Image Interpolation Technique by the PCA of the Gradient Distribution

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Outline

- Motivations
- PCA(Principal Component Analysis) for the Image Gradient
- Estimation of the Original Image from JPEG Image
- Numerical Experiments
- Conclusion

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Motivations

Original image

JPEG image

Point Because of Block wise DCT or something, We can not avoid annoying blocking artifact and edge artifact

Motivations

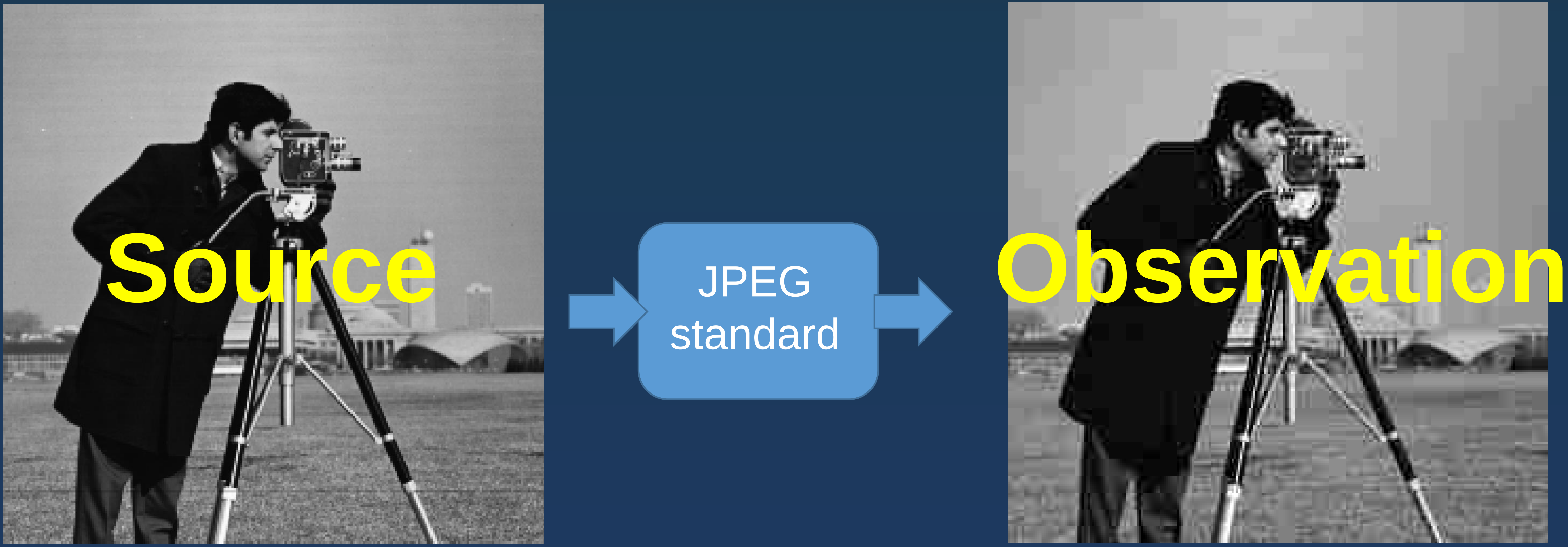
Original image

JPEG image

Point We can restore image quality lost by tradeoff between information reduction and quality.

Motivations

Direct problem



Original image

JPEG image

Point JPEG compression can be regarded as a direct problem

Motivations

Inverse problem

Original image

JPEG image

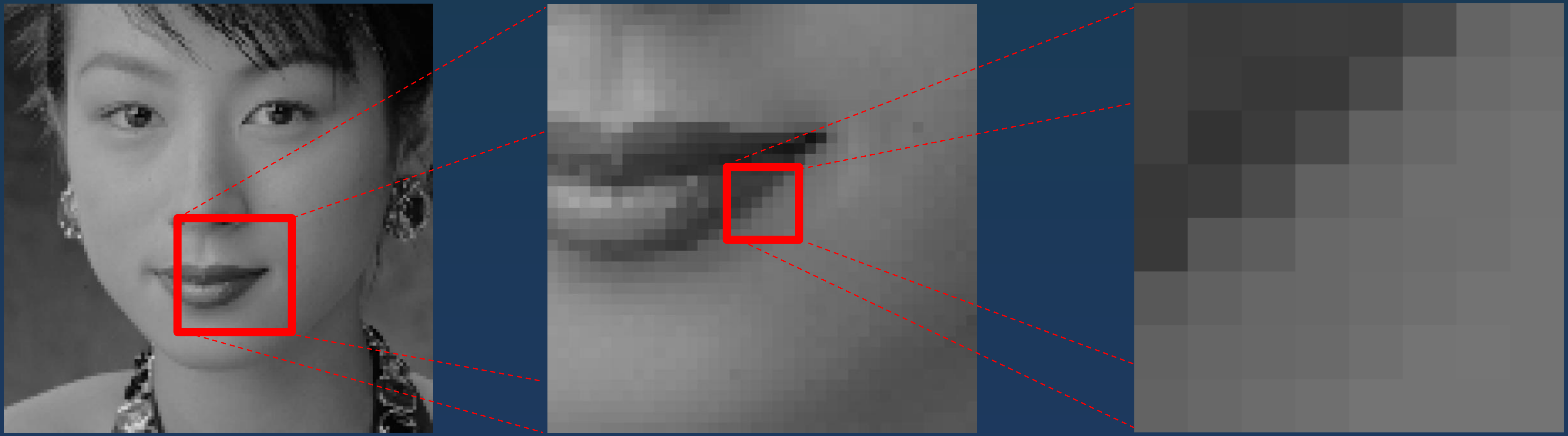
Point Propose a PCA-based estimation method of the original image from JPEG image

Related Previous Work

- N. Wiener(1949): Extrapolation, Interpolation, and Smoothing of Stationary Time Series with Engineering Applications (Wiener filter)
- S. Geman, D. Geman(1984): Stochastic Relaxation, Gibbs Distributions, and the Bayesian Restoration of images (Bayesian method)
- R. Kikushi, B. H. Soffer (1977): Maximum entropy image restoration. I. The entropy expression (Maximum entropy method)
- H. Murata, H. Nishikado, H. Yamauchi (2000): Image Restoration using Genetic Algorithm (Genetic Algorithm)

Point The main difference, between us and others is that we use PCA for estimation

Basic Ideas



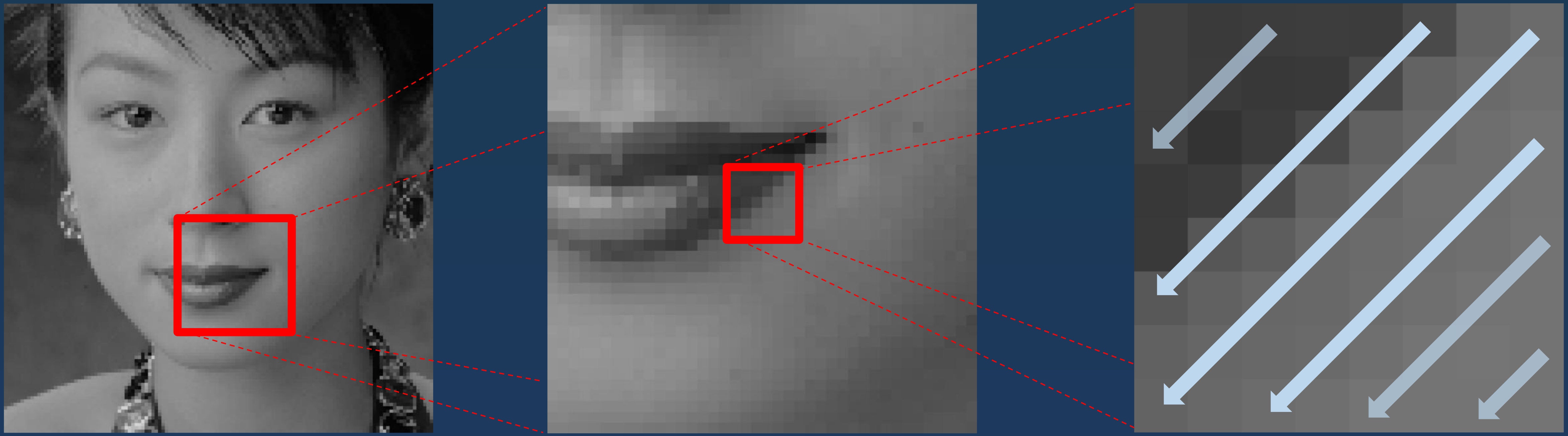
Test image

Zoomed image

8 × 8 pixels
(one block in JPEG)

Point There are only simple geometrical structures.

Basic Ideas



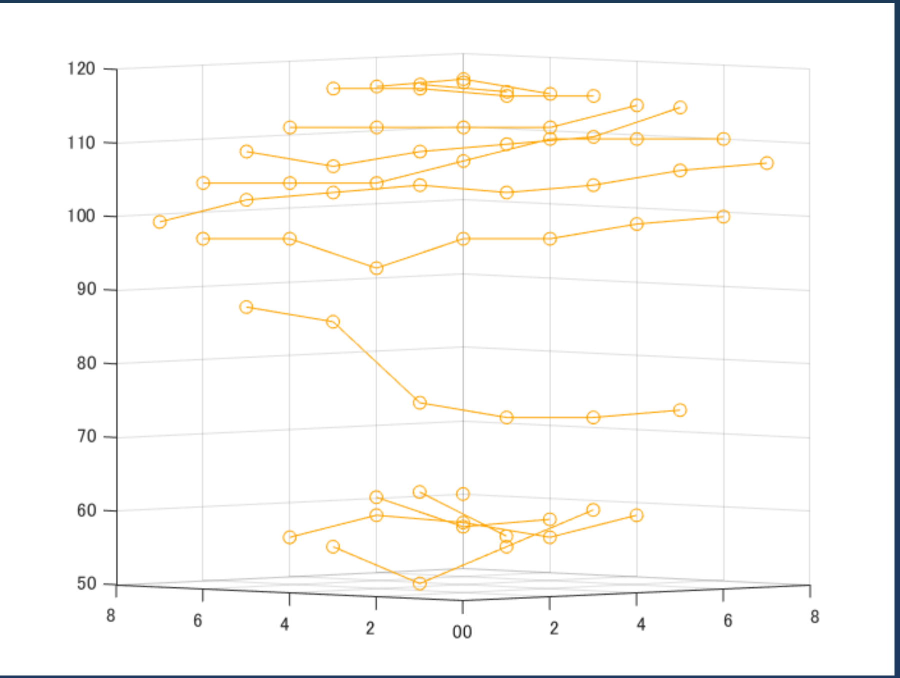
Test image

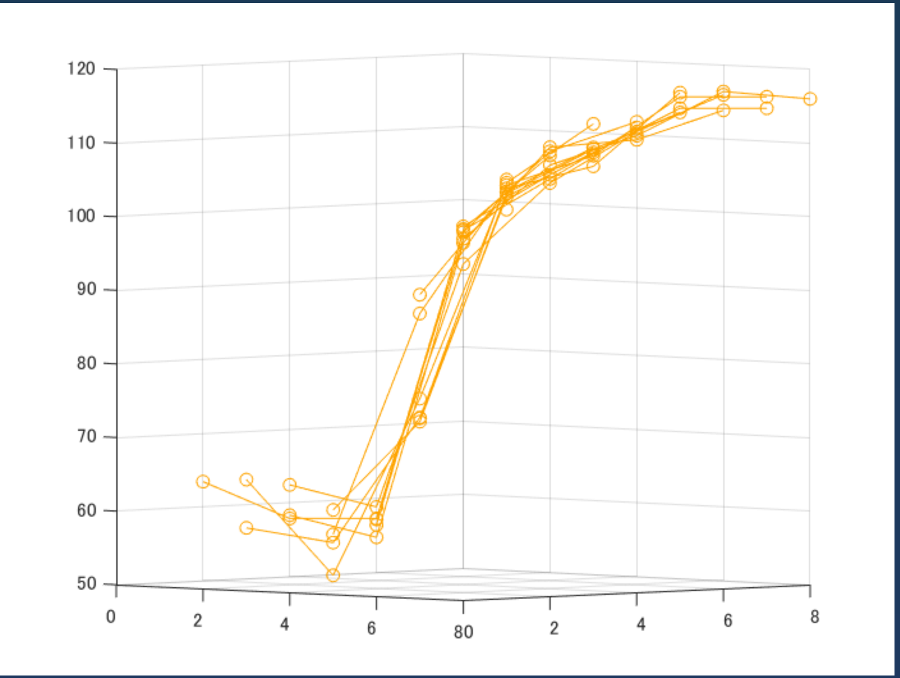
Zoomed image

8 × 8 pixel
(one block in JPEG)

Point Especially focus on the direction of image gradient.

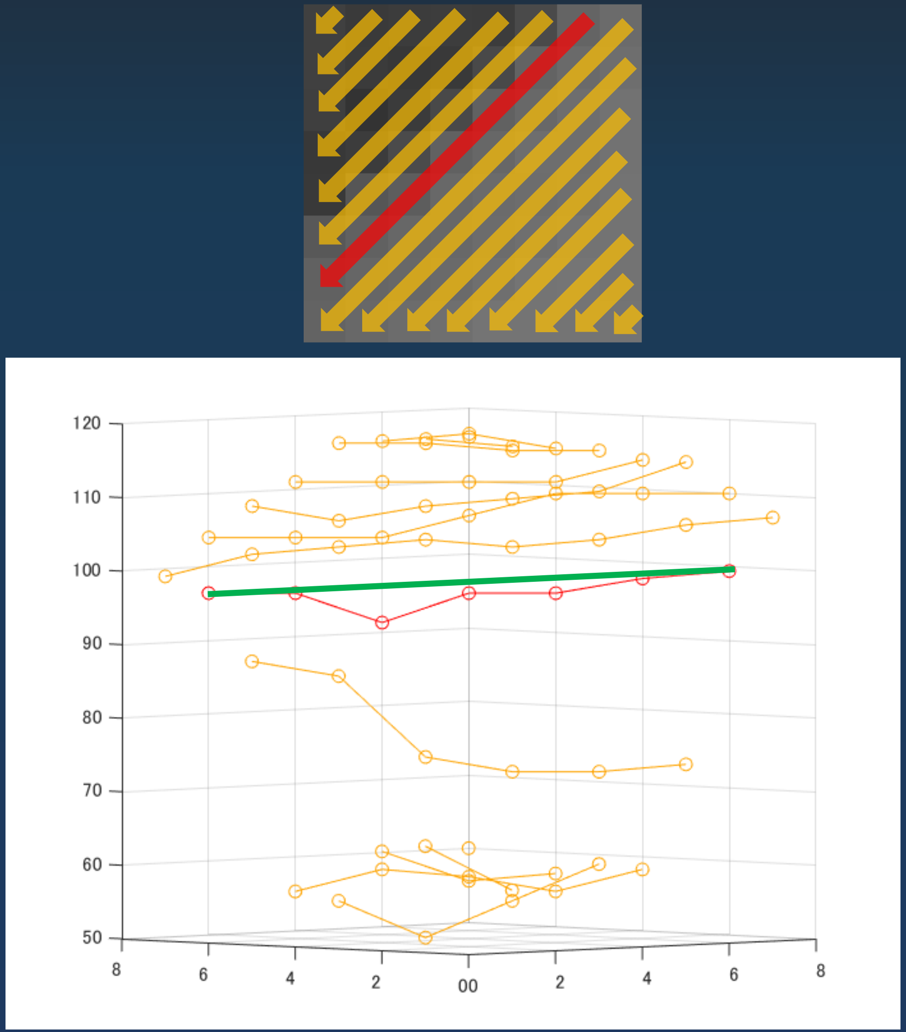
Basic Ideas

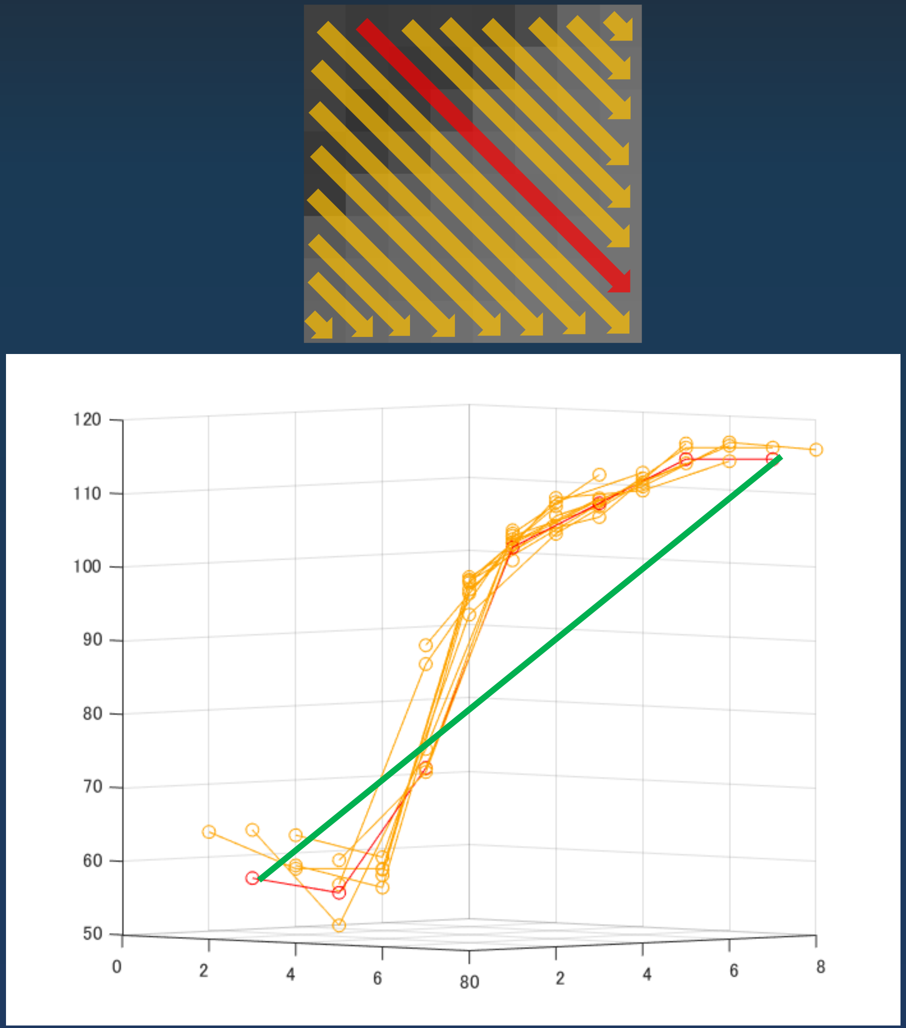




Point Depending on the direction,
the intensity can be a simple linear change.

Basic Ideas





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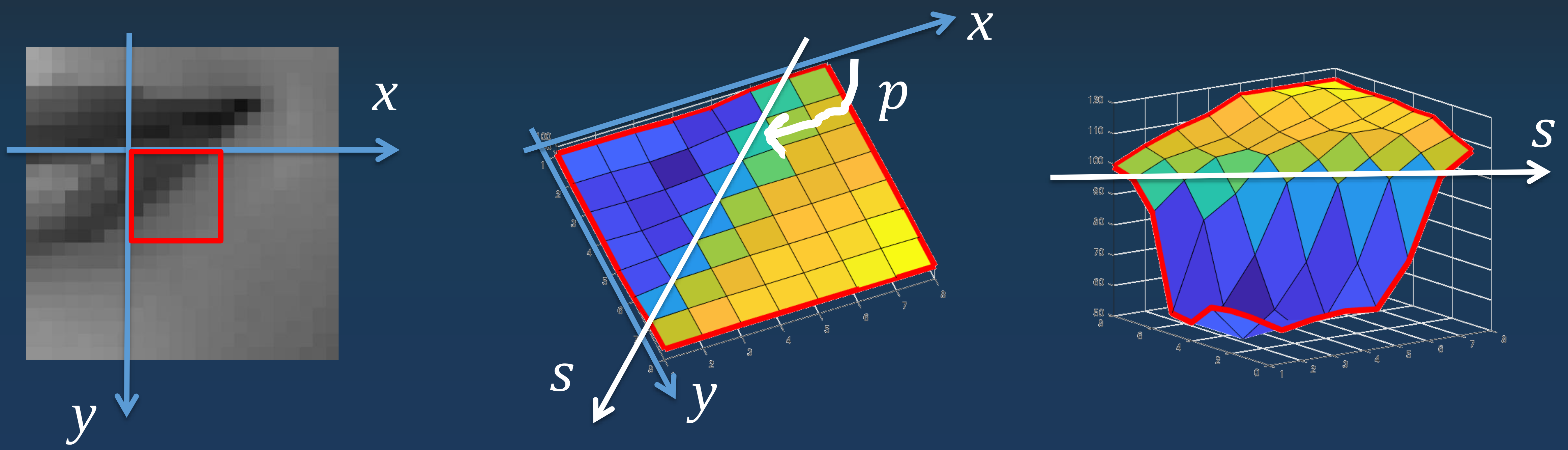
Basic Ideas



$$s = (\cos p)x + (\sin p)y$$

$f(s)$: Image intensity along s-axis

Point Want to find p so that $\frac{d}{ds}f(s)$ is locally constant

→ PCA of the image gradient

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Image Gradient

●: Target image point

Central difference

$$\frac{\partial f}{\partial y} = \begin{bmatrix} -1 \\ +1 \end{bmatrix} * A$$

$$\frac{\partial f}{\partial x} = \begin{bmatrix} -1 & +1 \end{bmatrix} * A$$

A: Image intensity function

64	58	60	61	60	74	100	107
64	59	56	57	73	99	106	110
63	51	59	73	97	104	110	114
56	60	75	97	103	110	110	114
57	86	93	104	107	109	111	115
88	97	103	104	108	111	115	115
97	102	104	106	111	116	117	115
99	104	108	111	116	116	116	116

Point The derivatives in the horizontal and vertical directions are given by the central difference

Image Gradient

●: Target image point

⬚: Target area

64	58	60	61	60	74	100	107
64	59	56	57	73	99	106	110
63	51	59	73	97	104	110	114
56	60	75	97	103	110	110	114
57	86	93	104	107	109	111	115
88	97	103	104	108	111	115	115
97	102	104	106	111	116	117	115
99	104	108	111	116	116	116	116

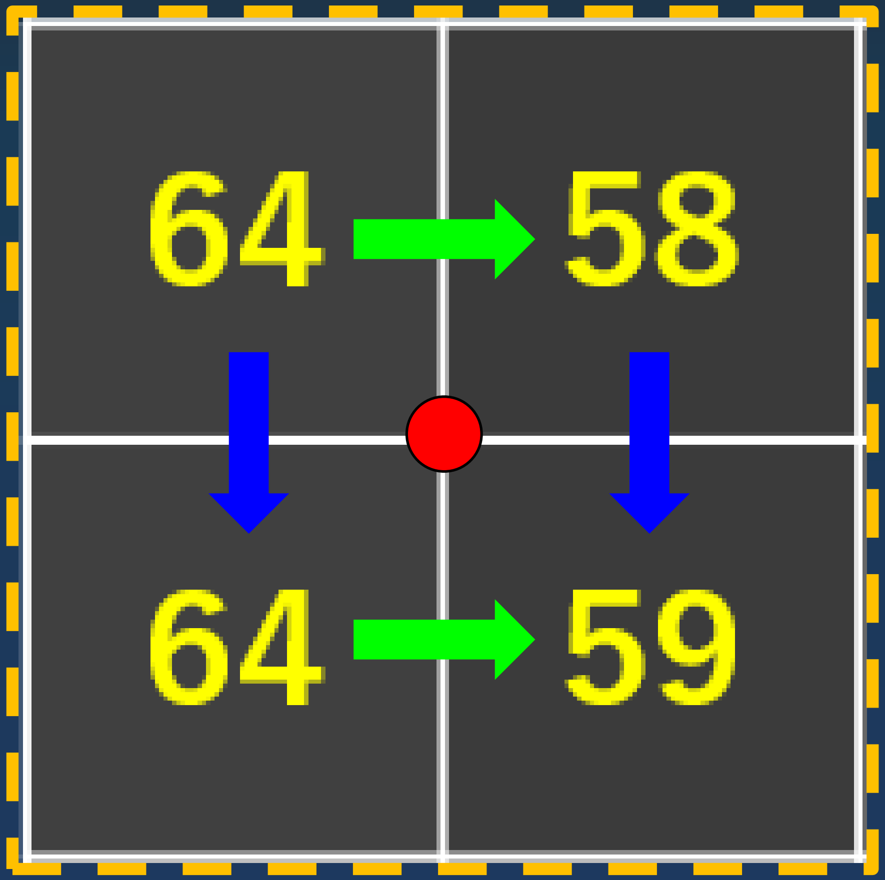
Image Gradient

●: Target image point

$$d_x = \frac{58 - 64 + 59 - 64}{2}$$

$$d_y = \frac{64 - 64 + 59 - 58}{2}$$

$$(d_x, d_y) = (-5.5, 0.5)$$



Point We consider that the approximately gradient of original image is not lost even in JPEG image

PCA for the Image Gradient

$N = 8$: $N \times N$ pixel

$(d_{x_i}, d_{y_j}) \quad i, j = 0,1, \dots, N-2$: Gradient Distribution

$$V = \begin{pmatrix} v_{xx} & v_{xy} \\ v_{xy} & v_{yy} \end{pmatrix}$$: Covariance Matrix

$$v_{xx} = \frac{1}{N-1} \sum_{n=0}^{N-2} \left(d_{x_n} - \bar{d}_x\right)^2, \quad v_{yy} = \frac{1}{N-1} \sum_{n=0}^{N-2} \left(d_{y_n} - \bar{d}_y\right)^2, \quad v_{xy} = \frac{1}{N-1} \sum_{n=0}^{N-2} \left(d_{x_n} - \bar{d}_x\right)\left(d_{y_n} - \bar{d}_y\right)$$

$$\bar{d}_x = \frac{1}{N-1} \sum_{n=0}^{N-2} d_{x_n}, \qquad \bar{d}_y = \frac{1}{N-1} \sum_{n=0}^{N-2} d_{y_n}$$

Point The covariance matrix based on the gradient distribution

PCA for the Image Gradient

$V = \begin{pmatrix} v_{xx} & v_{xy} \\ v_{xy} & v_{yy} \end{pmatrix}$: Covariance Matrix

$V\boldsymbol{e} = \lambda \boldsymbol{e}$: Eigen Equation

$\boldsymbol{e} = (e_x, e_y)$: Eigen Vector

$\begin{cases} \boldsymbol{e}_{\max} & \text{: direction first principal component} \\ \boldsymbol{e}_{\min} & \text{: direction second principal component} \end{cases}$

λ : Eigen Value

$\begin{cases} \boldsymbol{\lambda}_{\max} & \text{: maximum variance} \\ \boldsymbol{\lambda}_{\min} & \text{: minimum variance} \end{cases}$

Point Calculation of the direction for minimum variance $\boldsymbol{e}$.

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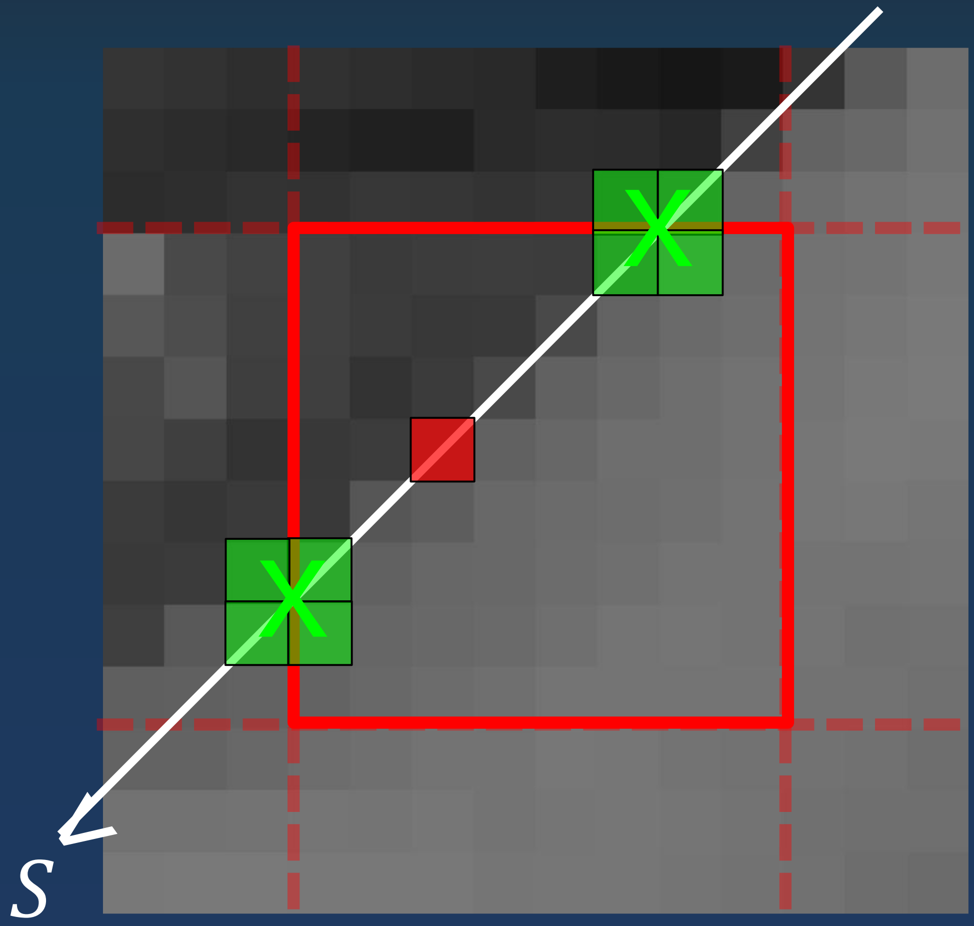
Linear interpolation for the direction of the minimum variance



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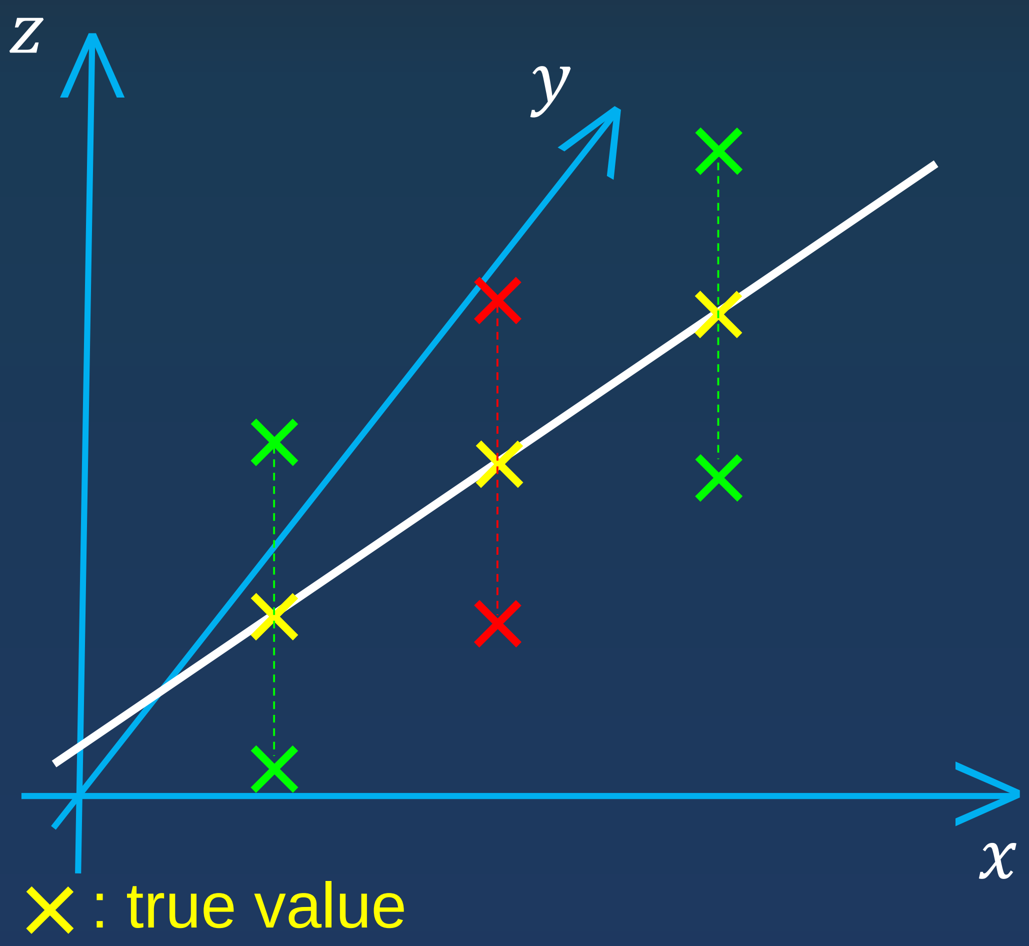
88	97	103	104	108	111	115	115
97	102	104	106	111	116	117	115
99	104	108	111	116	116	116	116

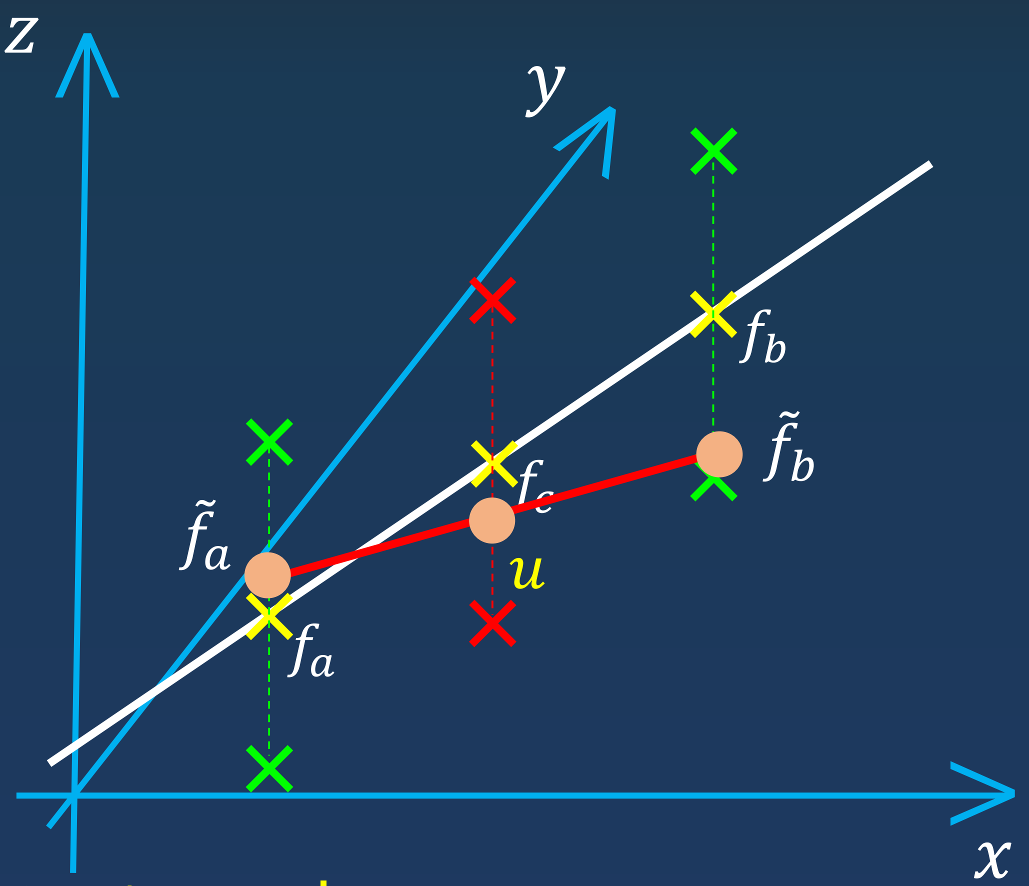
S

Point We consider to apply the Linear interpolation for the direction of the minimum variance

Linear interpolation for the direction of the minimum variance



Linear interpolation for the direction of the minimum variance



$$E[\tilde{f}_a] = f_a\ ,\quad E[\tilde{f}_b] = f_b$$

$$u = \alpha \tilde{f}_a + (1-\alpha)\tilde{f}_b$$

(Linear Interpolation)

$$E[u] = \alpha E[\tilde{f}_a] + (1-\alpha)E[\tilde{f}_b]$$
$$= \alpha f_a + (1-\alpha) f_b$$
$$\approx f_c$$

(If we are correct in our assumptions)

Point The expected value of u is approximately equal to f_c.

Estimation of the Original Image from JPEG Image

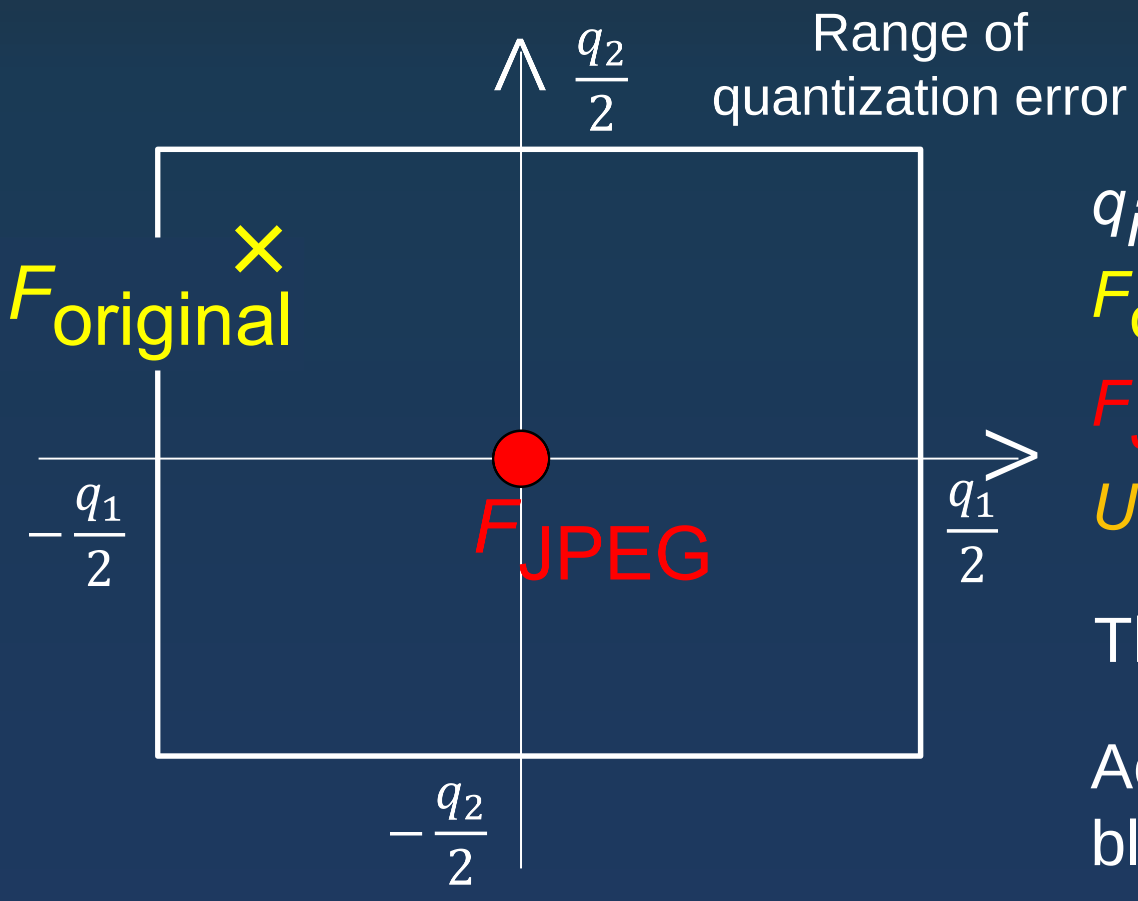
Original image

JPEG image

Linear interpolation

Point Linear interpolation alone is not enough.

Estimation of the original DCT coefficients by using DCT coefficients of interpolation function *u*



q_i : quantization step for i's DCT coefficient

F_{original} : DCT coefficients of original image

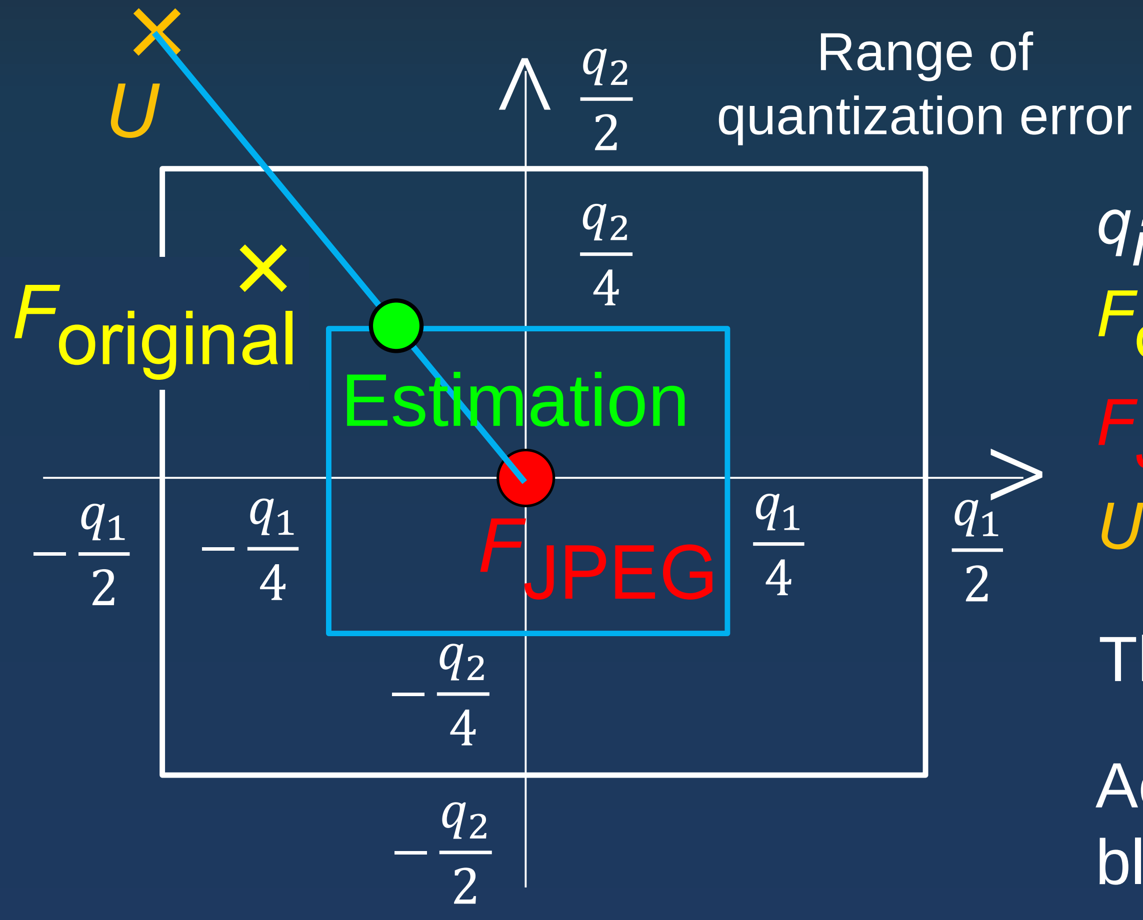
F_{JPEG} : DCT coefficients of JPEG image

U : DCT coefficients of interpolation function

This is a 2-dimensional rough sketch.

Actually, DCT coefficients for a 8×8 pixel block are represented as a 64-dimensional vector.

Estimation of the original DCT coefficients by using DCT coefficients of interpolation function *u*



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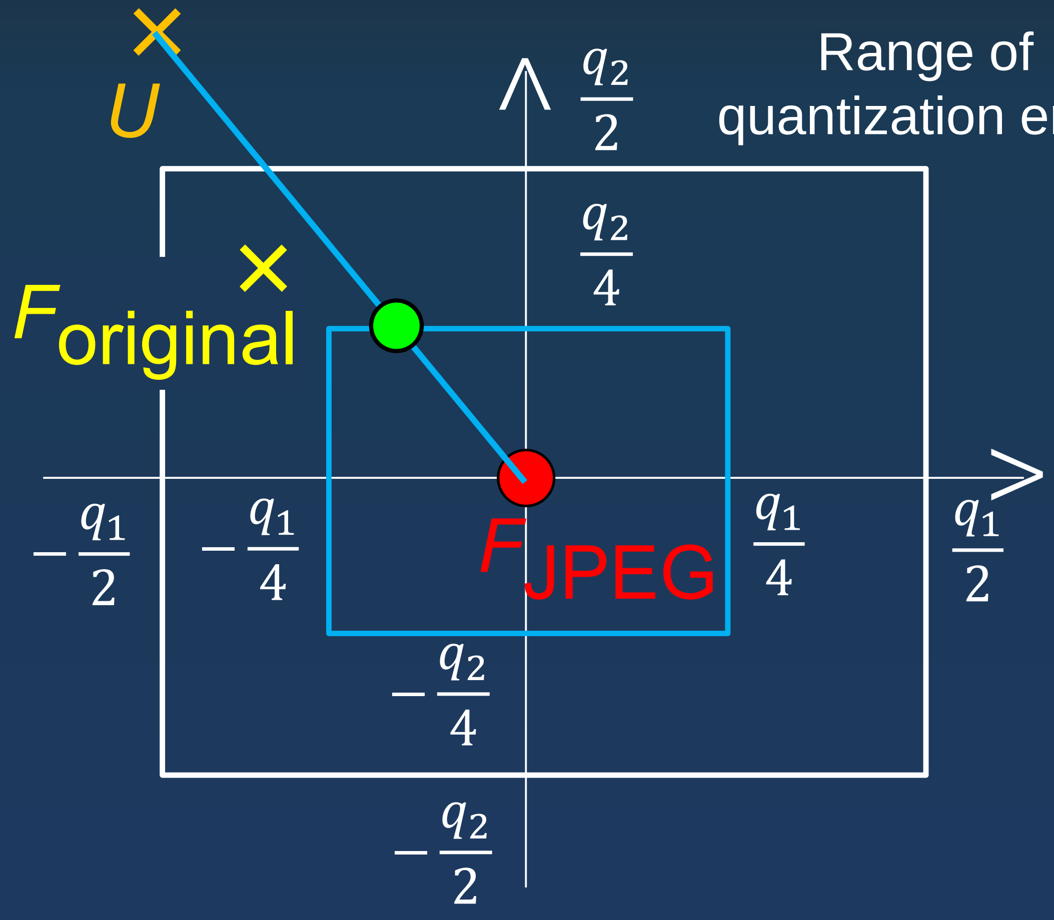
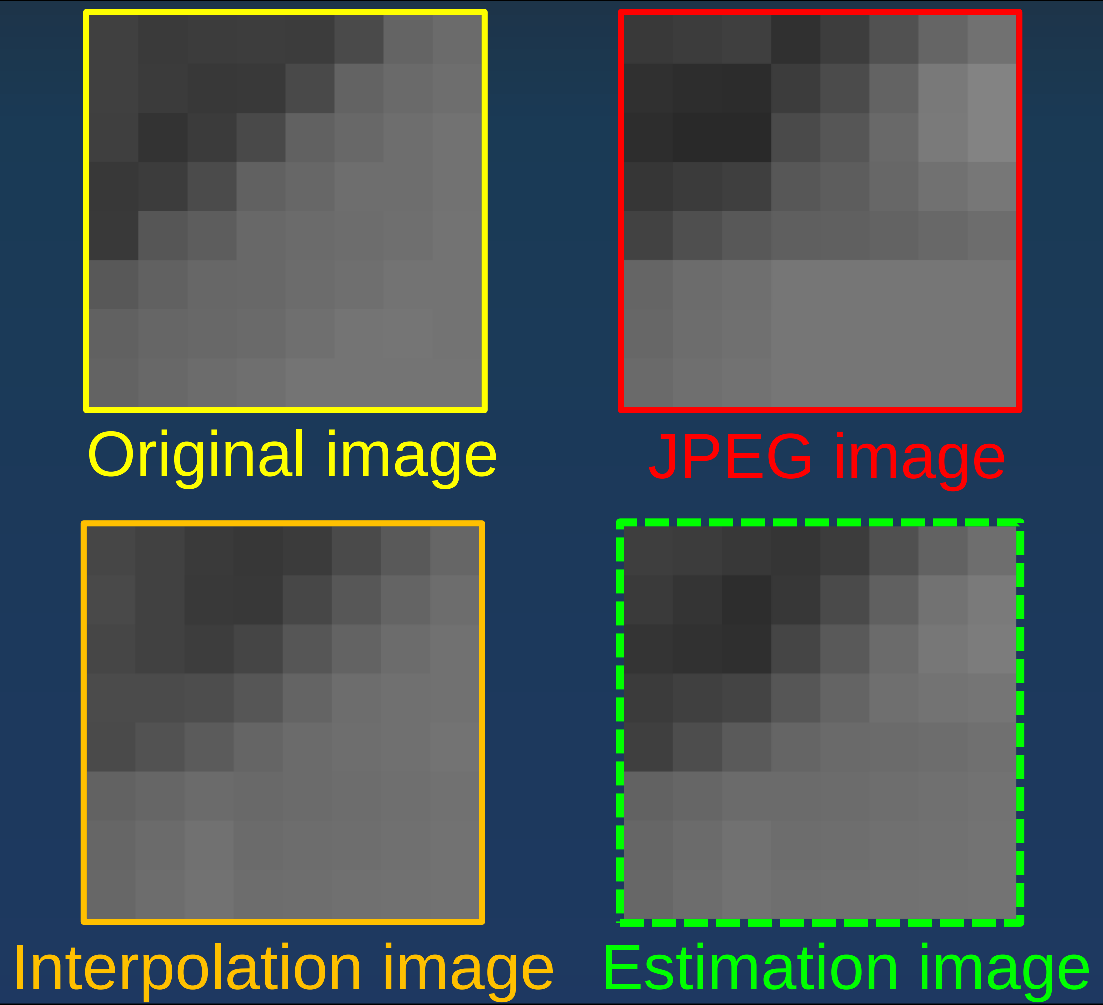
Point

Take the best of JPEG image and interpolation image.

Estimation of the original DCT coefficients by using DCT coefficients of interpolation function *u*





Point

Take the best of JPEG image and interpolation image.

Estimation of the Original Image from JPEG Image

Original image

JPEG image

Estimation image

Interpolation image

Point Estimation image is clearer and no blocking artifact or edge artifact.

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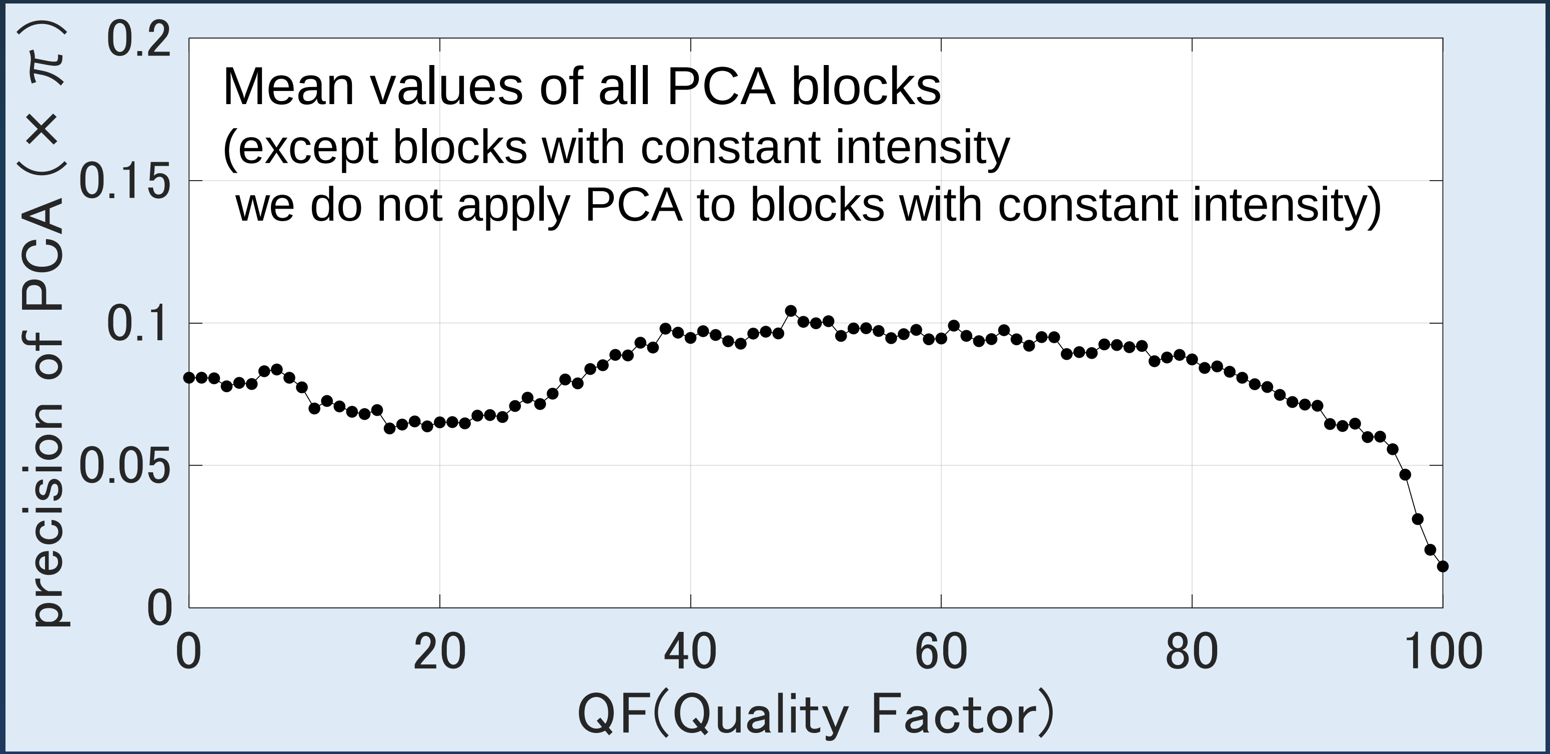
Numerical Experiments

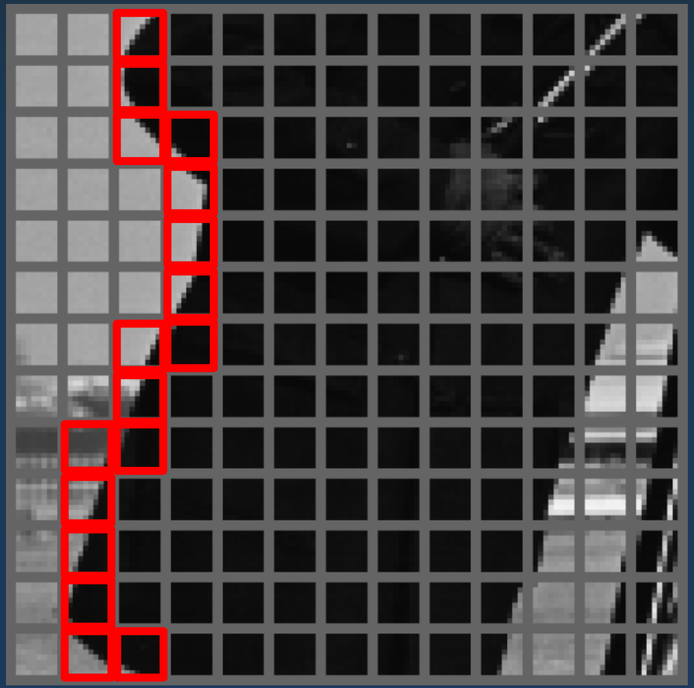
Test image



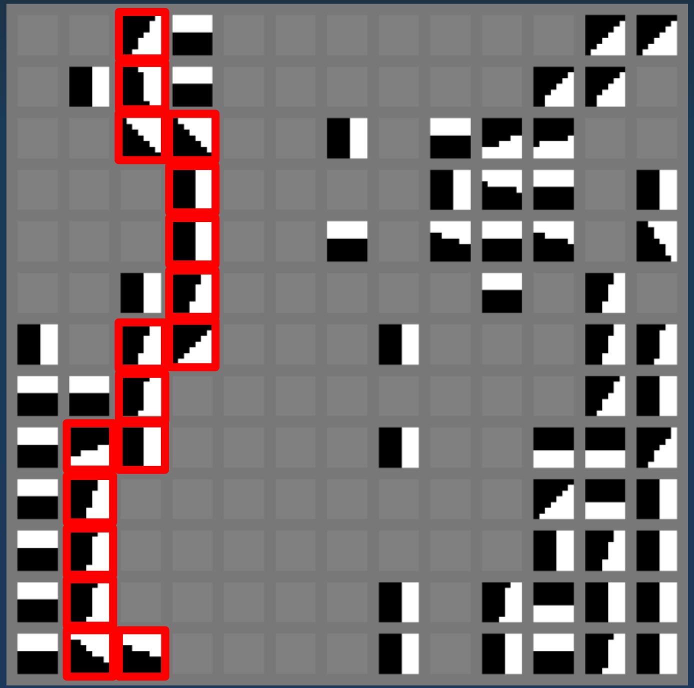
The difference of direction of minimum variance between original image and various QF JPEG image.

Point The accuracy of PCA is stable at about $\frac{\pi}{10}$ for all QF.

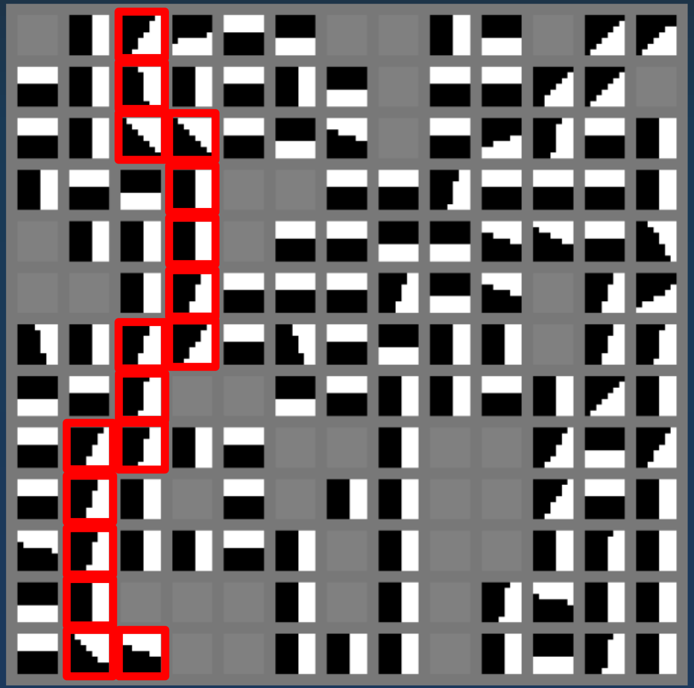
Numerical Experiments



Original image



QF=20



QF=60


: PCA results (direction of image gradient)


: flat block (PCA is not applied)

Point PCA can find out the direction of image gradient even if quality factor is very small.

Test image

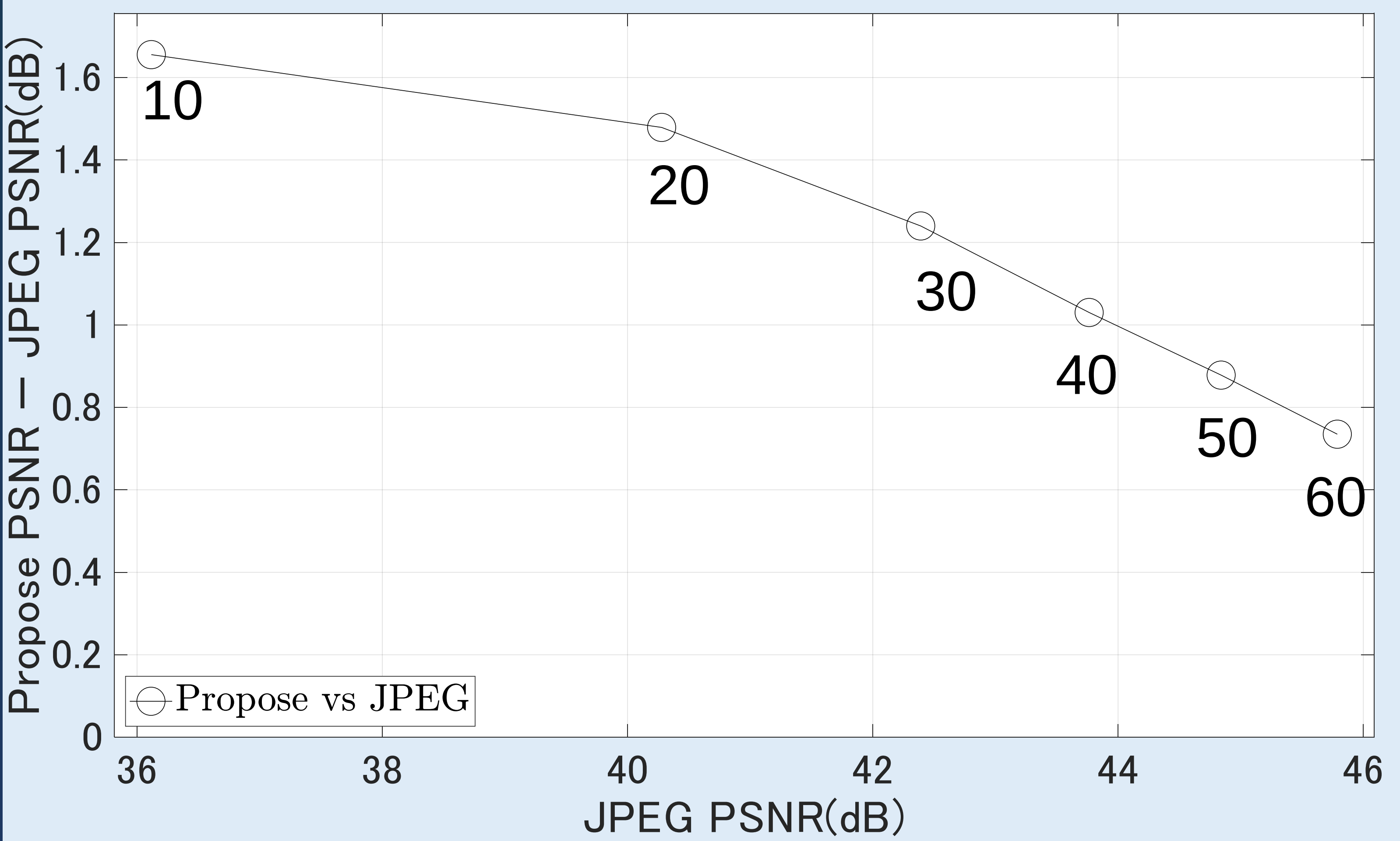
High-resolution image ($4096 \times 3072, 3072 \times 4096$, 8bit gray scale)

Numerical Experiments

The gradation like skin is dominant

Quality Factor = 10, 20, …, 60

Test image

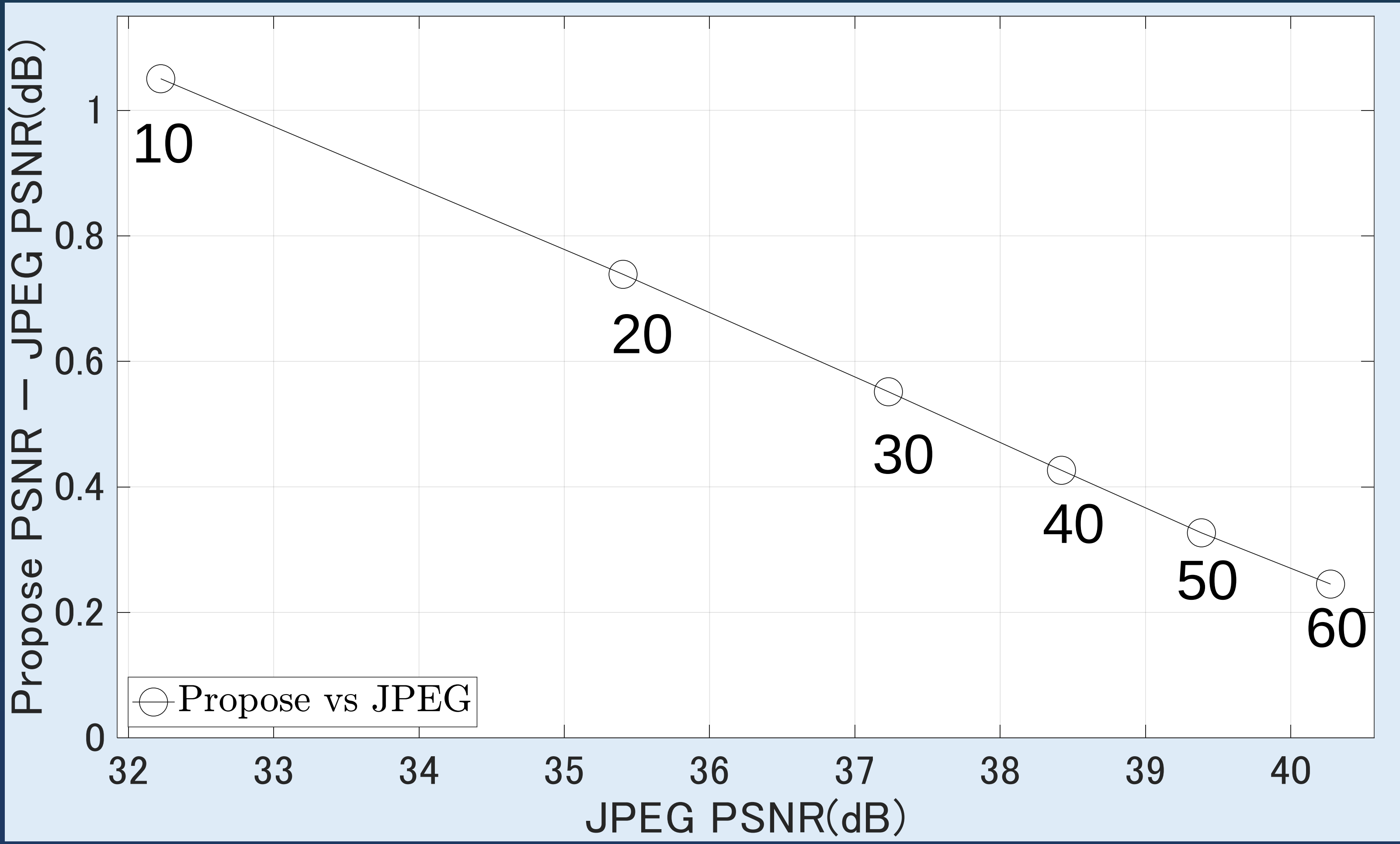


Numerical Experiments

Contain many fine patterns

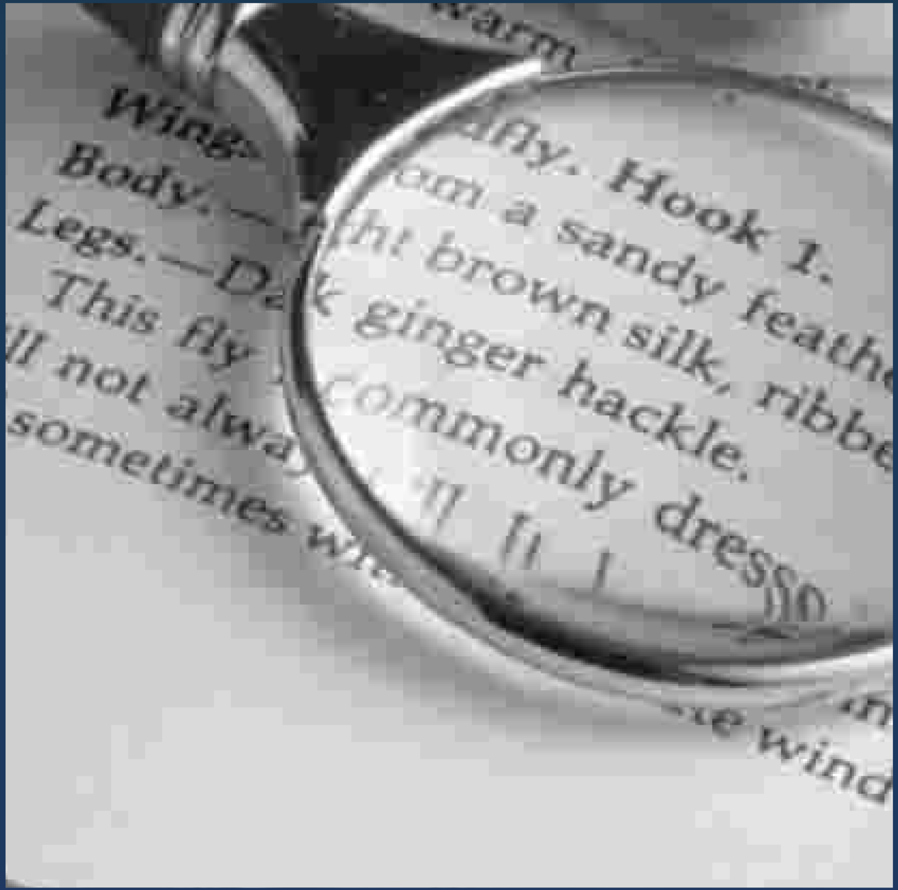
Quality Factor = 10, 20, …, 60

Test image



Numerical Experiments

Original image



JPEG image

Estimation image

Conclusion

- We proposed a PCA-based estimation method of the original image from JPEG image.
- We assumed that the pixel changes in a local region of an image are linear.
- We used PCA of the image gradient to identify the linearly changing directions.
- We confirmed that the estimation image with PCA applied is clearer than JPEG image, and there are no blocking artifacts or edge artifacts.

Thank you very much for your attention!