The Surprising Mathematics of Longest Increasing Subsequences
Cambridge University Press, 2015

Errata

1. Page 14: the random variables S_n defined here (and mentioned on lines 1, 2, 11 and 14) cause a conflict with the identical notation S_n used to denote the symmetric group of order n. They should be relabeled by a different letter, e.g., Z_n.

2. Page 72, Exercise 1.6: the fourth line of the exercise text is badly formatted due to an errant LaTeX command. The initial part of the exercise should read:

   > (Lifschitz-Pittel [75]) Let X_n denote the total number of increasing subsequences (of any length) in the uniformly random permutation σ_n. For convenience, we include the empty subsequence of length 0, so that X_n can be written as $X_n = 1 + \sum_{k=1}^{n} X_{n,k}$ where the random variables $X_{n,k}$ are defined in the proof of Lemma 1.4 (p. 9).

3. Page 76, Exercise 1.15: change "$Q(n) = \sum\limits_{k=1}^{n} q(n)$" to "$Q(n) = \sum\limits_{k=1}^{n} q(k)$".

4. Page 76, Exercise 1.15(a): change "Prove that $q(n,k) \le \frac{1}{k!}\binom{n+k-1}{k-1}$" to "Prove that $q(n,k) \le \frac{1}{k!}\binom{n-1}{k-1}$". (Note: the bound $q(n,k) \le \frac{1}{k!}\binom{n-1}{k-1}$ also implies that $q(n,k) \le \frac{1}{k!}\binom{n+k-1}{k-1}$, so the uncorrected version of the exercise is still technically correct.)

5. Page 90: in equation (2.13), change the numerator of the fraction on the right-hand side from "$\prod_{1\le i,j\le d}(p_i - p_j)(q_i - q_j)$" to "$\prod_{1\le i<j\le d}(p_i - p_j)(q_i - q_j)$".

6. Page 118: in the equation immediately above Lemma 2.24, a factor of $e^{-\frac{2}{3}x^{3/2}}$ is missing.

7. Page 118: in Lemma 2.24, display (2.75) should read:

$$|\mathbf{A}(x,y)| \le C \exp\left(-\tfrac{1}{2}(x^{3/2} + y^{3/2})\right) \tag{2.75}$$

Date last updated: January 15, 2021

8. Page 119: in the proof of Lemma 2.25, display (2.76) should read:

$$\begin{aligned}\left|\det_{i,j=1}^{n}\Big(\mathbf{A}(x_i,x_j)\Big)\right| &= \exp\left(-\sum_{j=1}^{n} x_j^{3/2}\right)\left|\det_{i,j=1}^{n}\Big(e^{(x_i^{3/2}+x_j^{3/2})/2}\mathbf{A}(x_i,x_j)\Big)\right| \\ &\le n^{n/2}C^n\exp\left(-\sum_{j=1}^{n} x_j^{3/2}\right), \qquad (2.76)\end{aligned}$$

9. Page 122: the display in which E_1 is bounded should read:

$$E_1 \le C\exp\left(-\tfrac{1}{2}(x+T)^{3/2}-\tfrac{1}{2}(y+T)^{3/2}\right),$$

10. Page 126: display (2.89) should read:

$$\begin{aligned}|a_n(T,\infty)| &\le \int_T^\infty \cdots \int_T^\infty C^n e^{-\sum_{j=1}^n x_i^{3/2}} \det_{i,j=1}^{n}\Big(e^{\frac{1}{2}x_i^{3/2}+\frac{1}{2}x_j^{3/2}}\mathbf{A}(x_i,x_j)\Big)\,dx_1\ldots dx_n \\ &\le \left(C\int_T^\infty e^{-x^{3/2}}\,dx\right)^n n^{n/2} \le C^n e^{-nT} n^{n/2}. \qquad (2.89)\end{aligned}$$

11. Page 128: In Lemma 2.31, change "Let $P_1 \ge P_2 \ge P_3 \ge \ldots$" to "Let $P_0 \ge P_1 \ge P_2 \ge \ldots$".

12. Page 129: the line following display (2.97) should be changed from "By Lemma 2.32, for fixed t the sequence $P_1(t), P_2(t), \ldots$ is nonincreasing." to "By Lemma 2.32 (and its trivial extension to cover the case $n = 0$), for fixed t the sequence $P_0(t), P_1(t), P_2(t), \ldots$ is nonincreasing."

13. Page 129: in the line following display (2.98), change "But by Theorem 2.47" to "But by Theorem 2.29".

14. Page 134: the first display in the proof of Lemma 2.34 should read:

$$|F_2(t)-1| \le \sum_{n=1}^\infty \frac{C^n n^{n/2}}{n!}\left(\int_t^\infty e^{-x^{3/2}}\,dx\right)^n \le \sum_{n=1}^\infty \frac{C^n e^n}{n^{n/2}} e^{-nt} = O(e^{-t}).$$

The display following it should read:

$$\begin{aligned}|\mathbf{H}(x,y,t)-\mathbf{A}(x,y)| &\le \sum_{n=1}^\infty \frac{(n+1)^{(n+1)/2}C^{n+1}}{n!} e^{-x^{3/2}-y^{3/2}}\left(\int_t^\infty e^{-u^{3/2}}\,du\right)^n \\ &\le e^{-x-y}\sum_{n=1}^\infty \frac{(n+1)^{(n+1)/2}C^{n+1}}{n!} e^{-nt},\end{aligned}$$

15. Page 155, Exercise 2.22: An accessible derivation of Nicholson's approximation is given in Section 4.4 of the lecture notes "Integrable probabilities: around the Longest Increasing Subsequence problem" by Jérémie Bouttier, available at `https://www.normalesup.org/~bouttier/coursM2Lyon/notes_coursM2Lyon.pdf`.

16. Page 251, line 10−: change "$u(x,0) = u_(x)$" to "$u(x,0) = u(x)$".

17. Page 292, line 4−: change $p_0 = \langle 1,1\rangle_w^{1/2}$ to $p_0 = \langle 1,1\rangle_w^{-1/2}$

18. Page 295: in line 11−,

$$p_n(x) = (A_n x + B_n)p_{n-1}(x) - D_n p_{n-2},$$

to

$$p_n(x) = (A_n x + B_n)p_{n-1}(x) - D_n p_{n-2}(x),$$

and in the sentence on the following line that starts "The value of D_n can be found by assuming inductively that (5.23) holds and writing", delete the words "assuming inductively that (5.23) holds and".

A few lines further down, the last line of the proof of Lemma 5.14 should be changed to: "which gives that $D_n = A_n \frac{\kappa_{n-2}}{\kappa_{n-1}} = \frac{\kappa_n \kappa_{n-2}}{\kappa_{n-1}^2} = C_n$ for $n \geq 2$, as claimed. For $n = 1$ it is not necessary to show that $D_1 = C_1$, since p_{-1} was defined as the zero polynomial. "

19. Page 296, line 4: change the equation

$$(\mathbf{K}_n f)(x) = \int_{\mathbb{R}} \mathbf{K}_n(x,y) f(y)\, dy$$

to

$$(\mathbf{K}_n f)(x) = \int_{\mathbb{R}} \mathbf{K}_n(x,y) f(y) w(y)\, dy$$

20. Page 296, equation (5.24): change "$\left(\sum_{i=1}^m u_i u_j p_k(x_i)\right)^2$" to "$\left(\sum_{i=1}^m u_i p_k(x_i)\right)^2$".

21. Page 302, equation (5.31): in this equation, change $L_n(x)$ to $L_n^\alpha(x)$ and $L_m(x)$ to $L_m^\alpha(x)$.