Edge enhancement with directional wavelet transform

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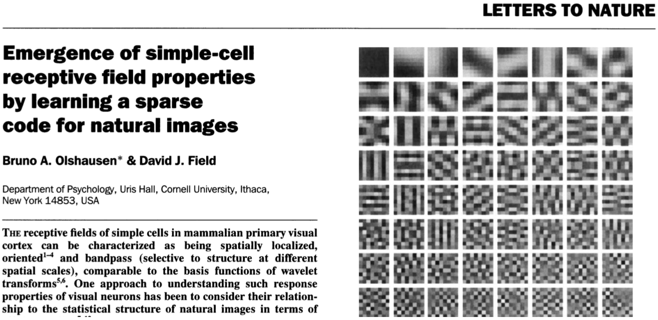
An efficient representation for an image

- In neuropsycological studies, the importance of directional sensitivity in the efficient processing of natural images by the human brain has been a major finding, as in a seminal work of Field and Olshausen (1996)[1].
- The research for efficient representations for images has also been developed in the both fields of harmonic analysis and signal processing.



LETTERS TO NATURE

Emergence of simple-cell receptive field properties by learning a sparse code for natural images

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The receptive fields of simple cells in mammalian primary visual cortex can be characterized as being spatially localized, oriented[1–4] and bandpass (selective to structure at different spatial scales), comparable to the basis functions of wavelet transforms[5,6]. One approach to understanding such response properties of visual neurons has been to consider their relationship to the statistical structure of natural images in terms of

[1] B. Olshausen and D. Field, Emergence of simple-cell receptive field properties by learning a sparse code for natural images, Nature **381**, 607–609 (1996).

Introduction

- Wavelet-based methods have shown substantial success in multiscale image analysis including edge detection, compression, and denoising.
- However, wavelets do not provide good directional selectivity, which results in some failure in geometrical image analysis.
- Several directional wavelet-based methods have been proposed, such as dual-tree complex wavelets, curvelets, contourlets, and shearlets.
- One of the key ideas behind these methods is to allow redundancy in their construction, which allows for more flexibility in the design of a wavelet transform, such as having good directional selectivity.
- Although they frequently outperform the traditional discrete wavelet transform (DWT) in geometrical analysis, the redundancy makes a transform or a system computationally expensive.

Our work

- We have proposed a new wavelet-based transform, DLWT (directional lifting wavelet transform).
  - Pure discrete setting on a plane.
  - 2D discrete wavelet transform with directional selectivity $N = 12$.
  - It has good tradeoff between directional selectivity and redundancy.

	SWT	DT$\mathbb{C}$WT	**DLWT**
Directions	$N = 3$	$N = 6$	$N = 12$
Redundancy	$N \times J + 1$	4	$(N \times J + 1)/4$

- We also have shown that the directional property of the DLWT plays an important role in the analysis of image edge components.

- In this talk, taking advantage of its directional selectivity, we show properties of edge analysis using the DLWT.

Preliminary

- Let $\{c_j[\boldsymbol{t}]\}_{\boldsymbol{t}\in\mathbb{Z}^2}$ be an image with resolution level $j \in \mathbb{N} \cup \{0\}$.
- The DWT decomposes an image $\{c_j[\boldsymbol{t}]\}_{\boldsymbol{t}\in\mathbb{Z}^2}$ into its coarse component $\{c_{j-1}[\boldsymbol{t}]\}_{\boldsymbol{t}\in\mathbb{Z}^2}$ and three detail components $\{d_{k,j-1}[\boldsymbol{t}]\}_{\boldsymbol{t}\in\mathbb{Z}^2,k=1,2,3}$, which consist of horizontal, vertical and diagonal degrees. This can also be written as

$$\{c_j[\boldsymbol{t}]\}_{\boldsymbol{t}\in\mathbb{Z}^2} \mapsto \{d_{k,j-1}[\boldsymbol{t}],\, c_{j-1}[\boldsymbol{t}]\}_{\boldsymbol{t}\in\mathbb{Z}^2,k=1,2,3}\,.$$

- We say that the directional selectivity is $N = 3$. Each component has a half resolution.
- This decomposition can be iterated to an arbitrary decomposition level $J \geq 1$ with the resulting coarse component as a new signal.

DLWT: Directional lifting wavelet transform

- As a result, the following sequences of coefficients are obtained by the DWT:

$$\{c_j[\boldsymbol{t}]\}_{\boldsymbol{t}\in\mathbb{Z}^2} \mapsto \{d_{k,j-1}[\boldsymbol{t}],\, d_{k,j-2}[\boldsymbol{t}],\, \ldots,\, d_{k,j-J}[\boldsymbol{t}],\, c_{j-J}[\boldsymbol{t}]\}_{\boldsymbol{t}\in\mathbb{Z}^2, k=1,2,3}\,.$$

- The DLWT also gives a similar decomposition but it has 12 directional components. The J-th level decomposition with the DLWT can be written by

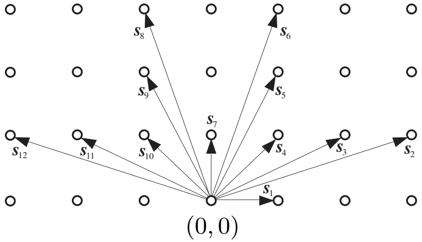
$$\boxed{\{c_j[\boldsymbol{t}]\}_{\boldsymbol{t}\in\mathbb{Z}^2} \mapsto \{d_{k,j-1}[\boldsymbol{t}],\, d_{k,j-2}[\boldsymbol{t}],\, \ldots,\, d_{k,j-J}[\boldsymbol{t}],\, c_{j-J}[\boldsymbol{t}]\}_{\boldsymbol{t}\in\mathbb{Z}^2, k\in D}\,,}$$

  where $D = \{\ell \in \mathbb{Z} \mid 1 \le \ell \le N\}$ with $N = 12$.

- This is a redundant transform, but an efficient computational algorithm owing to the lifting implementation is available.

- We define directional vectors $\{\boldsymbol{s}_m \in \mathbb{Z}^2\}_{m=0}^{N}$ $(\boldsymbol{s}_0 = \boldsymbol{0})$ that represent the following approximated angles

$$\boxed{\theta \approx \left(\frac{180\,(d-1)}{N}\right)^{\circ}, \quad d \in D}.$$



$$N = 12 \Rightarrow \theta \approx \{0^\circ, 15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ, 90^\circ, 105^\circ, 120^\circ, 135^\circ, 150^\circ, 165^\circ\}.$$

Modified lifting scheme in 2D

For a signal $\{x[\boldsymbol{t}]\}_{\boldsymbol{t}\in\mathbb{Z}^2}$, we define two bounded linear operators, **prediction operators** $\{\mathcal{P}_k\}_{k\in D}$ and an **update operator** $\{\mathcal{U}\}_{k\in D}$ by

$$(\mathcal{P}_k\, x)\,[\boldsymbol{t}] := (p * x)\,[\boldsymbol{t}] = \sum_{\boldsymbol{\ell}\in\mathbb{Z}^2} p_k[\boldsymbol{\ell}]\, x[\boldsymbol{t}-\boldsymbol{\ell}],$$

$$(\mathcal{U}_k\, x)\,[\boldsymbol{t}] := (u * x)\,[\boldsymbol{t}] = \sum_{\boldsymbol{\ell}\in\mathbb{Z}^2} u_k[\boldsymbol{\ell}]\, x[\boldsymbol{t}-\boldsymbol{\ell}].$$

- $\{p_k[\boldsymbol{t}] \in \mathbb{R} \mid k \in D,\, \boldsymbol{t} \in \mathbb{Z}^2\}$ are **prediction filters**.
- $\{u_k[\boldsymbol{t}] \in \mathbb{R} \mid k \in D,\, \boldsymbol{t} \in \mathbb{Z}^2\}$ are **update filters**.

Decomposition algorithm with the modified lifting scheme

Let $\{\mathcal{P}_k\}_{k\in D}$ and $\{\mathcal{U}_k\}_{k\in D}$ be convolution operators.

1. **Split**: Decompose a signal $\{c_j[\boldsymbol{t}]\}_{\boldsymbol{t}\in\mathbb{Z}^2}$ into an even component $\{c_{0,j-1}[\boldsymbol{t}]\}_{\boldsymbol{t}\in\mathbb{Z}^2}$ and directional components $\{c_{k,j-1}[\boldsymbol{t}]\}_{\boldsymbol{t}\in\mathbb{Z}^2,k\in D}$ by

$$c_{0,j-1}[\boldsymbol{t}] = c_j[2\boldsymbol{t}], \qquad c_{k,j-1}[\boldsymbol{t}] = c_j[2\boldsymbol{t}+\boldsymbol{s}_k].$$

2. **Prediction** : Calculate N–th detail components $\{d_{k,j-1}[\boldsymbol{t}]\}_{\boldsymbol{t}\in\mathbb{Z}^2,k\in D}$ with prediction operators $\{\mathcal{P}_k\}_{k\in D}$ by

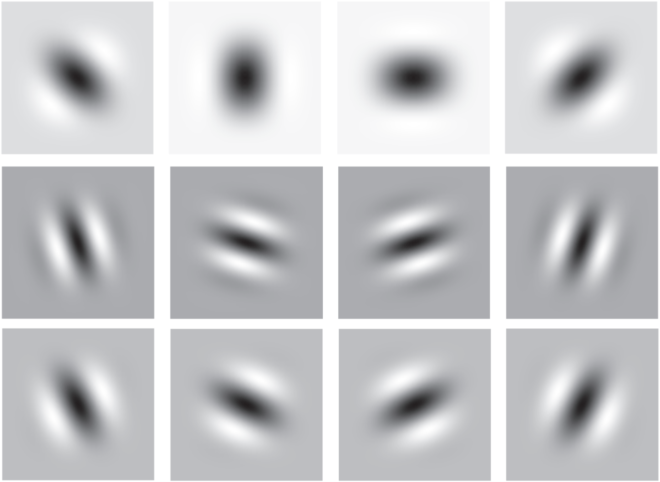
$$d_{k,j-1}[\boldsymbol{t}] = c_{k,j-1}[\boldsymbol{t}] - (\,\mathcal{P}_k\, c_{0,j-1}\,)\,[\boldsymbol{t}].$$

3. **Update** : Calculate a coarse component $\{c_{j-1}[\boldsymbol{t}]\}_{\boldsymbol{t}\in\mathbb{Z}^2}$ with update operators $\{\mathcal{U}_k\}_{k\in D}$ by

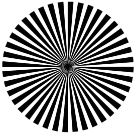
$$c_{j-1}[\boldsymbol{t}] = c_{0,j-1}[\boldsymbol{t}] + \sum_{n=1}^{3}\left(\alpha_n \sum_{k\in D_n} (\mathcal{U}_k\, d_{k,j-1})\,[\boldsymbol{t}]\right).$$

4. **Scaling** : Normalize $\{c_{j-1}[\boldsymbol{t}], d_{k,j-1}[\boldsymbol{t}]\}_{\boldsymbol{t}\in\mathbb{Z}^2,k\in D}$.

Frequency responses of 12 HP filters

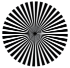


Application to image decomposition

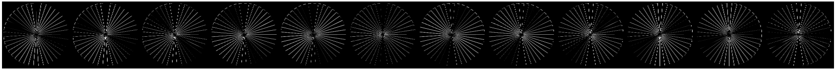


$c_9 \in \mathbb{R}^{512\times 512}$

$\downarrow$ decompose



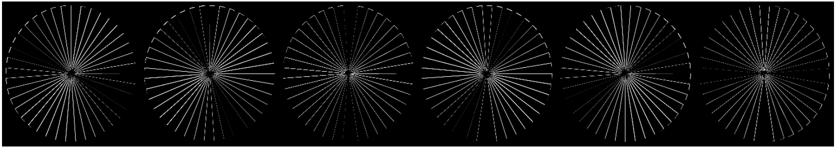
$c_8 \in \mathbb{R}^{256\times 256}$



d_{12}	d_{11}	d_{10}	d_9	d_8	d_7	d_6	d_5	d_4	d_3	d_2	d_1
$165°$	$150°$	$135°$	$120°$	$105°$	$90°$	$75°$	$60°$	$45°$	$30°$	$15°$	$0°$

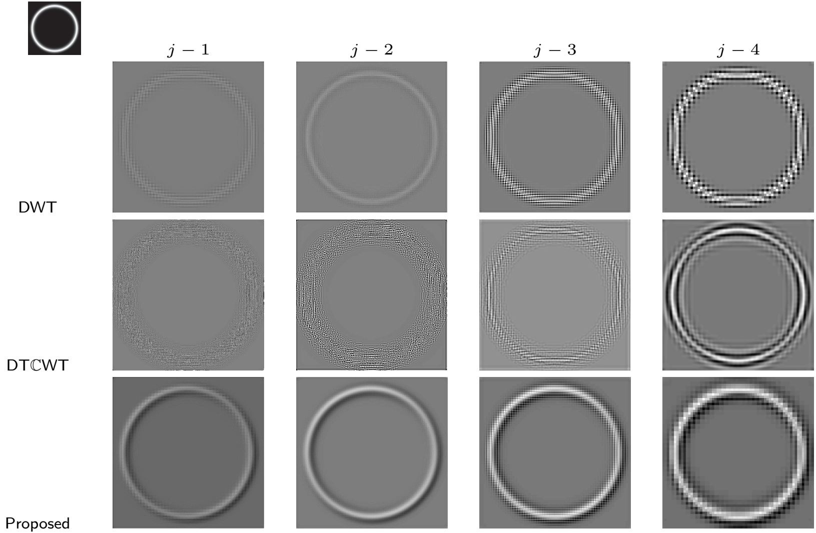
Image decomposition

$N = 6$



d_{11}	d_9	d_7	d_5	d_3	d_1
150°	120°	90°	60°	30°	0°

Reconstruction from each level of detail component $\{d_{k,j}[\boldsymbol{t}]\}_{k\in D}$.



Edge enhancement

The procedure of our edge enhancement method is described as follow:

1. Apply the J-th level decomposition of the DLWT to an image to get a collection of the coarse component and the directional components $\{d_{k,j-L}[\boldsymbol{t}],\, c_{j-J}[\boldsymbol{t}]\}_{\boldsymbol{t}\in\mathbb{Z}^2,\, k\in D,\, 1\leq L\leq J}$.
2. Set zero to all the elements of the coarsest component $\{c_{j-J}[\boldsymbol{t}]\}_{\boldsymbol{t}\in\mathbb{Z}^2}$.
3. Among the 12 detail components at each resolution level, select the detail components that contain the most energy for reconstruction.
4. Reconstruct the decomposed images from the components calculated in above steps 1 and 2 to get an image $\{\tilde{c}_j[\boldsymbol{t}]\}_{\boldsymbol{t}\in\mathbb{Z}^2}$ that only consists of essentially edge components.
5. Apply a Gaussian filter to $\{\tilde{c}_j[\boldsymbol{t}]\}_{\boldsymbol{t}\in\mathbb{Z}^2}$ for smoothing in order to reduce small edges as well as noise.
6. Finally, do binarization process.

Edge enhancement

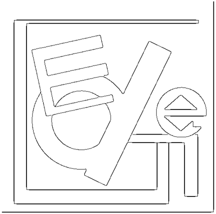
We evaluated the quantitative measure of these edge images by using the Pratt's figure of merit (FOM), which is one of the major metrics for an edge detector. The FOM is calculated by

$$F = \frac{1}{\max\left(N_e, N_d\right)} \sum_{k=1}^{N_d} \frac{1}{1 + \alpha d(k)^2},$$

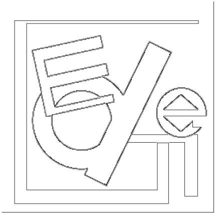
where N_e and N_d are the number of actual edge points and detected edge points respectively, and $d(k)$ is the distance between the k-th detected edge points and actual edge points. The scaling constant α was taken as $1/9$.

Edge enhancement

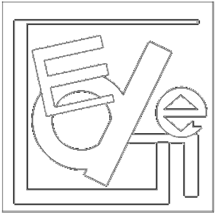
(a) Original



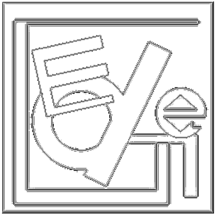
(b) Canny ($F = 0.52$)



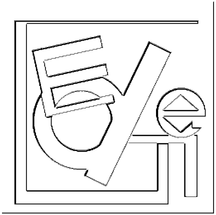
(c) DWT ($F = 0.61$)



(d) DT$\mathbb{C}$WT ($F = 0.63$)



(e) Shearlet ($F = 0.64$)



(f) DLWT ($F = 0.66$)

Edge enhancement

(a) Original

(b) Canny ($F = 0.38$)

(c) DWT ($F = 0.66$)

(d) DT$\mathbb{C}$WT ($F = 0.67$)

(e) Shearlet ($F = 0.68$)

(f) DLWT ($F = 0.69$)

Edge enhancement

(a) Original

(b) Canny ($F = 0.43$)

(c) DWT ($F = 0.44$)

(d) DT$\mathbb{C}$WT ($F = 0.45$)

(e) Shearlet ($F = 0.49$)

(f) DLWT ($F = 0.51$)

Computational cost

- We show the computational cost of the proposed method by measuring an execution speed.
- We ran each edge enhancement/detection by MATLAB2022a with Apple M1 MAX 10 cores, and took the average over 100 trials.
- We see that our method is the fastest wavelet-based edge detector compared here. Notably, it is almost as fast as the Prewitt or Sobel method which is one of the simplest filtering-based edge detector, and thus it is sufficient for practical use.

Detector	Time [sec]
Sobel	0.0108
Prewitt	0.0122
Canny	0.0223
DWT	0.0156
DT$\mathbb{C}$WT	0.0450
Shearlet	0.3587
DLWT	0.0142

Summary

- We introduced the directional lifting transform (DLWT).
  1. Pure discrete setting on a 2D lattice.
  2. Directional selectivity: $N = 3 \longrightarrow N = 12$.
  3. Angles: $0^\circ \leq \theta < 180^\circ$, every 15°.
  4. The transform is designed and implemented on a real plane.
  5. Fast computation is possible by the lifting-based implementation.
- Numerical experiments on edge analysis involving a comparison with several conventional edge detection methods demonstrated the advantages of the proposed method in terms of capturing both global and local edge structures well.