Graph based similarity: going beyond personal Page Rank

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Outline

1. Graph based similarity
2. Graph transduction:
   Probability propagation Vs. Diffusion propagation
3. Structural graph similarity
4. Application to shape matching

Motivation

John Donne
“no man is an island”

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“no man is an island”

“no data point is an island”

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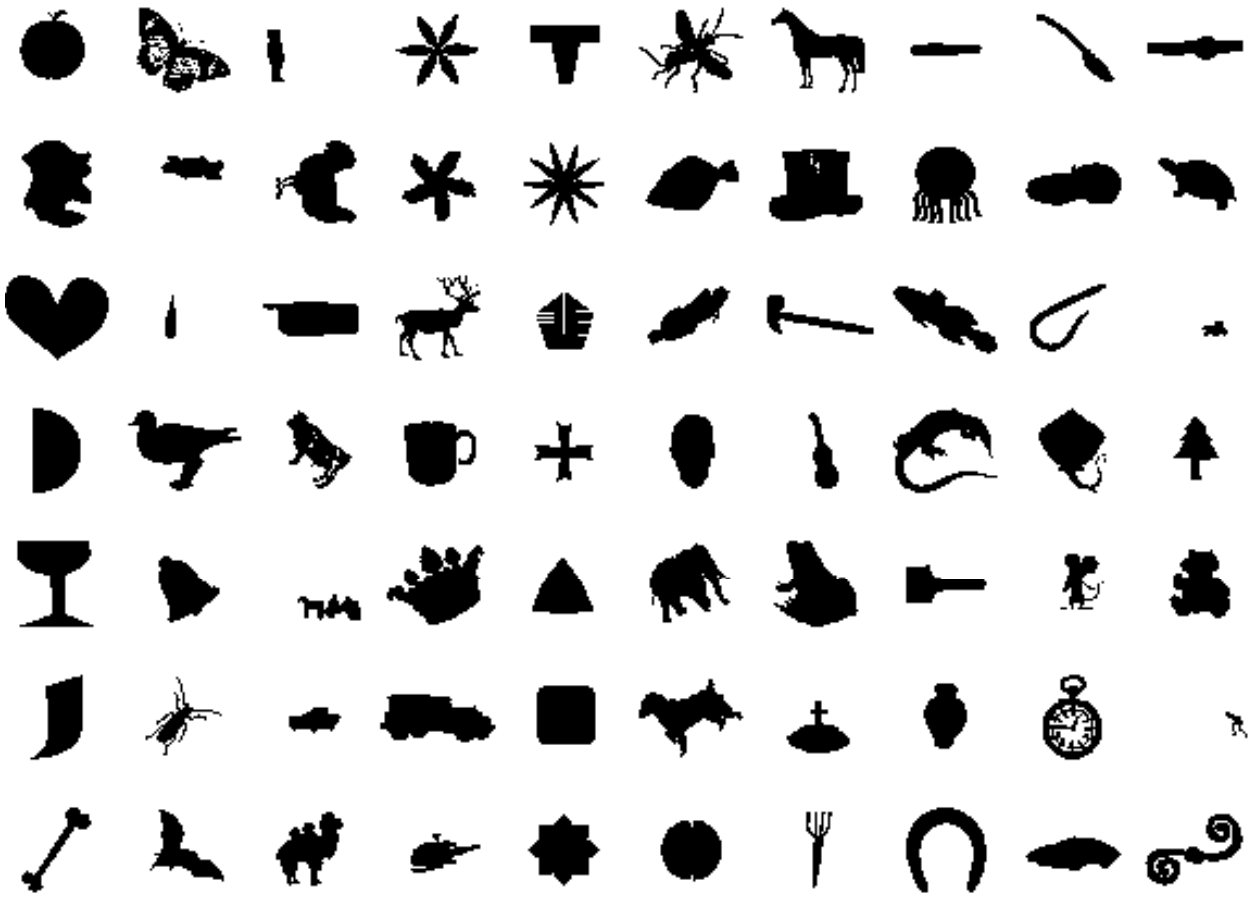
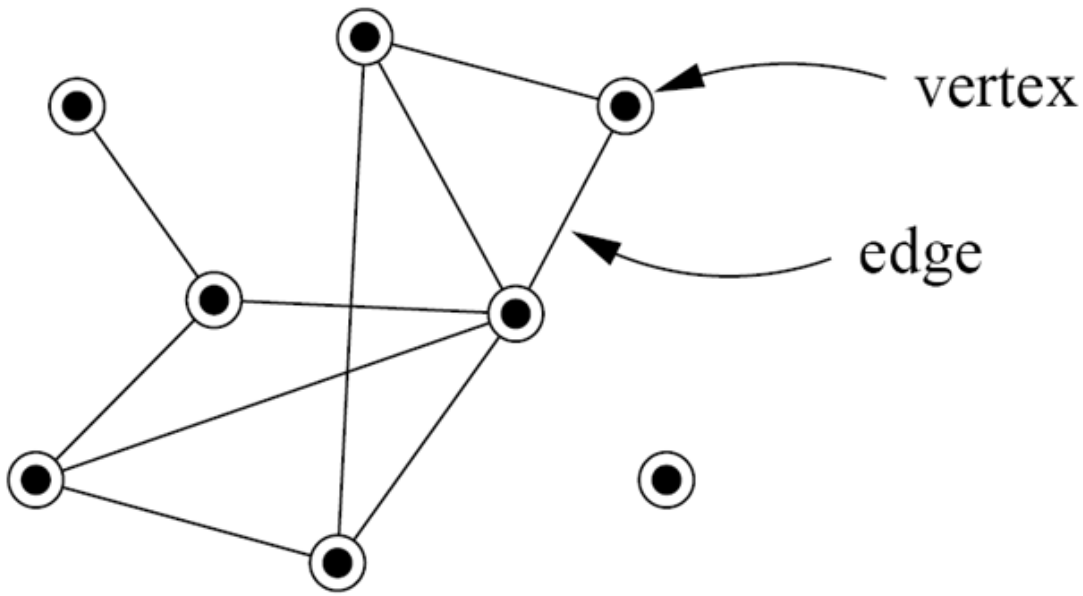
In most applications data points are part of a set/manifold



Data manifolds can be represented by graphs

Data sets as graphs



Nodes $\{x_i\}_1^n$

Edges $e_{ij} = d(x_i, x_j)$

$d(\cdot, \cdot)$ is an application specific distance/affinity measure

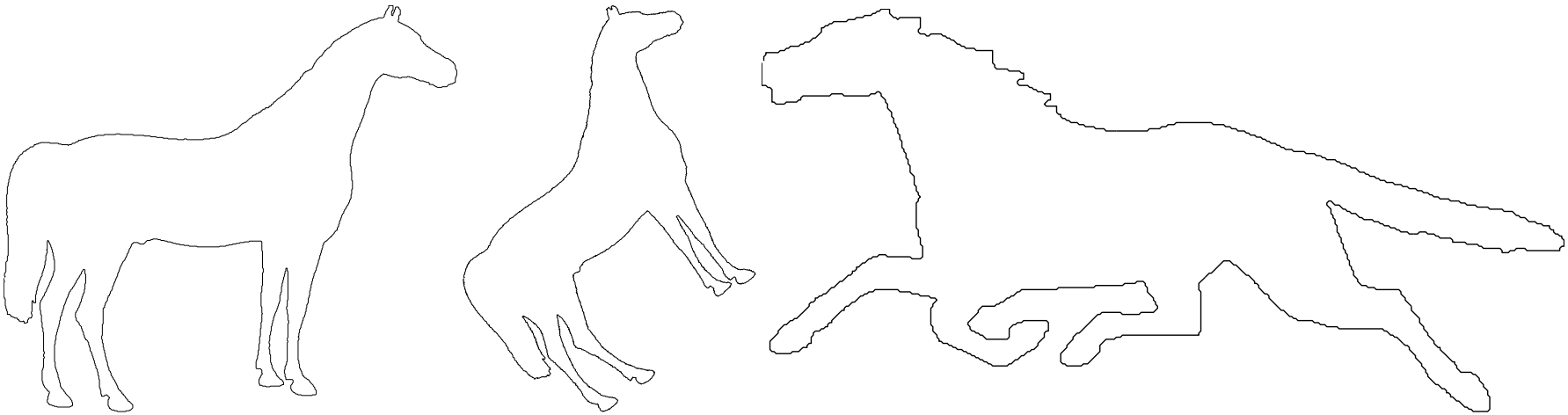
Recognition I

Given a query sample y and a set $\{x_i\}_1^n$, find

$$i^* = \arg \min_i d(y, x_i)$$

The problem:

- we can only approximate $d(\cdot, \cdot)$
- $d(\cdot, \cdot)$ might be corrupted by noise



Recognition II

- Useful for a gamut of applications:
  - Shape and object recognition
  - Target acquisition
  - HMM (state space) modeling
  - Soft Vs. Hard recognition
  - Can be part of a multi-layer system

Low level one-to-one shape matching

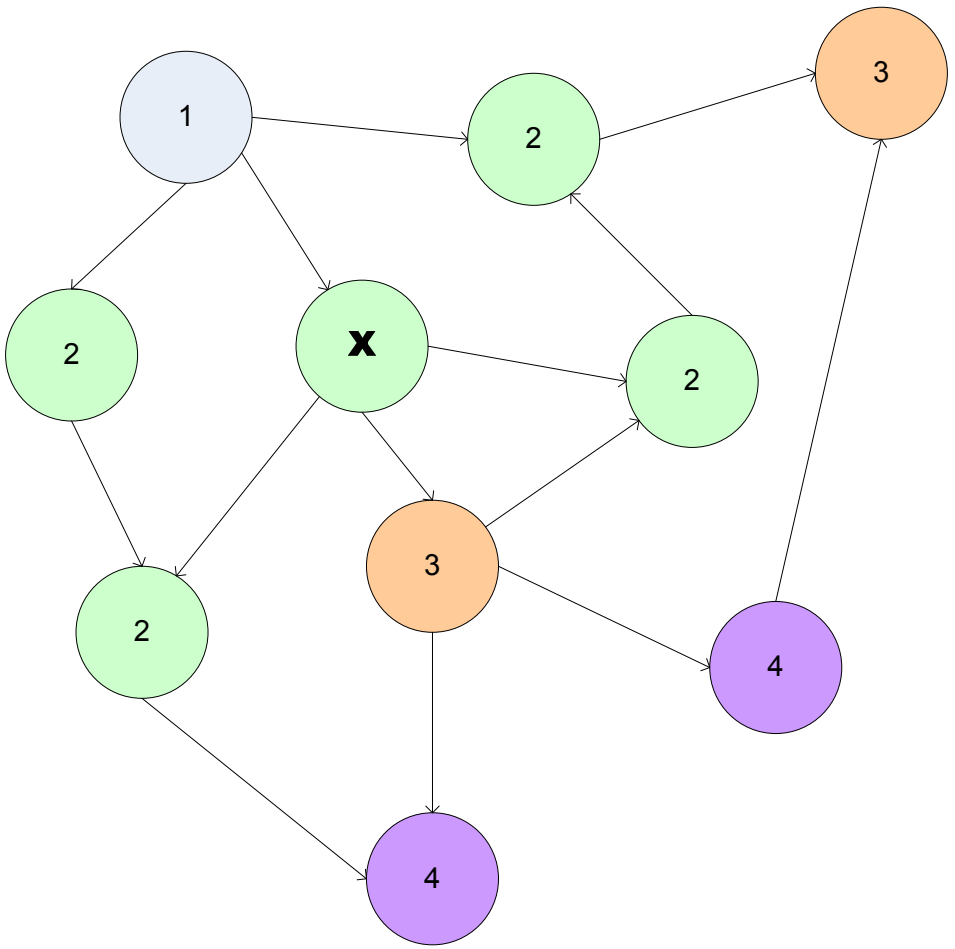
Algorithm	Performance
CSS Mokhtarian96	75.44%
Visual Parts Latecki00	76.45%
Shape Contexts Belongie03	76.51%
Curve Edit Distance Sebastian03	78.17%
MDS+SC+DP Ling07	84.35%
Planar Graph cuts Schmidt-cvpr09	85%
IDSC+DP Ling07	85.40%
IDSC+DP+EMD-L_1	86.56%
Shape-tree	87.7%
GM+IDSC	87.47%
GM+SC	88.11%
Contour Flexibility	89.31%

- Application specific
- Mediocre accuracy

Graph based recognition

Basic idea

My friend's friend is my friend



Mathematical formulation

The graph is analyzed by simulating a Markov random walk and analyzing its properties:

1. Represent the graph by an affinity matrix A
2. Row-normalize A to compute the Markov matrix $M = D^{-1}A$, where $d_{ii} = \sum_j a_{ij}$

$\mathbf{x}^T M = \lambda \mathbf{x}^T$ $\mathbf{x}$ is the asymptotic Markov state *probability*

$M\mathbf{x} = \lambda \mathbf{x}$ $\mathbf{x}$ a *diffusion* vector

Probability propagation

Personalized PageRank (Brin,Page,'98)

$$\mathbf{x}_{n+1}^T = (1-c)\,\mathbf{x}_n^T M + c\mathbf{r},\ c \in [0,1]$$

Fast Random Walk with Restarts (C. Faloutsos's group , CMU)

repeat the above N times

Theoretical analysis Fan Chung (UCSD) and Dan Spielman (Yale)

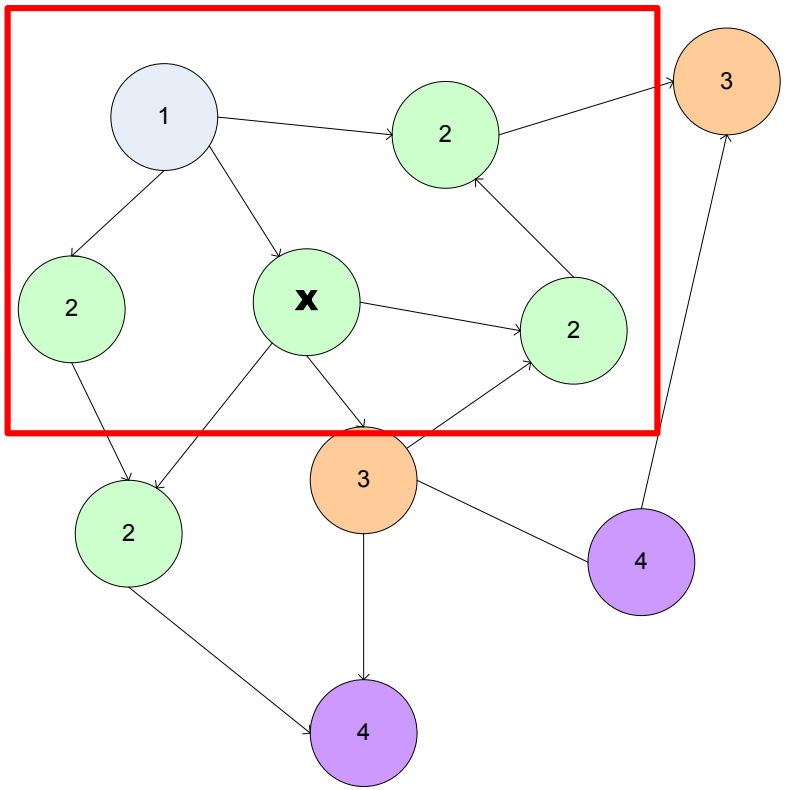
Diffusion propagation I

Xiang *et al.* , *Learning Context Sensitive Shape Similarity by Graph Transduction*, PAMI2009.

Algorithm	Performance
IDSC+DP+EMD-L_1	86.56%
Graph Transduction	92.3%

The algorithm

$$\mathbf{x}_{n+1} = M\mathbf{x}_n$$
$$\mathbf{x}_{n+1}(i_0) = 1$$



Xiang *et al*. actually compute the leading *constrained* eigenvector of the sub-graph.

Commute time/diffusion distance

Commute time/diffusion distance

$C(u,v)$ is the expected time for a random walk to travel between u and v and back

Commute time: Qiu and Hancock PAMI2007
Diffusion distance: Lafon and Coifman ACHA2006

$$C(u,v) = \|\Psi_t(u) - \Psi_t(v)\|_{L_2}^2 = \sum_{l=0}^{n-1} \lambda_l^{2t}(\psi_l(u) - \psi_l(v))^2$$

Our approach: structural similarity

Instead of node similarity

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use structural similarity

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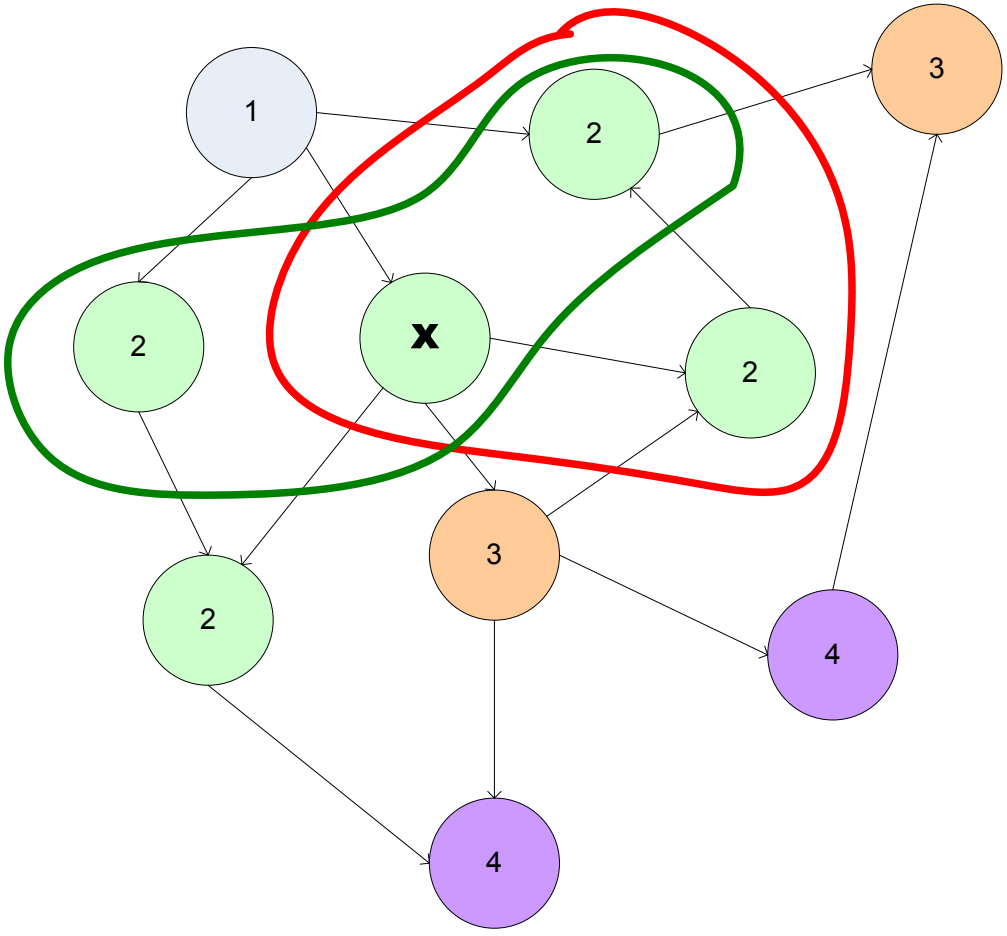


Formally

Let $G_1 \subseteq G$ and $G_2 \subseteq G$, where G_1 and G_2 are $K-NN$ graphs. We aim to compute $d(G_1, G_2)$.

First idea: K-NN graph intersection

$$d(G_1, G_2) = |G_1 \cap G_2|$$

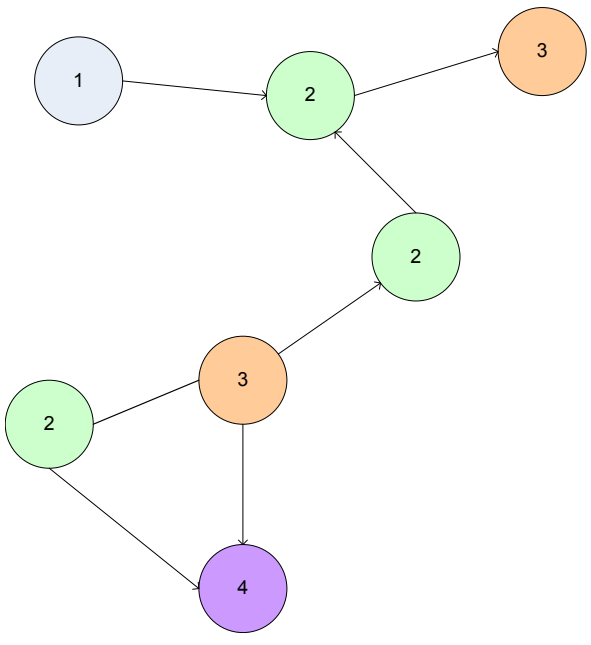
A. Egozi and Y. Keller2009

Algorithm	Performance
IDSC+DP+EMD-L_1	86.56%
Graph Transduction	92.3%
K-NN intersection	93.1%

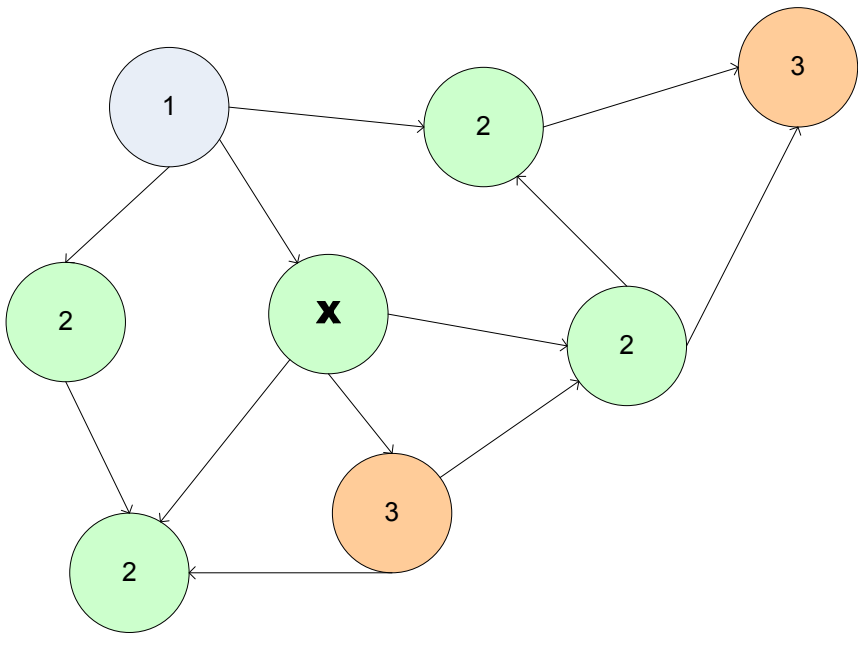
The mixing time of Markov chains I

Mixing time of a Markov chain

The time a Markov chain takes to converge to its stationary distribution



τ_1



τ_2

The mixing time of Markov chains II

$$\tau_1 \gg \tau_2$$

Second idea: use the mixing time of $G_i \cup G_j$

- Compute $\tau_{ij} = \tau\left(G_i \cup G_j\right)$
- $\tau_{ij} \approx \tau_{ii}$: x_i and x_j are similar
- $\tau_{ij} \gg \tau_{ii}$: x_i and x_j are dissimilar

Computing the mixing time

Given a graph G and a corresponding Markov matrix M, the mixing time $\tau(G)$ can be approximated by

$$\tau(G) = \frac{1}{1-\lambda_1}$$

where $\lambda_0 = 1 \geq \lambda_1 \geq \lambda_2$ are the eigenvalues of M.

Algorithm	Performance
IDSC+DP+EMD-L_1	86.56%
Graph Transduction	92.3%
K-NN intersection	93.1%
K-NN mixing time	95.2%

Conclusions

- Many data sources are derived from data manifolds
- Manifolds can be represented by graphs
- The graph structure can be used to improve general-purpose recognition
- Graphs can be analyzed by Markov walk theory
- Probability and diffusion can be propagated
- Intrinsic Markov walk properties such as the commute and mixing times can be used

Thanks You!