

Computer Assignment 2 (due Wednesday, May 10)

Runge-Kutta method, Runge-Kutta Fehlberg method, Adams-Bashforth method

Problem 1:

Use the Runge-Kutta-Fehlberg algorithm with tolerances 10^{-4} and 10^{-8} to approximate the solution to the following initial value problem:

$$\frac{dy}{dt} = 1 + y^2, \quad 0 \leq t \leq 1.4, \quad y(0) = 0$$

with $h_{max} = 0.1$, $h_{min} = 0.01$. The actual solution is $y(t) = \tan(t)$. Analyze the error of your approximation compared to the actual solution. Compare your results to the solutions obtained by using the Matlab procedures *ode23*, *ode45*. Discuss the results.

Problem 2:

Use the Adams-Bashforth Four-Step method solve the initial value problem:

$$\frac{dy}{dt} = 1 + \frac{y}{t}, \quad 1 \leq t \leq 2, \quad y(1) = 2$$

using $h = 0.1$ and $h = 0.01$. Compute the starting values using the Runge-Kutta method. The actual solution is $y(t) = t \log t + 2t$. Use this information to estimate the local truncation error of this method. Does it agree with the experimental results? Compare the approximation to the approximation obtained by Runge-Kutta-4 and to the actual solution. Comment on your results.

Note: Your programs should be written such that they can handle general initial value problems, not only the ones given above. Make use of graphics to illustrate your results.

Format for Computation Problems

Your task in each of the programming assignments is to write a brief paper which answers the given questions and illustrates your ideas in clear and concise prose. The report should separate the required tasks and document each in the appropriate section: *Analysis, Computer Program, Results*.

Analysis (30%): Brief statement of the problem. Mathematical derivations necessary to solve the problem. Brief description of all algorithms you plan to use in your code. Discussion of numerical considerations (if applicable)

Computer Program (30%): The source code in matlab should be readable and printed with margins. Internal comments should describe algorithms and variables, relating them to those described in your Analysis section. Briefly describe input and output to and from your code. Do not expect bugs to be found during the grading process.

Results (40%): Output of your program and explanation of the results. Answers on qualitative questions. Discussion (why it worked, why it did not work, comparison to the predictions, error bounds)

Computer assignments may be done individually or in groups of up to three students (but not more!).