubins was offering a seminar based on his book (coauthored with Jim Savage) *How to Gamble if You Must: Inequalities in Stochastic Processes.* Blackwell was attending that seminar, as well as a fair number of students, including myself. At the end of each lecture, Blackwell would suggest a number of open questions for possible thesis topics. It was such a question that he brought to my attention and insisted that I look into. Indeed, I did, and in a couple of months I asked for an appointment with him to report accordingly. At that time, he was advising a large number of students, and his appointments were limited by necessity to a half-hour block of time. Apparently, he was pleased by what he read, was very encouraging, and also made a number of concrete suggestions. After another couple of sessions like this, he decided that the solution that I arrived at was what he expected. Encouraged by this, I asked whether I could combine this piece of work with another paper on asymptotics, which was already accepted for publication in the *Annals of Mathematical Statistics*, to make up my thesis. Blackwell's response was as always brief and clear-cut. "To me, this by itself is more than enough."

And that is how I had the privilege and honor to be David Blackwell's student.

From the courses I took from him (one in game theory and the other in coding/information theory) I saw firsthand how a great scientist can also be an inspiring and superb teacher. From my interaction with him, as his advisee, I could not help but admire the clarity of his thoughts, articulated in an amazingly brief and simple way. At the same time, his polite disposition and abundant kindness had absolutely no match.

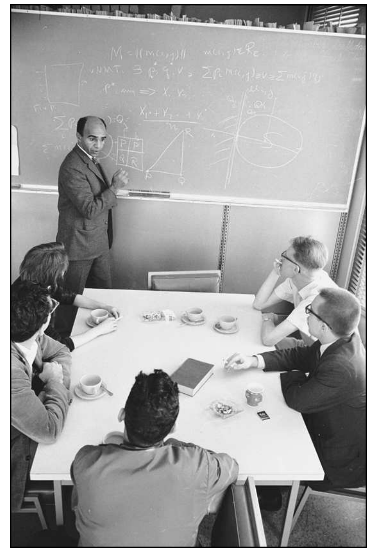


Photo courtesy of Getty Images.

Blackwell teaching a class in game theory at UC-Berkeley.

Soon after I was conferred my Ph.D. degree in 1964, I had the opportunity to host a dinner party for Professor and Mrs. Blackwell in a rather original and upscale restaurant in the Bay area (I believe it was called the Nero's Nook), located in the Los Gatos area. It was apparent that all three of us had a truly enjoyable time.

After I moved to the University of Wisconsin-Madison (UWM), the first time that I met him was in 1972 during the Sixth (and last) Berkeley Symposium in Statistics and Probability. The next ten years or so I spent overseas, at the University of Patras, Greece, and I more or less lost contact with him except for a Christmas card. In 1984 I dropped by UCB after I returned to the West Coast (at the University of California, Davis). In Berkeley, I had the opportunity to have lunch with Edward Barankin (in the restaurant of the Durant Hotel), who, unfortunately, succumbed to cancer soon thereafter. David Blackwell received me very warmly and invited me for lunch at the canonical fish restaurant in Berkeley (Spenger's Fresh Fish Grotto). Also, he expressed his satisfaction that one of his old students did fairly well as a senior faculty member now (full professor at UWM and chair of applied mathematics at the University of Patras) and also as an academic administrator (dean of the School of Physical and Mathematical Sciences at Patras and also chancellor of the same university). However, for me, David Blackwell remained "Professor Blackwell" as an expression of my utmost respect for him and also because of my European early upbringing. But this would not do anymore for Blackwell. On the spot he put me in a difficult dilemma; "Either you call me David or I will never talk to you again!" So, Professor Blackwell became David for me henceforth.

Once at UCD, I was given the opportunity to drop by UCB, but not as often as I would have liked. From what remained of the old guard, Blackwell and Le Cam were the people that I would always visit.

It was in early June 2010 when we received an email message at UCD from Bin Yu, the current chair of the Department of Statistics at UCB, that David was not doing well. I was about to depart for the annual pilgrimage to Greece (on June 13), but I did make a concerted effort to obtain a brief visit with David before my departure. That effort was not successful. I resolved to try again soon after my return (July 7). Unfortunately, that effort never came to be; David Blackwell departed on July 8.

His memory will remain alive among all those who were fortunate enough to know him and to profit from his wisdom and his gentle and kind disposition. For me, David Blackwell was and will remain the role model of a great mathematician, an inspiring teacher, and a superb human being.

Howard Tucker

David Blackwell was a considerable influence in my life. This influence is best summarized in my dedication of a joint paper I was invited to submit for the *I.M.S. Lecture Notes Monograph* series in his honor, which stated, "To David Blackwell, who with his characteristically concise sentences taught me, among other things, how to write a mathematics paper, how to look at mathematics, how to welcome responsibility and how to face one's more mature years, this paper is affectionately dedicated."

When David arrived at Berkeley in 1954 I was beginning my last year as a graduate student. My hazy recollection of my first interaction with him was that I was appointed as his teaching assistant for the graduate course in probability at the measure-theoretic level. When I asked him what he wanted me to do during the two one-hour sessions per week for the semester, his instructions were for me to do what I felt I should do for the six or seven students in the class. Since he wanted to cover other topics, I had a very enjoyable time for the semester or the year (I forget which) going through the recently published Gnedenko and Kolmogorov book on limit distributions.

I received my Ph.D. in mathematics in June 1955 and am listed as David's first doctoral student. This occurred as follows. The problem that I was working on starting in 1953 was one suggested by Professor J. Neyman. However, some time after David arrived at Berkeley, Neyman asked him if he would be available to advise me in the throes of putting the dissertation into its final shape. Among other things during that period, he showed me how to write a mathematics paper, which is recalled in the dedication quoted above. So somehow David was appointed chairman of my committee, and I am listed as his first doctoral student. This was and is a great honor for me.

Howard Tucker is professor emeritus of mathematics at the University of California, Irvine. His email address is `htucker@math.uci.edu`.

Roger J.-B. Wets

I first thought I would devote this short contribution to a couple of remarkable technical achievements of David Blackwell and how they influenced subsequent research. This would have included the deep insight provided by his elegant proof of Lyapunov's theorem about the range of a vector measure, about his seminal articles laying the foundations of dynamic programming, and so on. But it is in his role as a lifelong advisor and model that his influence turned out to be most significant.

It is impossible to find any information about David that does not refer to him as an outstanding teacher, and indeed he was. He liked his classes to be scheduled as early as reasonable. The first course I took with him was an undergraduate course on dynamic programming, in which he mostly covered his own development of the field. It was listed as an undergraduate course, I suppose, on the basis that he didn't require much more than a decent background in real analysis and linear algebra. But one could never have guessed that it was an undergraduate class on the basis of the student body. There might have been one or two smart undergraduates lost in the audience, but the rest consisted mostly of graduate students in operations research and statistics and a not insignificant number of faculty members. In addition to remembering that homework assignments were extensive, instructive, and relatively hard, I was fascinated by the constructive approach; not just whether it exists or might be done but the fact that the results were derived in such a way that suggested the potential of solution procedures. I didn't realize at the time how strongly it would eventually influence my own research strategy.

After I took a couple more courses with him and chose to work in stochastic optimization, David became a natural coadvisor of my thesis. The subject stochastic programming (decision making under uncertainty) had been proposed by G. B. Dantzig.

Roger J.-B. Wets is Distinguished Research Professor in the Department of Mathematics at the University of California, Davis. His email address is `rjbwets@ucdavis.edu`.

I was pleased but not surprised by David's acceptance to act as coadvisor. But his advice/comments could be quite candid and to the point. The first time I went to discuss what I was planning to do, I gave a too-succinct version of the class of questions I was going to consider, and David bluntly told me "but that's just finding the minimum of an expected function", and he definitely was not impressed. When, a bit later, I explained that this "function" was not a simple one but involved not just an objective but also (complex) constraints, he revised his assessment to "Oh, that, make sure you first handle some manageable cases", and he immediately started with a couple of suggestions that eventually turned up as illustrations in my thesis.

He had played, more than once, the role of the "wise uncle" for students interested in optimization who were concerned about getting a degree in a field whose mathematical standing wasn't yet well established or recognized. They somehow felt that they could confide their concerns to him and would then receive the appropriate advice. He could be quite plainspoken in such situations and simply told the hesitating student, "You are telling me that you are interested in area A, but would consider getting a degree in statistics, how can this make sense?" For one of my friends, this advice turned out to be exactly what was needed, and it resulted in a brilliant, mathematically rich career.

I didn't return to statistics until it became difficult to ignore the ubiquitous lack of statistical data available to construct reliably the distribution of the random quantities of a stochastic optimization problem. My approach was based on the idea of incorporating in the estimation problem all the information available about the stochastic phenomena, not just the observed data but also all nondata information that might be available, and relying on variational analysis for the theoretical foundations and optimization techniques to derive nonparametric, as well as parametric, estimates. This didn't look like an easy sale to either frequentist or Bayesian statisticians. So, I went to see David, by then professor emeritus. After all, this could be fitted in the framework of the theory of games and statistical decisions. This time, it didn't take him more than a few minutes to understand and encourage me to pursue this approach. Of course, he also immediately suggested further possibilities and reserved a place for a lecture in the Neyman Seminar, as well as time for further discussions.

On repeated occasions, David provided this steady anchor that made you feel that what you were trying to do was or was not worthwhile, and, given the wide scope of his interests and knowledge, this always turned out to be an invaluable resource. Thanks, professor extraordinaire, David Blackwell.

Peter Bickel

I first met David Blackwell when I took his course on information theory during my first year as a doctoral student. David had chosen as a text Jack Wolfowitz's *Information Theory for Mathematicians*, which, as the title suggests, was somewhat dry. David made the subject come to life. His style was well established. Strip the problem of all excess baggage and present a solution in full elegance. The papers that I read of his, such as those on the Blackwell renewal theorem and on Bayesian sequential analysis/dynamic programming, all have that character. I didn't go on in information theory, but I didn't foreclose it. My next memorable encounter with David, or rather the strength of his drinks, was at a party he and Ann gave for the department. When I declined his favorite martini he offered Brandy Alexanders. I took two and have trouble remembering what happened next!

And then I had the great pleasure and good fortune of collaborating with David. I was teaching a decision theory course in 1966, relying heavily on David and Abe Girshick's book, *Theory of Games and Statistical Decisions*. I came across a simple, beautiful result of theirs that, in statistical language, can be expressed as: If a Bayes estimator is also unbiased, then it equals the parameter that it is estimating with probability one. In probabilistic language this says that if a pair of random variables form both a forward and a backward martingale, then they are a.s. equal.

Unbiasedness and Bayes were here specified in terms of squared error loss. I asked the question "What happens for L_p loss for which a suitable notion of unbiasedness had been introduced by Lehmann?" I made a preliminary calculation for p between 1 and 2 that suggested that the analogue of the Blackwell-Girshick result held. I naturally then turned to David for confirmation. We had essentially an hour's conversation in which he elucidated the whole story by giving an argument for what happened when p equals 1, and, in fact, the result failed. He then sent me off to write it up. The paper appeared in 1967 in the *Annals of Mathematical Statistics*.

It is still a paper I enjoy reading. It led to an interesting follow-up. In a 1988 *American Statistician* paper, Colin Mallows and I studied exhaustively what happens when the underlying prior is improper, which led to some surprises.

Peter Bickel is emeritus professor of statistics and professor at the Graduate School of the University of California, Berkeley. His email address is `bickel@stat.berkeley.edu`.

David was a Bayesian belonging, I think, to the minority who believed that axioms of rational behavior inevitably lead to a (subjective) prior. He was essentially alone in that point of view in the department but never let his philosophical views interfere with his most cordial personal relations.

Sadly, our collaboration was the last of my major scientific contacts with David. We were always on very friendly terms, but he would leave the office at 10 AM, which was my usual time of arrival.

After we both retired, we would meet irregularly for lunch at an Indian restaurant, and I got a clearer idea of the difficulties as well as triumphs of his life. Despite having grown up in the segregated South, David always viewed the world with optimism. As long as he could do mathematics, "understand things", rather than "doing research", as he said in repeated interviews, he was happy.

It was my fortune to have known him as a mathematician and as a person. He shone on both fronts.

Thomas S. Ferguson

It was my good fortune to have been a graduate student in statistics at U. C. Berkeley when David Blackwell joined the faculty there in 1954. The distinguished statisticians who were there already—Neyman, Lehmann, Le Cam, Scheffé, Loève, and others—constituted the most approachable faculty I've seen anywhere. We students shared coffee and conversation with them in the afternoons. When Blackwell joined the group, he fit right in with his warm humor, his winning smile, his modesty and his congeniality with the students.

He had an outstanding mathematical reputation by that time, having been invited to give an address in probability at the ICM meetings in Amsterdam in 1954. In 1955 he was elected president of the Institute of Mathematical Statistics. Important for me personally was his book with M. A. Girshick, *Theory of Games and Statistical Decisions*, which came out in 1954. At that time, I was working on my thesis under the direction of Lucien Le Cam. I took a course from Blackwell and read his book, which views statistics as a subset of the art of making decisions under uncertainty. The beauty of this view influenced me to such an extent that my subsequent work did not go so much in the direction of the topics of my thesis but more in the direction of the areas that interested Blackwell—game theory, probability, and sequential decisions.

Thomas S. Ferguson is emeritus professor of mathematics at the University of California, Los Angeles. His email address is tom@math.ucla.edu.

Dave was one of the early Bayesian statisticians, that is, he considered statistics, and life as well, as a process of observation, experiment, information gathering, and, based on one's prior beliefs and the outcomes of the observations, modifying one's opinions and acting accordingly. Although his views certainly influenced me, I was never a complete Bayesian—no student of Le Cam could be—but of all the Bayesians I know, he was the most persuasive. It was characteristic of him to spread his interests over several areas rather than to specialize in one. It is amazing how he managed to produce deep and original results in several fields. The underlying theme of his work springs from his Bayesian perspective: probabilistic, sequential decision making and optimization.

Let me mention just a few of his achievements. In probability, there is a basic renewal theorem that goes by his name. There is his work in Markov decision processes in which he conceived the concepts of positive and negative dynamic programs and in which the notion of Blackwell optimality plays an important role.

In statistics, there is the famous Rao-Blackwell theorem and its association with a simple method of improving estimates now called Rao-Blackwellization. There is a fundamental paper of Arrow, Blackwell, and Girshick that helped lay the foundation for Bayesian sequential analysis. The subject of comparison of experiments was introduced by Blackwell and Stein in 1952. The notion of merging of opinions with increasing information was introduced by Blackwell and Dubins in 1962.

In game theory, he has initialized several areas: games of timing, starting with Rand reports on duels; games of attrition; the vector-valued minimax theorem, leading to the notions of approachability and excludability, etc. He has had a long interest in set theory and analytic sets. This led to his study of conditions under which certain infinitely long games of imperfect information have values. This has had a deep impact in the field of logic; logicians now call such games Blackwell games.

My own professional interaction with him came in 1967–1968. He suggested working on a problem in the area of stochastic games. In 1958 Gillette had given an example of a stochastic game that did not have a value under limiting average payoff if the players are restricted to using stationary strategies. Dave called this example game the "Big Match". He wondered if the game had a value if all strategies were allowed. After working on the problem together for a while, we simultaneously and independently came up with different proofs of the existence of a value. To me, it was just an interesting problem. But Dave somehow knew that the problem was important. It was the first step in showing that all stochastic games under limiting average payoff have a value. This

Photo courtesy of Statistics Dept., U. C. Berkeley.

David Blackwell (center) at his birthday party at the Dept. of Statistics at Berkeley in 2009. On his right is Erich Lehmann, on his left Peter Bickel.

took another fourteen years, with many scholars contributing partial results before the result was finally completely proved.

Dave Blackwell is one of my role models. He influenced me in my professional work and in my personal life. He was a great teacher, both in the classroom and in conversations on general subjects. He had a way of cutting through massive detail to get to the heart of a problem. He had over sixty Ph.D. students. But if you count people like me, he had many more students. His spirit and his works are still alive in all of us.

Albert Lo

David Blackwell wrote one of the first comprehensive treatments of Bayesian statistics, and his insistence on the Bayesian approach is legendary. I approached Blackwell for a Ph.D. thesis topic in the mid-1970s. He told me to look in the *Annals of Statistics*, find a topic I liked, and come back. After some searching, I reported that kernel density estimation interested me. Blackwell, staring at me with his piercing big eyes, said "The topic is fine, yet it *must* be done the Bayesian way." This was exactly what he said to me. Later, when I presented to him a result on the consistency of the posterior distribution of a location parameter with respect to a Lebesgue prior, he concluded matter-of-factly, "It is good since it is *almost* Bayesian." Again, these were his exact words. On another occasion he stated that all the non-Bayesian papers in the *Annals* have to be rewritten using a Bayesian approach, and I myself found this "Bayesianization" a good source of research topics.

Albert Lo is professor of statistics in the Department of Information Systems, Business Statistics and Operations Management at the Hong Kong University of Science and Technology, Clearwater Bay, Hong Kong. His email address is `imaylo@ust.hk`.

Blackwell always insisted on the exactness and clarity of solution. For all his undoubted mathematical ability, his preference was for simplicity over mathematical abstraction. On the density estimation problem, he suggested modeling a density by a location mixture of uniform kernels and putting a Dirichlet process prior on the mixing distribution. The problem was hard then, and after a year and a half of futile searching, I had to present an alternative, yet more standard, approach based on expanding the square root of the density in an orthogonal series with a prior on the infinite sequence of coefficients that lies on the shell of a Hilbert sphere. Upon hearing the proposed approach, Blackwell simply commented "Al, you are not ready." To this day, I can still hear his devastating voice! His opinion about the maturity/readiness of students was perceptive; two years later his Bayesian mixture density problem was resolved with an explicit solution that he had anticipated, presumably at a time when the student was ready.

Though there are suggestions of a good-natured rivalry between Blackwell and some of his famous colleagues, he was not one for direct confrontation. He was very quiet about the racial injustice that he endured and overcame, never mentioning the subject in my hearing. It brings to mind how he handled me as a student, who had been expertly trained by Berkeley frequentists. His only advice to me on how to learn Bayesian statistics was to read Part III of De Groot's text. Though he made some extremely valuable suggestions in his nice and gentlemanly way, he never really discussed or showed me *how* to approach a research problem, except by example. I had to find my own way by observing him and others (mostly others) in the department. The discrimination Blackwell experienced may have given him the philosophy that one should also be able to fight his own way up, or perhaps that if one is worthy, one will eventually be able to make it on one's own. Or perhaps he understood that this was the right approach to take with certain students individually.

David Blackwell was an intellectual giant. But he was modest and unassuming on a personal level. He always dressed properly in an aged jacket/suit, and he drove an old car that often invited jokes from students. While graduate students all over the world were learning about the Rao-Blackwell theorem, I never saw him teaching a graduate course. He enjoyed teaching undergraduate courses, and he placed great emphasis upon spending time on preparation to improve classroom teaching.

A great mind and a great spirit has departed. The world is a richer place because of his writings, but those of us who had the privilege of meeting him personally have benefited even more.

Madan L. Puri

If ever a definition of gravitas was sought, one need look no further than Professor David Harold Blackwell—the first black admitted to the National Academy of Sciences—who died Thursday, July 8, 2010, at age ninety-one. David had substance; he had weight (intellectual weight); he had depth; he was compassionate to the core; and he was genuine. In the course of a career marked by great accomplishments in a number of areas in statistics, probability, and mathematics, he had earned the reputation for intellectual rigor and integrity, and he commanded deep respect in the global academic community. He was an outstanding person, both intellectually and morally, and it is a pleasure to say a few words about this noble man.

I will not talk about his scientific accomplishments. First, they are too many, and second, they are well known. I will concentrate on David as a man.

On the basis of the personal association that I was fortunate to have had with Professor Blackwell, first as a student, and then as a colleague, I say with a sense of pride that David was a rare individual who possessed warmth, integrity, humility, intellectual passion, a commitment to students, faculty, colleagues, and friends alike. He had the courage to take a stand on important issues—the qualities that a creative, gifted scholar imbued with high moral sense is supposed to possess—and we were fortunate that we had such a person as our colleague. I am doubly fortunate to have had him as my teacher as well.

Good stories always invite us to slip into the shoes of other people—a crucial step in acquiring a moral perspective. Stories about friendships require taking the perspective of friends, taking them seriously for their own sake. In the best friendship, we see in perhaps its purest form a moral paradigm for all human relations. Professor Blackwell was a good friend. He had a unique talent, a rare gift of making everybody and anybody feel as though they were his best and most intimate friends. His steadfast friendship, his counsel, his magnanimity, and his example over many years placed me forever in his debt.

David was a living legend whose work not only influenced probability, statistics, and mathematics but has also had far-reaching implications for many fields, including economics. To quote him, "I've worked in so many areas—I'm sort of a dilettante. Basically, I'm not interested in doing research and I never have been. I'm interested in understanding, which is quite a different thing. And often to understand something you have to work it out yourself because no one else has done it." He received his Ph.D. in 1941 at the age of twenty-two from the University of Illinois under the direction of Professor J. L. Doob, and he directed sixty-five Ph.D.s. It is well known that in 1942 Jerzy Neyman of the University of California at Berkeley asked Doob if he was interested in going west. "No, I cannot come, but I have some good students, and Blackwell is the best," he replied. "But of course he's black," Doob continued, "and in spite of the fact that we are in a war that's advancing the cause of democracy, it may not have spread throughout our own land." Neyman then wanted to offer Blackwell a position, but the idea met with protest from the wife of the mathematics department chairman. She was a Texas native who liked to invite the math faculty to dinner occasionally, and she said she "was not going to have that darky in her house", according to Dr. Blackwell's recollection in an oral history interview. The job offer never came. Neyman had never forgotten Blackwell and finally hired him in 1954, and Blackwell would stay at Berkeley for the remainder of his career.

As a teacher he kept his expectations high. When the students walked into his class, they felt the spirit of excellence. He saw to it that no student was left behind. He made every effort to see that at the end of the day, the poor student became good and the good student became superior. Students were his audience. He never walked away from them as long as they did not walk away from him. As long as they were buying what he was selling, he kept on selling. He was the shining light.

Professor Blackwell received many honors in his lifetime, which include, among others, elected membership in the National Academy of Sciences (the first and the only black mathematician) and the American Academy of Arts and Sciences, president of the Institute of Mathematical Statistics, vice president of the American Statistical Association, vice president of the American Mathematical Society, and twelve honorary degrees of doctorate of science from Harvard, Yale, Carnegie Mellon, and other universities.

We live in a difficult world; we live in a complicated world; a demanding, unforgiving world, a world in which honesty and integrity are becoming rare commodities; where malice, jealousy, and self-centeredness motivate people to act in unprofessional, unethical, and undesirable manners. Ironically, and painfully, these things are happening even in the academic world, which is supposed to be the moral voice of humanity, but during these difficult and complicated times, in this demanding and unforgiving world, Professor Blackwell mastered the art of living the difficult life with integrity and style, and he made it look easy

Madan L. Puri is College of Letters and Sciences Distinguished Research Scholar and emeritus professor at Indiana University and Chair Professor at the King Fahd University of Petroleum and Minerals, Saudi Arabia. His email address is `puri@indiana.edu`.

and certainly desirable. He showed his strength by siding with the weak and helping the downtrodden, and with his loyal heart and with his purest hands, he executed faithfully the university, public, and professional trusts. Anybody who knew him, or met with him, respected him, revered him for his bright sunny nature and the saintly unselfishness by which or with which he discharged his responsibilities and earned the respect and trust of his friends and colleagues. The mathematical community in general, and those who knew him outside the mathematical community, loved him while he was living; they love him even now when he is gone.

Stephen Stigler

David Blackwell's research work places him in the pantheon of twentieth-century probability, game theory, and statistical inference, but it is as a teacher that I best recall him. To hear Blackwell lecture was to witness a master of the art. He was not charismatic; he spoke slowly and deliberately, and, when not writing on the board, used slight hand gestures with his palms toward the audience, to conjure up a shape or an entire space in our minds. His mastery came from the way he was thinking through the material with us at our speed and making his thoughts our thoughts. With simple gestures he could create an infinite-dimensional space in our minds and let us see with startling clarity how a result could follow—or, more accurately, how it would be absurd that it could fail to follow. It was magical—but it was lasting magic, since the knowledge imparted remained with us.

Even in a classroom the work he presented acquired a new flavor in the process. When he presented a wonderful generalization of von Neumann's minimax theorem designed for statistical games in function spaces, he named it after the author of an article he cited, but when I consulted that article later I could see that Blackwell had without comment recast it in a new form; the sparkling clarity of that form was a hallmark of this extraordinary mathematician's mind and style.

For a few years in the 1960s Blackwell taught an extremely elementary Bayesian statistics course to a very large audience of undergraduates. The book he wrote for that class is almost unknown today, but in his hands it was a gem. The insights he brought to a tired old syllabus were a wonderful reward to the students and teaching assistants alike. At the end of the term he invited the large team of TAs to his home at 5 PM and served soft drinks and a large pitcher of martinis. Many of the foreign TAs were new to this libation, and some took to it too enthusiastically, but the gracious host reined them in, and all felt the warmth of his collegial fellowship.

Blackwell was a second-generation beneficiary of an early 1930s grant from the Carnegie Corporation to Harold Hotelling at Columbia University. In 1932 Hotelling had supported Joe Doob, fresh from a Ph.D. in mathematics at Harvard and with no good job prospects, and introduced Doob to probability and statistics. A decade later Doob did the same for Blackwell at the University of Illinois. No Carnegie money was ever better spent.

Stephen Stigler is the Ernest DeWitt Burton Distinguished Service Professor of Statistics at the University of Chicago. His email address is stigler@uchicago.edu.

W. Sudderth

My first encounter with David Blackwell was as a student in his course on dynamic programming at Berkeley in the fall of 1965. There were, as I recall, about forty or so students from various applied areas, together with a few math types like me. The class met once a week in the evening for about two hours. David always arrived right on time, nattily dressed and sporting a bow tie. He would take a small piece of paper from his shirt pocket, glance at it briefly, and then, with no additional notes, lecture for about an hour. There was then a short break, after which David would look at the other side of the piece of paper before lecturing for the second hour.

The lectures were so clear that the applied students could understand and we math types could easily see that the arguments were airtight. David would often give an intuitive explanation for why a result should be true and then follow it with a rigorous proof. I still have my notes from the course and consult them almost every year to remind myself of an argument or a key example.

David held office hours at 8 AM. Since few students showed up at this early hour, I was able to see him a number of times with questions about dynamic programming and later on about my thesis problem and other matters. These meetings were always fruitful for me. David could always see quickly to the heart of a problem. Sometimes he knew the solution and, if he did not, he always had a good idea about where to look.

My thesis adviser Lester Dubins was a good friend of David's. Lester liked to work with finitely additive probability measures, and, following his lead, I worked with them, too. David was quite dubious of this because of the nonconstructive nature of purely finitely additive measures. He once remarked that he was impressed by all the

William D. Sudderth is professor of statistics at the University of Minnesota. His email address is bill@stat.umn.edu.

interesting results we were able to prove about these measures that do not exist.

On another occasion, when Roger Purves and I had been working a long time on an obscure measurability problem, we asked David whether he thought our endeavor was worthwhile. He said that when a problem arises naturally in a theory and is difficult to solve, its solution may well require new mathematical tools that will be useful for other purposes as well. Indeed, when, with the aid of Lester Dubins and Ashok Maitra, we finally found the answer to our problem, it did require new techniques that we were able to apply elsewhere.

David made seminal contributions to mathematical statistics, probability theory, measure theory, and game theory. He also found deep connections between game theory and descriptive set theory. As already suggested, he was a great teacher. His only failing, which I observed while serving on search committees at the University of Minnesota, was that he was too kind to ever write anything but a good letter of recommendation for a job candidate.

Yannis Yatracos

I came to meet David Blackwell in the 1978–1979 academic year, when I commenced my graduate work in statistics at UC Berkeley. I found him to be a very warm individual, exhibiting a positive attitude toward all students, and in particular newcomers. He made himself available to answer all kinds of questions regardless of time. During my stay at Berkeley, I had the opportunity to hold several discussions with him about the subject of statistics and the profession, the department there, academic careers, and life outside academia. He was always straightforward, informative, helpful, and generous in sharing his vast knowledge and experiences. In social and student-related issues and in departmental issues shared with students, he was in general more liberal than most of his colleagues, usually in agreement with Lucien Le Cam. Here is a token of remembrance of some instances of personal interaction with him. As a member of my Ph.D. thesis committee, he provided in my mailbox the solution to one of the questions I had asked him about related references. With regard to the potential employment of undergraduates as teaching assistants in statistics courses, the Statistics Graduate Students Association (SGSA), expressing serious concerns, created an ad hoc committee to handle the issue. I also participated, and a meeting of the ombudsman was arranged with the entire department. Professor Blackwell unequivocally stated to the ad hoc committee that the department had a financial obligation toward Ph.D. students until completion of the degree and that this obligation should be addressed. In 1983, before my graduation, I had an extensive discussion with him about the various models of academic careers and, not to my surprise, he supported the British model. At that time faculty ranks in the British universities were lecturer, senior lecturer, reader, and professor. Lecturers became permanent after a probationary period that normally required no more than three years. Promotion to senior lecturer was often based on prowess in teaching and administration. Promotion to reader was based on achievements in research and would usually precede promotion to professor. In a more recent email contact with him in June 2008, I sent a greeting note with some of my papers that he might be interested in. He replied immediately with kind and warm words, as he always did during the last thirty years. Berkeley students who came to know Professor David Blackwell will always remember him as the generous, kind, and warm person he was; he will be greatly missed.

Yannis Yatracos is professor of statistics at the School of Management and Economics of Cyprus University of Technology, Cyprus. His email address is `yannis.yatracos@cut.ac.cy`.

David Brillinger

I am one of the many whose careers and lives David Blackwell influenced in important ways.

My first contact with David was in 1958. I bought a copy of the Blackwell and Girshick (1954) book using part of my Putnam Prize money. From that work I learned the decision theory and Bayesian approaches to statistical problems. I also remember liking the group theory material.

My next contact with David came in spring 1961. He telephoned me at Princeton inviting me to Berkeley. The conversation ended with "If ever..." I didn't accept the invitation then as I had a postdoctoral fellowship to spend the following year in London, but I did not forget it. What happened eventually is that I became a lecturer and then a reader at the London School of Economics (LSE) for most of the 1960s. I did follow David's work, and I did think about Berkeley from time to time. One thing that I noticed was that David was typically spoken of with awe both in the United Kingdom and United States. I particularly remember that, in the mid-1960s, I went through David's 1951 paper "The range of certain vector integrals" when I was preparing a 1967 *Proceedings of the AMS* paper, "Bounded polymeasures and

David Brillinger is professor of statistics at the University of California, Berkeley. His email address is `brill@stat.berkeley.edu`.

associated translation commutative polynomial operators".

During the academic period 1967–1968, I spent nine months in Berkeley's Statistics Department on sabbatical leave from LSE. I found David to be larger than life, and also I finally met a nonaggressive Bayesian! Just after that visit he further helped my career when he communicated my 1969 paper, "An asymptotic representation of the sample distribution function", to the *Bulletin of the AMS*.

I became David's colleague in January 1970 when I joined the Berkeley faculty. There his collegiality, teaching, research, power-packed talks, committee work, treatment of students, and social conscience were role models for academic behavior. To mention one personal research example, his work with Lester Dubins that appeared in the 1983 *Proceedings of the AMS*, "An extension of Skorokhod's almost sure representation theorem", surely influenced my 1980 work, "Analysis of variance problems under time series models", *Handbook of Statistics 1*. In that paper Skorokhod representation results allowed formal development of asymptotic noncentral chi-squared and F distributions for various time series statistics.

David Blackwell has been there my whole academic life, and his contributions and style remain. It was a privilege to share conversations and experiences with him, for he was a major reason why I came to Berkeley. He helped me out continually when I was department chair.

I wish to end by mentioning that, in an encounter, David seemed always to have a pungent quip to offer. One I remember from the early 1980s is "Ronald Reagan likes strong trade unions—in Poland."

Leo A. Goodman

This statement, due to space constraints, will describe only two experiences that I had with David Blackwell. The first experience took place a very long time ago, and the second took place more recently.

After David received his Ph.D., he was given a one-year appointment as a postdoctoral fellow at the Institute for Advanced Study at Princeton. When his tenure at the Institute was drawing to a close, he applied for teaching positions at 105 historically black colleges and universities. He didn't apply to institutions that were not black institutions because it was assumed at that time that such institutions would not accept him because of his race. His first teaching job was at Southern University in Baton Rouge, Louisiana, and his second was at Clark College in Atlanta, Georgia. In 1944 he joined the mathematics department at Howard University in Washington, D.C., and he was promoted to full professor and head of the department in 1947. He stayed there until 1954. I was a faculty member in the statistics department at the University of Chicago beginning in 1950, and in 1951 or 1952 we invited David to become a professor in our department. We made him a good offer. I believe we were the first university that was not a black university to offer him a job. This was, as I have just noted, in 1951 or 1952.

Leo A. Goodman is Class of 1938 Professor in the departments of statistics and sociology at the University of California, Berkeley. His email address is `lgoodman@berkeley.edu`.

Photo courtesy of Statistics Dept., U. C. Berkeley.

Also at the 2009 birthday party, with (left to right) Erich Lehmann, Ching-Shui Cheng, Bin Yu, Cari Kaufman, and Juliet Shaffer. Others in the background are staff members and students.

He turned us down. Here is why. This is what he told me: He was born and grew up in a small town, Centralia, in southern Illinois right on the borderline of segregation. If you went a bit south of Centralia to the southern tip of Illinois, the schools were completely segregated in those days. Centralia had one school only for blacks, one school only for whites, and a few "mixed" schools. He attended one of the "mixed" schools. His family would sometimes travel north in Illinois from Centralia to visit relatives living in Chicago; and he could see, when he visited his relatives living there, what life was really like for black people living in Chicago. He told me that he would definitely prefer to live with his wife and children in Washington, D.C., where Howard University is located, than to live with them in Chicago.

David didn't accept our University of Chicago offer; but in 1954, he accepted an appointment at the University of California at Berkeley as a visiting professor for the 1954–1955 academic year. And starting with the 1955–1956 academic year, he was a professor in the statistics department at the University of California at Berkeley.

The second experience that I had with David, which I will describe next, dates from the late 1990s and early 2000s. I moved from being a faculty member at the University of Chicago to being a faculty member at the University of California at Berkeley in 1987, so David and I were colleagues from then on. In 1998 a best-selling book called *A Beautiful Mind* was published, and it inspired the making of a movie with the same name in 2001 that won four Academy Awards. The book was a biography of John Nash, the winner of a Nobel Memorial Prize in Economic Sciences. The prize was for the research that Nash had done on game theory when he was a graduate student in the mathematics department at Princeton University. Because of the book, the movie, and the Nobel Memorial Prize, interest in Nash was high for quite a few years—even, it seems, for example, among San Francisco's social elite, members of the Bohemian Club and Bohemian Grove. Albert (Al) Bowker, a devoted member of the Bohemian Club and Bohemian Grove—and also a well-known statistician and former chancellor at the University of California at Berkeley and a friend of David's and a friend of mine—invited David and me to speak about Nash at the Bohemian Club. Al invited David because David was an expert in game theory, and he invited me because John Nash and I had been graduate students at the same time in the mathematics department at Princeton. John and I were friends then, and we continued to be friends after leaving Princeton. On the evening when David and I spoke at the Bohemian Club, David spoke beautifully—as he always did. It was striking to see how well he was able to speak on game theory to this audience—members of the Bohemian Club—who were largely unfamiliar with this rather arcane subject. I think that the audience did gain some understanding of what game theory was about and why Nash's research was important. David and I had a good time, and our talks were well received. David was, simply, a great lecturer and teacher, as well as a gracious and interesting colleague and a sterling human being. We all miss him very much.

Juliet Shaffer

With the death of David Blackwell, following the death last year of Erich Lehmann, both at ninety-one years old, the senior level of statisticians at U.C. Berkeley is gone.

Erich and I were reasonably close to David. For many years we drove him to the joint Berkeley-Stanford colloquia when they were held at Stanford, giving us time for much conversation.

Erich told me that, when David first came to Berkeley, his large family could not find any place that would accommodate them, and they lived for some months camping out in a park. In David's interview in *Statistical Science* in 1986, he stated that the early discrimination he faced (before Berkeley) "never bothered me". Since the early Berkeley experience wasn't mentioned, it's not clear that he was similarly unbothered by this early Berkeley discrimination. I know that he was keenly aware of such issues later.

I talked with him from time to time about discrimination, mentioning things that had happened to me both as a woman and a Jew. I had the impression that he thought a lot about discrimination in general (not necessarily against him personally) in later years. I understand his initial unconcern very well. When I was first looking for jobs there were many ads, at least half, that stated "men only". It didn't bother me then—it was the way things were. Only later with the rise of feminism did I begin to see things differently. David was usually very unruffled, but I saw a rare strong reaction when I told him how the Georgia flag, which resembles the Confederate flag, bothered me as I saw it flying over a hotel in which I had just stayed in Atlanta. He mentioned that a white beggar on Telegraph Avenue had approached him for money while wearing a Confederate flag costume and how angry he had felt about that.

It must have been somewhat difficult being a Bayesian in the strongly-non-Bayesian Berkeley Statistics Department. I once mentioned to David that I was not very sympathetic to the Bayes approach but did have some interest in empirical Bayes. He noted that he didn't believe in empirical Bayes and showed that it didn't make sense when applied to a single inference. I remarked that it made sense in the context of a large number of similar inferences, to which there was no reply. I interpreted his reaction as illustrative of an aspect of his approach to statistics. He liked elegance and simplicity. Issues had to be clear in the very simplest of situations. Empirical Bayes didn't meet this test. He felt a Bayesian prior was necessary. His ability to clarify issues in simple and elegant ways was presumably what made him such an outstanding lecturer and teacher.

David was a kind and wonderful person, but he was also a very private person, and there was always the feeling of an inner core that couldn't be penetrated. I urged him many times to write his memoirs, but he never did.

Juliet Popper Shaffer is senior lecturer emerita of statistics at the University of California, Berkeley. Her email address is shaffer@stat.berkeley.edu.

Herman Chernoff

I first met David Blackwell in 1951 when we were both invited to visit the new Stanford University

Herman Chernoff is emeritus professor in the Department of Statistics at Harvard University and in the Department of Mathematics at MIT. His email address is chernoff@hustat.harvard.edu.

Department of Statistics. By then he was a recognized force in statistics, having contributed the Rao-Blackwell theorem on the use of sufficient statistics to derive efficient unbiased estimates and the Arrow-Blackwell-Girshick derivation of the optimality of the sequential probability ratio test.

The latter derivation was based on a plan proposed by Wald and Wolfowitz, the details of which suffered from a serious measurability difficulty. Arrow, Blackwell, and Girshick bypassed that problem by employing a backward induction argument, the success of which depended on the fact that a decision to be made in the distant future would have a negligible effect on the current expected value of the overall strategy. This backward induction argument was essentially the origin of dynamic programming. Blackwell used to claim that sequential analysis and dynamic programming were the same subject.

At the time we met, David and his wife Ann already had five of their eight children. Transporting his family by car across country was a major challenge requiring considerable discipline and planning at a time when professorial salaries were extremely limited. We were disappointed when David chose to go to Berkeley rather than to Stanford.

Ann and I had a vice in common. We both loved ice cream. My wife Judy noticed that at picnics, Ann had a definite tendency to overcount the consumers, as a result of which we always had an extra portion, which Ann would gracefully consume to avoid a battle among her children.

I have known many very smart people, including some Nobel laureates, but David had the greatest ability to take a complicated situation, scientific or personal, and explain the issues clearly and simply. This gift made him a great expositor and advisor. His book, *Basic Statistics*, was an extraordinary illustration of his ability to clearly and concisely explain the subject to beginners.

We had one major misunderstanding. He maintained that I had introduced him to the secretary problem, and I just as distinctly claim that he had introduced me to it. Judy and I both enjoyed visiting with David and Ann, and we were honored to be invited by David to the special dinner Harvard had for its recipients of the honorary doctorate.

Photo courtesy of Statistics Dept., U. C. Berkeley.

Blackwell at the 2009 birthday party. On the left behind him is Juliet Shaffer, and David Aldous is at the right. The others are staff members and students.

Persi Diaconis

David Blackwell was a brilliant, gentle man. I don't think I ever met anyone with so many IQ points fused into such an agreeable exterior. We had many areas of contact: Bayesian statistics, descriptive set theory, and his damned triangle problem. Let me briefly comment on each.

Persi Diaconis is professor of mathematics and statistics at Stanford University. His email address is `diaconis@math.stanford.edu`.

David had been converted (his phrase) to the Bayesian view during a walk with Jimmy Savage. Bayes made sense to David, and he made sense of it to others. Since not everyone is a Bayesian, some of us learn to speak both classical and Bayesian languages. Once, David heard me give a colloquium using both languages. Afterward, he gave me a really hard time! "Persi, it sounded as if you were apologizing for being a Bayesian. You don't have to apologize, it's the only sensible statistical theory."

Berkeley was a hotbed of descriptive set theory—analytic, coanalytic, and universally measurable sets were friends of Blackwell, Dubins, and Freedman. I learned the subject from Freedman and wrote a small paper with Blackwell: the *American Mathematical Monthly* had published a longish paper constructing a nonmeasurable tail set in coin-tossing space. We noticed that the standard construction of a nonmeasurable set due to Vitali also had this property. We sent a one-page note to the *Monthly* and received a scathing referee's report in return: "Why would you send a paper about such junk to the *Monthly*?" Clearly, not everyone likes measure theory. We published our paper elsewhere. I recently gave a talk on it at Halloween (when the monsters come out).

I had dinner with David, Erich Lehmann, Julie Schaffer, and Susan Holmes about a year before he died. You don't ask someone in his high eighties if he's still thinking about math. However, David had taken up computing late in life, and I asked if he was still at it. He answered with a loud pound on the table and "Yes, and damnit I'm stuck." He explained: "Take any triangle in the plane. Connect the three vertices to the midpoints of the opposite sides; the three lines meet in the middle to give the barycentric subdivision into six triangles. If you do it again with each of the six little triangles, you get thirty-six triangles, and so on." He noticed that most of the triangles produced get flat (the proportion with largest angle greater than 180 degrees minus epsilon tends to one). He was trying to prove this and had gotten stuck at one point. His proof idea was brilliant: he turned this geometry problem into a probability problem. Construct a Markov chain on the space of triangles by picking one of the six inside, at random. He defined a function on triangles that was zero at perfectly flat triangles and that he believed was superharmonic. Nonstandard theory shows the iteration tends to zero; hence the average tends to zero, and so "most triangles are flat". The proof of superharmonicity was a trigonometric nightmare, and he had not been able to push it through. I couldn't believe that such a simple fact about triangles was hard (it was). I tricked several colleagues into working it through by various complex arguments. After a lot of work, we found the result in a lovely paper by Barany, Beardon, and Carne (1996). Blackwell's approach remains a tantalizing possibility.

Murray Rosenblatt

I had only occasional contact with David Blackwell through the years. But I always found him to be a warm, gracious person with a friendly greeting. He entered the University of Illinois at Urbana–Champaign in 1935 at the age of sixteen and received a bachelor's degree in mathematics in 1938 and a master's degree in 1939. Blackwell wrote a doctoral thesis on Markov chains with Joseph L. Doob as advisor in 1941. Two earlier almost-contemporary doctoral students of J. L. Doob were Paul Halmos, with a doctoral degree in 1938, and Warren Ambrose, with the degree in 1939.

Blackwell was a postdoctoral fellow at the Institute for Advanced Study for a year from 1941 (having been awarded a Rosenwald fellowship). There was an attempted racist intervention by the then-president of Princeton, who objected to the honorific designation of Blackwell as a visiting fellow at Princeton (all members of the Institute had this designation). He was on the faculty of Howard University in the mathematics department from 1944 to 1954. Neyman supported the appointment of David Blackwell at the University of California, Berkeley, in 1942, but this fell through due to the prejudices at that time (see [4]). However, in 1955 David Blackwell was appointed professor of statistics at UC Berkeley and became chair of the department the following year.

Blackwell wrote over ninety papers and made major contributions in many areas—dynamic programming, game theory, measure theory, probability theory, information theory, and mathematical statistics. He was an engaging person with broad-ranging interests and deep insights. He was quite independent but often carried out research with others. Interaction with Girshick probably led him to research on statistical problems of note. Researches with K. Arrow, R. Bellman, and E. Barankin focused on game theory. Joint work with A. Thomasian (a student of his) and L. Breiman was on coding problems in information theory. He also carried out researches with colleagues at UC Berkeley, such as David Freedman, Lester Dubins, J. L. Hodges, and Peter Bickel. The Rao–Blackwell theorem dealing with the question of optimal unbiased estimation is due to him.

He was elected the first African American member of the National Academy of Sciences, USA, and received many other awards. He was a distinguished lecturer. We're thankful that he survived the difficulties that African Americans had to endure in a time of great bias (in his youth). He was a person of singular talent in the areas of statistics and mathematics.

I shall describe limited aspects of the research of Blackwell and Dubins [2] on regular conditional distributions (see Doob [1] for a discussion of conditional probability). This was an area that Blackwell often found of interest. Given a measurable space $(\Omega, \mathcal{B})$ with $\mathcal{A}$ a sub σ-field of the σ-field $\mathcal{B}$, call P defined on $\Omega \times \mathcal{B}$ a *regular conditional distribution* (r.c.d.) *for* $\mathcal{A}$ *on* $\mathcal{B}$ if for all $\omega \in \Omega$, $B \in \mathcal{B}$,

(1) $P(\omega, \cdot)$ is a probability measure on $\mathcal{B}$.
(2) For each $B \in \mathcal{B}$, $P(\cdot, B)$ is $\mathcal{A}$-measurable and related to the probability distribution via

$$\int_A P(\omega, B)\, \mathrm{d}P(\omega) = P(A \cap B)$$

for $A \in \mathcal{A}$, $B \in \mathcal{B}$.

Such regular conditional distributions do not always exist. But assuming existence, call it *proper* if

$$P(\omega, A) = 1$$

whenever $\omega \in A \in \mathcal{A}$.

The probability measure P on $\mathcal{A}$ is called extreme if $P(A) = 0$ or 1 for all $A \in \mathcal{A}$. An $\mathcal{A}$-atom is the intersection of all elements of $\mathcal{A}$ that contain a given point of Ω. If for $A \in \mathcal{A}$, $P(A) = 1$, P is said to be *supported* by A.

Then we have:

Murray Rosenblatt is emeritus professor of mathematics at the University of California, San Diego. His email address is mrosenblatt@ucsd.edu.

Theorem. *Assume $\mathcal{B}$ is countably generated. Then each of the conditions implies the successor.*

(a) *There is an extreme countably additive probability measure on $\mathcal{A}$ that is supported by no $\mathcal{A}$-atom belonging to $\mathcal{A}$.*
(b) *$\mathcal{A}$ is not countably generated.*
(c) *No regular conditional distribution for $\mathcal{A}$ on $\mathcal{B}$ is proper.*

This result shows that, for Ω the infinite product of a separable metric space containing more than one point, neither the tail field, the field of symmetric events, nor the invariant field admit a proper r.c.d (regular conditional distribution). They weaken the countable additivity condition of an r.c.d. to finite additivity and add (1) to obtain the notion of a *normal conditional distribution* and arrive at sufficient conditions for existence. Later related research by Berti and Rigo [3] considers the r.c.d.s with appropriate weakenings of the concept of proper.

References

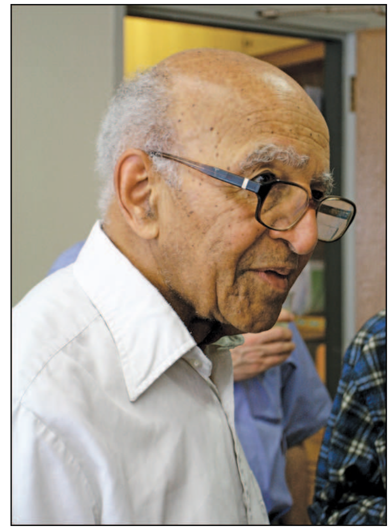
[1] J. L. Doob, *Stochastic Processes*, John Wiley, 1953.
[2] D. Blackwell and L. Dubins, On existence and non-existence of proper, regular conditional distributions, *Ann. Prob.* **3** (1975), 741–752.
[3] P. Berti and P. Rigo, 0–1 laws for regular conditional distributions, *Ann. Prob.* **35** (2007), 649–662.
[4] National Visionary Leadership Project, Oral History Archive, David Harold Blackwell.

Francisco J. Samaniego

Because of David Blackwell's widely recognized genius, as evidenced in his path-breaking research, the many creative ideas he generously shared with students and colleagues, his election to the National Academy of Sciences, and his receipt of the coveted Berkeley Citation upon his formal retirement from the faculty, it is perhaps understandable that another facet of his remarkable career would be less known and less universally celebrated. This facet was his extraordinary ability to teach mathematics and statistics in new, clear, and compelling ways. There is a good deal of evidence that may be advanced in support of the proposition that Blackwell was an exceptional teacher. We would be remiss if this aspect of his wonderfully successful career was overlooked in the present overview of his work. While our discussion of Blackwell's teaching is necessarily brief, we hope that we will leave no doubt among readers of this piece that Blackwell was a preeminent teacher and mentor.

The most telling evidence of Blackwell's teaching prowess is simply the testimonials from students and colleagues that exist in a number far too great to attempt a comprehensive summary. Suffice it to say that many of his students considered him to be the finest instructor that they ever had the privilege to study with.

Francisco J. Samaniego is Distinguished Professor of Statistics at the University of California, Davis. His email address is `fjsamaniego@ucdavis.edu`.



David Blackwell, 2009.

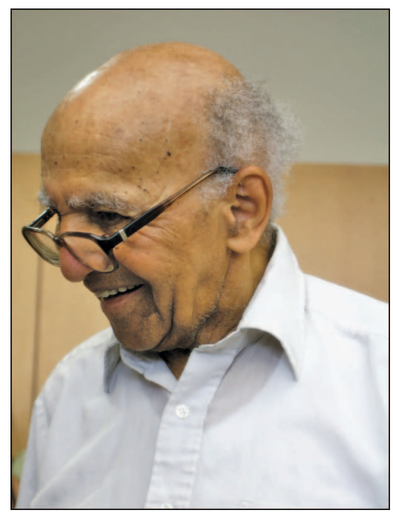


Photo courtesy of Statistics Dept., U. C. Berkeley.

His style was unfailingly engaging, as it was his custom to share his natural curiosity with his students, explaining not just the "how" associated with a statistical procedure but the "why" as well, along with the motivation for the ideas involved and the (often surprising) connections with other ideas usually of interest in their own right. It was a pleasure to hear him speak. One generally came away from a lecture by David Blackwell both impressed with his mastery of the subject and intrigued by questions he had left his audience to think about. His colleagues at Berkeley looked to him as a model and often sought his advice on the best way to present a given topic (as well as on a host of other matters, personal and professional). In spite of his wonderful gifts as a teacher, Blackwell was very modest about his skills and would give his advice as if it was a tentative, off-the-cuff suggestion. Once, in a reception prior to a seminar he presented at UC Davis, a former student of his asked him, "David, what do you do when you've presented an idea in a way that you consider to be 'just right', and a student raises his hand and says 'I didn't get that'?" Without missing a beat, David answered, "Well, I just repeat exactly what I just said, only louder."

David Blackwell's well-honed teaching instincts were as evident in his writings as they were in the classroom. Two of his many published "notes" come to mind in this regard. These notes appeared in the *Annals of Statistics* volume in which Thomas Ferguson presented his now celebrated paper on Bayesian nonparametrics (an idea, by the way,

About the Cover

Very special functions

The cover was suggested by this issue's article by Daniel Lozier and others on the National Institute of Standards and Technology's (NIST) mathematical functions project. It amounts to an assembly of screenshots taken from the associated online Digital Library of Mathematical Functions.

Graphics are a distinctive part of this project. When we asked Bonita Saunders of the NIST Mathematical Software Group to say something about its production, she replied:

> We first thought the development of graphics for the DLMF would be fairly straightforward: Create the graphs using a commercial or free package and export the data to a format that could be viewed on the Web. While this worked well for 2D graphs of function curves, it did not work for 3D graphs. A considerable amount of user input was needed to plot accurate graphs of function surfaces in most packages. This has improved considerably in recent years, but in many cases it is still difficult to export the data to suitable formats, especially if the goal is interactive viewing on the Web.
>
> To eliminate or lessen the severity of our plotting problems, we designed custom-made computational meshes to properly clip the surfaces and capture key function features such as zeros, poles, branch cuts, and other singularities. To ensure the accuracy of all function data, we computed the functions using at least two different methods. We designed our own translators to export the 3D data to file formats such as VRML (Virtual Reality Modeling Language) and X3D, which allow a user to manipulate 3D scenes and objects on the Web with a free downloadable viewer. We wrote code to supplement the standard rotate, zoom, and pan capabilities with user options—to change the color map, vary the scaling of the surface, create density plots, change the look of the axes, and interactively move a cutting plane through the surface in each coordinate direction.
>
> We expect to continually enhance the graphics on the website. In particular, sometime in the future we hope to offer an option that allows users to view the interactive graphics inside a webpage without the need of a special viewer.

Our thanks also to Brian Antonishek, also of NIST, for much help in assembling the cover.

—Bill Casselman
Graphics Editor
(notices-covers@ams.org)

that he acknowledged as grounded in his discussions with Blackwell). In a note entitled "Discreteness of Ferguson Selections", Blackwell gave an elementary proof of the discreteness of draws from a Dirichlet process, shedding much light on this particular characteristic of Dirichlet processes (which had been proven by Ferguson in an *Annals of Probability* paper using much more complex arguments). In the same AoP issue, Blackwell and MacQueen presented an alternative derivation of the Dirichlet process using a lovely and quite intuitive construction involving Pólya urn schemes. The latter paper has led to much fruitful research in Bayesian nonparametrics. Both papers contained useful techniques, but their greatest contribution was, without doubt, the clarification of the properties and potential of Ferguson's Dirichlet process.

Blackwell published the elementary textbook *Basic Statistics* in 1969. The book is unique in the field and is recommended reading both for students just being exposed to the subject and, we dare say, for the statistics community as a whole. It is no exaggeration to refer to the book as a "gem". In the book, Blackwell covered the "standard topics" found in an introductory course—elementary probability, the binomial and normal models, correlation, estimation, prediction, and the chi-square test for association. The treatment of these topics was, however, fresh and crisp, with most of the ideas motivated by thinking about drawing balls from urns. For example, he chose to introduce the idea of Bayesian point estimation through the problem of estimating the number of fish in a pond via a mark-recapture experiment. Although the mathematical level of the book was intentionally low, the conceptual reach was much broader than what one usually finds at the introductory level. In his preface, Blackwell describes his approach as "intuitive, informal, concrete, decision-theoretic and Bayesian". He took on the notions of probability densities, mean squared error, multiple correlation, prior distributions, point estimation, and the normal and chi-square approximations, all with the very modest expectation that the students reading the book "could do arithmetic, substitute in simple formulas, plot points and draw a smooth curve through plotted points". He was true to his promise of making statistics accessible to anyone who had only these skills. Perhaps the most remarkable thing about this book is that Blackwell managed to pack a treasure trove of ideas into 138 pages, divided into sixteen chapters and containing 118 problems and their solutions. He had a gift for getting to the core of the topics he wrote or taught about. This book is a lovely example of that gift in action.