Foreword: Special Issue on Variational Analysis, Optimization, and their Applications

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This issue of *Mathematical Programming, Series B*, is dedicated to the life and work of R. Tyrrell Rockfellar on the occasion of his 70th birthday. Terry (as he is know to his friends) was born in Wisconsin in 1935, and received his Bachelor and Doctoral degrees at Harvard University. The title of his 1963 Ph.D. thesis, "Convex functions and dual extremal problems," indicated the area of his first major contributions to mathematics. In 1967, after short stays in Copenhagen, Texas, and Princeton, he accepted an appointment at the University of Washington, an institution that has ever since been his "fixed point."

The presence of such mathematical luminaries as V. Klee, R. Phelps, A. Goldstein, L. Hewitt, and I. Namioka must have played a role in his decision to move to Seattle, but I suspect that the proximity of the Cascade Mountains and the Olympic Peninsula provided an equally strong enticement. However, he didn't quite stay put! Over the years, he visited and, in some cases, developed strong relationships with other institutions: Université de Grenoble, University of Colorado (Boulder), the International Institute for Systems Analysis (Laxenburg, Austria), Universidad de Chile (Santiago) and University of Florida (Gainesville) where he is now an Adjunct Professor.

Terry had been a prodigiously productive mathematician. He has presented hundreds of conference talks, often as a plenary speaker, served on the editorial board of nearly all the leading journals in the fields in which he has been active, and written nearly two hundred papers and five books. In 1982, the Mathematical Programming Society and the Society for Industrial and Applied Mathematics (SIAM) honored him with the first G. B. Dantzig Prize. In 1992, he received the von Neumann prize from SIAM, and in 1998 the Lanchester prize from INFORMS for his book *Variational Analysis* that I co-authored. In 2004, he was recognized as a Pioneer of Stochastic Programming at the Tenth International Conference on Stochastic Programming. He has received Honorary Doctorates from the Universities of Groningen (The Netherlands), Montpellier (France), Santiago (Chile) and Alicante (Spain).

Terry's book *Convex Analysis*, published in 1970, has had an enormous impact not only in mathematics, but also in economics, engineering, and even physics. Duality played an inportant role in Terry's approach and it continues, to this day, to be one of his favorite tools when dealing with a variety of problems.

At the time, few were aware, perhaps not even the author, that it was the first step in a mathematical revolution. The superb differential calculus tools pioneered by Fermat, and reaching its full development in the writings of Newton and Leibniz in the 17th century, have been an indispensable component of the scientific and technological advances of the last three centuries. However, these tools come with certain limitations: The functions had to be differentiable and their domain of definition must be an open set or a differentiable manifold. Many mathematical models arising from tehnological advances of the past century no longer fit the Newton-Leibniz paradigm. Starting with a subdifferential calculus for convex functions, Rockafellar went on to develop a calculus for nonsmooth functions whose domain of definition could be practically any set, but in particular a closed set defined by both equations and inequalities.

Rockafellar's first stab at developing this theory came by proxy. In the early 1970s, he suggested to one of his student, F. Clarke, a definition that might be appropriate for Lipschitz continuous functions. This definition lead to Clarke's thesis and this new paradigm was then exploited by a number of researchers in a variety of settings, including optimal control theory. To this point, convexity had played a central role, but in a series of articles written in the late 1970s, Rockafellar laid down the foundations of subdifferential calculus that placed no such restrictions on the properties of the functions or on their domain of definition; the umbilical cord with convexity was irremediably cut when the work of B. Mordukhovich came to his attention in the late 80's. There remained only to make the connection with variational approximation (convergence), and this he did in his work on second-order subdifferentiability in the early 1980s. There remained many applications and extensions to explore: improving the calculus rules, making the connection with the classical theory, and so on. To this day, Rockafellar continues his role of scientific locomotive in this area and in its applications and related fields.

Those not familiar with Rockafellar's work might expect that we can conclude a review of his work at this high point, lauding him as the one who, after three centuries, liberated us from the restrictions that came with the Fermat-Newton-Leibniz calculus. But even such acclaim shortchanges the overall impact of his work. He has also made crucial contributions in many other important areas, including the theory and applications of integral functionals, the calculus of variations and optimal control, network flows, nonlinear optimization (including augmented-Lagrangian and proximal-point algorithms), stochastic optimization, maximal monotone operators and, most recently, mathematical finance and the study of risk measures.

I should add, on a more personal note, that I have known Terry for more than forty years, and it has been a great pleasure to have him as a friend and an occasional collaborator. Many happy returns, Terry!

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