Book review: Topics in Complex Analysis, by Dan Romik

The optimal sphere packing problem in $\mathbb{R}^n$ concerns the following issue: How can one pack solid balls of radius r inside a sphere of radius R so that the packed space occupied by the balls is maximal? In 1611, Johannes Kepler conjectured that in three dimensions the maximal packing was achieved by placing the centers of the balls on a face-centered cubic lattice (the familiar stacking of cannon balls, or oranges in the market), resulting in a packing density of $\pi/3\sqrt{2}$ (as $R \to \infty$). This conjecture was proved eventually nearly four hundred years later by Thomas Hales. In two dimensions, optimal packing is achieved on a hexagonal lattice with packing density $\pi/\sqrt{12}$, a result finally proved in 1940.

What about packing in other dimensions n? In one of those mysterious and magical moments in mathematics, calculations of Cohn and Elkies indicated that dimensions 8 and 24 were special, and that a computation of the optical packing densities in these dimensions was in the cards. In remarkable work in 2016, Maryna Viazovska proved that the optical packing density in $\mathbb{R}^8$ was $\pi^4/384$, with the centers of the balls placed on the lattice E_8. An analogous result for $n = 24$ was obtained soon afterwards. A key role in Viazovska's proof is played by modular functions, one of the most beautiful constructs emerging from elliptic function theory.

Romik's text is a well-written, easily accessible introduction to the basics of complex function theory, but with a focused eye on elliptic function theory, and from then on to the theory of modular functions. The high point of the book is a very elegant presentation of the proof of Viazovska's theorem, using the modular function theory developed in earlier chapters. The text also has a useful Appendix explaining the basic of lattice packing theory.

Romik's book is a great service to the mathematical community, making Viazovska's wonderful result, for which she won the Fields Medal in 2022, easily accessible to the mathematical public. Romik's book is close in spirit to his earlier very successful text, "The Surprising Mathematics of Longest Increasing Subsequences", in which he broke down and clarified the intricacies of the final resolution of Ulam's longest increasing subsequence problem.

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