## **Computer Assignment 3** (due Wednesday, May 24)

**Predictor-corrector methods, stiff differential equations**

**Problem 1:**
Find out numerically at which values of $h$ Euler's method applied to the initial value problem

$$\frac{dy}{dt} = -30y, 0 \leq t \leq 1, y(0) = 1$$

becomes unstable. Compare this result with the analytical estimate. Use the Runge-Kutta-4 method to solve this problem with $h = 0.1$.

**Problem 2:**
Using Adams fourth order predictor-corrector algorithm solve the initial value problem:

$$\frac{dy}{dt} = -2y + e^t, 0 \leq t \leq 1, y(0) = 1$$

with $h = 0.1$ and $h = 0.01$. Compute the numerical errors as functions of time. How would you compare this method with Runge-Kutta order 4?

**Problem 3:**
The differential equation

$$y' = y^2(1 - y)$$

is a simple model of flame propagation. The initial condition is

$$y(0) = 10^{-4}, \quad 0 \leq t \leq 2 \cdot 10^4 .$$

Solve this problem with the Implicit Trapezoidal method and with Runge-Kutta-Fehlberg method (choose "reasonable" step sizes and tolerance). Compare and comment on the results (in particular observe the choice of step sizes for RKF). Which type of IVP is this? How does Matlab's function *ode23s* perform in this case?

**Note:** Your programs should be written such that they can handle general initial value problems, not only the ones given above. Make use of graphics to illustrate your results.

**Format for Computation Problems**

Your task in each of the programming assignments is to write a brief paper which answers the given questions and illustrates your ideas in clear and concise prose. The report should separate the required tasks and document each in the appropriate section: *Analysis, Computer Program, Results.*

Analysis (30%): Brief statement of the problem. Mathematical derivations necessary to solve the problem. Brief description of all algorithms you plan to use in your code. Discussion of numerical considerations (if applicable)

Computer Program (30%): The source code should be readable and printed with margins. Internal comments should describe algorithms and variables, relating them to those described in your Analysis section. Briefly describe input and output to and from your code. Do not expect bugs to be found during the grading process.

Results (40%): Output of your program and explanation of the results. Answers on qualitative questions. Discussion (why it worked, why it did not work, comparison to the predictions, error bounds)

Computer assignments may be done individually or in groups of up to three students (but not more!).