HW1 | MATH 121

1.2

10.) first lets put it into the form $Ae^{i\omega t}$

$$z = -4e^{i(2t+3\pi)} = -4e^{i2t}e^{i3\pi} \quad \text{but } e^{i3\pi} = -1$$

$$= 4e^{i2t}$$

or $x = Re(z) = 4\cos 2t$ and $y = Im(z) = 4\sin 2t$

and we have simple harmonic motion

lets use $z = 4e^{i2t}$

$\omega = 2$

$\omega = 2\pi f$ so $f = \frac{\omega}{2\pi} = \frac{\cancel{2}}{\cancel{2}\pi} = \frac{1}{\pi}$

$T = \frac{1}{f} = \frac{1}{\frac{1}{\pi}} = \pi$

$A = 4$

$i = e^{i\frac{\pi}{2}}$

$$v = Im\left(\frac{dz}{dt}\right) = Im\left(8ie^{i2t}\right) = Im\left(8e^{i\frac{\pi}{2}}e^{i2t}\right) = Im\left(8e^{i(2t+\frac{\pi}{2})}\right)$$

$$v = 8\sin\left(2t + \frac{\pi}{2}\right)$$

note: $\sin(x + \frac{\pi}{2}) = \cos x$

the velocity amplitude will be 8

7.2

14.) pendulum 1 $X_1 = 4 \sin \frac{\pi}{3} t$ $w_1 = \frac{\pi}{3}$

pendulum 2 $X_2 = 3 \sin \frac{\pi}{4} t$ $w_2 = \frac{\pi}{4}$

now $T = \frac{1}{f}$ and $w = 2\pi f$

so $T = \frac{2\pi}{w}$

$T_1 = \frac{2\cancel{\pi} 3}{\cancel{\pi}} = 6$ $T_2 = \frac{2\cancel{\pi} 4}{\cancel{\pi}} = 8$

the period is the time it takes to complete 1 cycle

pendulum 1 is at $x = 0$ at times 3, 6, 9, 12, 15, 18, ...

pendulum 2 is at $x = 0$ at times 4, 8, 12, 16, 20, ...

so ~~nt~~ the next time they are together again

is at $t = 12$

7.3)
9.)

$$y = \left(A + B\sin 2\pi f t\right)\sin 2\pi f_c\left(t - \frac{x}{V}\right)$$

$$= A\sin 2\pi f_c\left(t - \frac{x}{V}\right) + B\underbrace{\sin 2\pi f t \sin 2\pi f_c\left(t - \frac{x}{V}\right)}$$

break up using

$$\cos(A' - B') = \cos A'\cos B' + \sin A'\sin B'$$

$$- \quad \cos(A' + B') = \cos A'\cos B' - \sin A'\sin B'$$

$$\cos(A' - B') - \cos(A' + B') = 2\sin A'\sin B'$$

for us $A' = 2\pi f t$, $B' = 2\pi f_c\left(t - \frac{x}{V}\right)$

so $$y = A\sin 2\pi f_c\left(t - \frac{x}{V}\right) + \frac{B}{2}\cos(A' - B') + \frac{B}{2}\cos(A' + B')$$

$$y = A\sin 2\pi \underline{f_c}\left(t - \frac{x}{V}\right) + \frac{B}{2}\cos\left(2\pi\underline{(f - f_c)t} + 2\pi f_c\frac{x}{V}\right)$$

$$+ \frac{B}{2}\cos\left(2\pi\underline{(f + f_c)t} - 2\pi f_c\frac{x}{V}\right)$$

for our graph we plot $\underline{y = (3 + \sin 2\pi t)\sin 40\pi t}$

from $t = 0$ to $t = 1$

graph on next page

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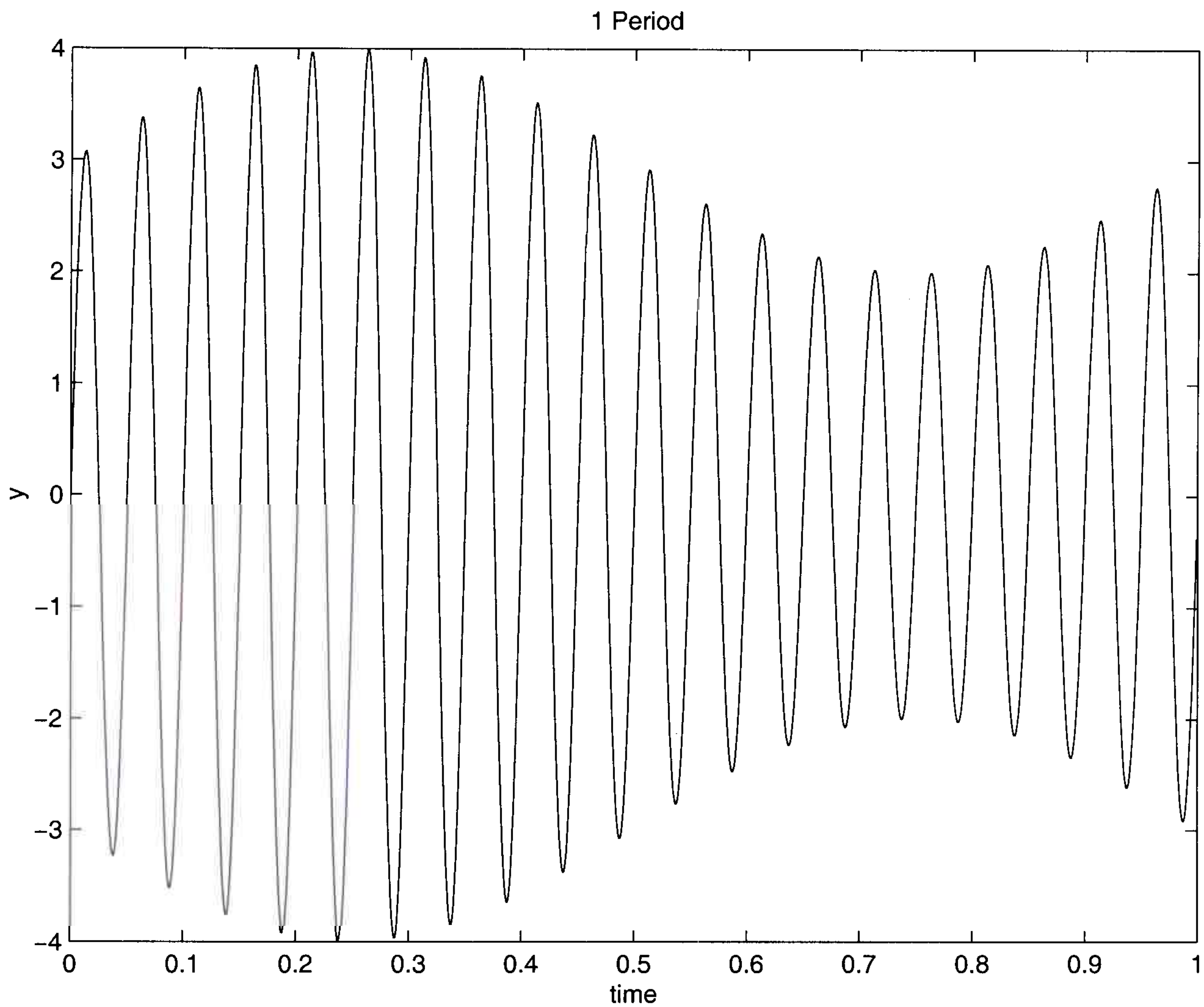

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7.4

13.) $\boxed{\int_a^b \sin^2 kx + \cos^2 kx = \int_a^b 1 = b-a}$ (A)

$$\int_{-n\frac{\pi}{2}}^{n\frac{\pi}{2}} \sin^2 x = \int_{-n\frac{\pi}{2}}^{n\frac{\pi}{2}} \cos^2 x$$ n is any natural number

now $\int_a^b \sin^2 kx = \frac{1}{k}\int_{ak}^{bk} \sin^2 x$

and $\int_{ak}^{bk} \sin^2 x = \int_{-n\frac{\pi}{2}}^{n\frac{\pi}{2}} \sin^2 x$ if $bk-ak=n\pi$

also $\int_{ak}^{bk} \cos^2 x = \int_{-n\frac{\pi}{2}}^{n\frac{\pi}{2}} \cos^2 x$ if $bk-ak=n\pi$

(B) thus $\boxed{\int_{a}^{b} \sin^2 kx = \int_a^b \cos^2 kx \quad \text{if } bk-ak=n\pi}$

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finally $\int_a^b \sin^2 kx + \cos^2 kx = 2\int_a^b \sin^2 kx$ by (B)

and $2\int_a^b \sin^2 kx = b-a$ by (A)

so $\boxed{\int_a^b \sin^2 kx = \frac{b-a}{2} \quad \text{if } bk-ak=n\pi}$ (note: same for $\int \cos^2$)

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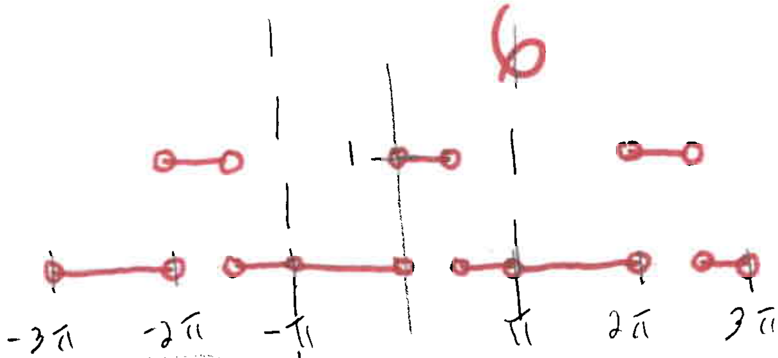
2.) 7.5



$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\int_0^{\pi/2} 1\,dx = \frac{1}{2}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx = \frac{1}{\pi}\int_0^{\pi/2} \cos nx\,dx$$

$$= \frac{1}{n\pi}\Big|_0^{\pi/2} \sin nx = \frac{1}{n\pi}\left(\sin n\left(\frac{\pi}{2}\right)\right) \qquad \underline{n \geq 1}$$

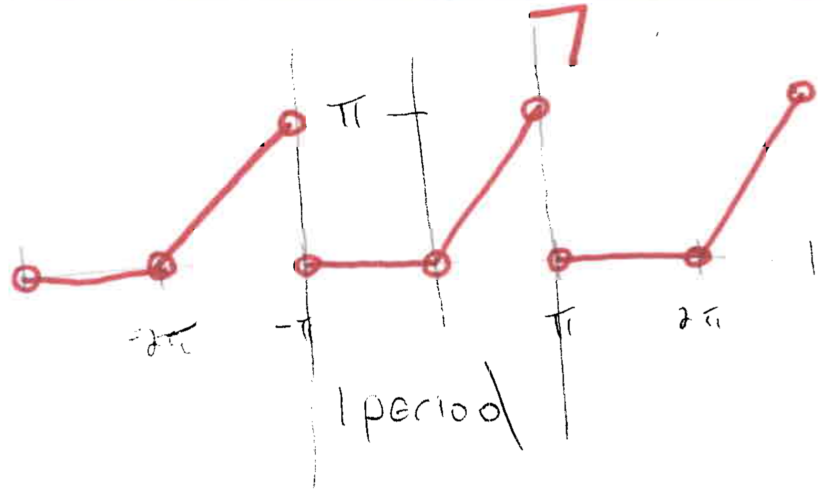
$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\int_0^{\pi/2} \sin nx\,dx = \frac{-1}{n\pi}\Big|_0^{\pi/2} \cos nx$$

$$= \frac{-1}{n\pi}\left(\cos n\left(\frac{\pi}{2}\right) - 1\right) \qquad \underline{n \geq 1}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$$

$$= \frac{1}{4} + \frac{1}{\pi}\left(\frac{\cos x}{1} - \frac{\cos 3x}{3} + \cdots\right) + \frac{1}{\pi}\left(\sin x + \frac{2\sin 2x}{2} + \frac{\sin 3x}{3} + \cdots\right)$$

7)
7.5



$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\int_0^{\pi} x\,dx = \frac{1}{\cancel{\pi}}\frac{\pi^{\cancel{2}}}{2} = \frac{\pi}{2}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx = \frac{1}{\pi}\int_0^{\pi} x\cos nx\,dx$$

$$\left(\text{use } (x\sin nx)' = \sin nx + nx\cos nx\right)$$

$$= \frac{1}{\pi n}\left[\Big|_0^{\pi} x\sin nx - \int_0^{\pi} \sin nx\,dx\right]$$

(use $\sin n\pi = 0$ for $n \in \mathbb{N}$)

$$= \frac{1}{n^2\pi}\Big|_0^{\pi} \cos nx = \frac{1}{n^2\pi}\left(\cos n\pi - 1\right)$$

$$a_n = \begin{cases} \frac{-2}{n^2\pi} & n \text{ odd} \\ 0 & n \text{ even} \end{cases} \qquad n \geq 1$$

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$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\int_{0}^{\pi} x\sin nx\,dx$$

Use $(x\cos nx)' = \cos nx - nx\sin nx$

$$= \frac{1}{n\pi}\left[\int_0^{\pi}\cos nx\,dx - \Big|_0^{\pi} x\cos nx\right]$$

$\cos nx$ is an odd fn. about $x = \frac{\pi}{2}$

so integral 0

$$= -\frac{\cancel{\pi}\cos n\pi}{n\cancel{\pi}} = \frac{(-1)^{n+1}}{n} \qquad n \geq 1$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos nx + b_n\sin nx$$

$$= \frac{\pi}{4} - \frac{2}{\pi}\left(\cos x + \frac{\cos 3x}{9} + \cdots\right) + \left(\sin x - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} + \cdots\right)$$

13.) find b_n

7.5

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$$

multiply both sides by $\sin mx$ and integrate from $-\pi$ to π

$$\int_{-\pi}^{\pi} f(x) \sin mx = \int_{-\pi}^{\pi} \frac{a_0}{2} \sin mx + \sum_{n=1}^{\infty} \int_{-\pi}^{\pi} a_n \cos nx \sin mx$$

③ ②

$$+ \int_{-\pi}^{\pi} b_n \sin nx \sin mx$$

①

use the fact that

① $\int_{-\pi}^{\pi} \sin nx \sin mx = \begin{cases} \pi & m=n \\ 0 & m \neq n \end{cases}$

② $\int_{-\pi}^{\pi} \cos nx \sin mx = 0$

③ $\sin x$ is an odd function

$$\int_{\pi}^{\pi} f(x) \sin nx = b_n \pi$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx$$

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7.12

2.) $f(x) = \sum_{n=-\infty}^{\infty} c_n e^{2\pi i n x}$

$$C_n = \int_0^1 f(x) e^{-2\pi i n x}$$

$$C_0 = \int_0^1 (x - \tfrac{1}{2})\, dx = 0$$ since $(x - \frac{1}{2})$ is an odd function about $x = \frac{1}{2}$

$$C_n = \int_0^1 (x - \tfrac{1}{2}) e^{-2\pi i n x} dx = \underbrace{\int_0^1 x e^{-2\pi i n x}}_{(A)} - \frac{1}{2} \underbrace{\int e^{-2\pi i n x}}_{(B)}$$

look at (B)

$$\int_0^1 e^{-2\pi i n x} = \frac{-1}{2\pi i n}\left(e^{-2\pi i n} - 1\right) = 0$$ since $e^{-2\pi i n} = 1$ for all $n \in \mathbb{Z}$

look at (A)

(use $\left(x e^{-2\pi i n x}\right)' = e^{-2\pi i n x} - 2\pi i n x e^{-2\pi i n x}$)

$$\int_0^1 x e^{-2\pi i n x} = \frac{1}{2\pi i n} \underbrace{\int_0^1 e^{-2\pi i n x}}_{0} \; \frac{-1}{2\pi i n} \Big|_0^1 x e^{-2\pi i n x} = \frac{-e^{-2\pi i n}}{2\pi i n} = \frac{-1}{2\pi i n}$$

so $C_0 = 0$

$C_n = \dfrac{i}{2\pi n}$