Computer Assignment 1 (due Monday, April 24)

Euler's, Midpoint, Modified Euler and Runge-Kutta order four method

Consider following two initial value problems:

$$\frac{dy}{dt} = 2y, \qquad 0 \leq t \leq 2, y(0) = 1, \tag{1}$$

$$\frac{dy}{dt} = y - t^2 + 1, \qquad 0 \leq t \leq 2, y(0) = 0.5 \tag{2}$$

(i) Solve the initial value problems (1) and (2) numerically using three step sizes, $h_1 = 0.1, h_2 = 0.01, h_3 = 0.001$ using:

(a) Euler's method
(b) Midpoint method
(c) Modified Euler method
(d) Runge-Kutta order four method .

Here solving numerically means that you write programs in matlab and compute the solution and plot it as a graph. Do not print out the sequence of numbers! It is a waste of papers, in particular, for small step sizes such as $h_3 = 0.001$.

(ii) Solve the initial value problems (1) and (2) analytically and compute the errors between the actual solution and your approximations as function of time in all considered cases.

(iii) Discuss the results and compare the methods. Use graphics to illustrate your results (plots of the actual solutions, approximations, errors,...).
Note: Your programs for Euler's method, midpoint method, etc. should be written such that they can handle general initial value problems, not only the ones given above.
Do not use functions like *ode23*,...

Format for Computation Problems

Your task in each of the programming assignments is to write a brief paper which answers the given questions and illustrates your ideas in clear and concise prose. The report should separate the required tasks and document each in the appropriate section: *Analysis, Computer Program, Results*.

Analysis (30%): Brief statement of the problem. Mathematical derivations necessary to solve the problem. Brief description of all algorithms you plan to use in your code. Discussion of numerical considerations (if applicable)

Computer Program (30%): The source code in matlab should be readable and printed with margins. Internal comments should describe algorithms and variables, relating them to those described in your Analysis section. Briefly describe input and output to and from your code. Do not expect bugs to be found during the grading process.

Results (40%): Output of your program and explanation of the results. Answers on qualitative questions. Discussion (why it worked, why it did not work, comparison to the predictions, error bounds)

Computer assignments may be done individually or in groups of up to three students (but not more!).