Computer Assignment 4 (due Monday, June 5)

Boundary value problems

Problem 1:
Use the Linear Shooting method with Runge-Kutta order 4 method with the steps $h = 0.1$ and $h = 0.01$ to solve the boundary value problem:

$$y'' = -\frac{2}{x}y' + \frac{2}{x^2}y + \frac{\sin(\ln x)}{x^2}, \qquad 1 \le x \le 2, y(1) = 1, y(2) = 2.$$

The exact solution is

$$y = c_1 x + \frac{c_2}{x^2} - \frac{3}{10}\sin(\ln x)) - \frac{1}{10}\cos(\ln x),$$

where

$$c_1 = \frac{11}{10} - c_2,$$

and

$$c_2 = \frac{1}{70}[8 - 12\sin(\ln 2) - 4\cos(\ln 2)].$$

Compute the errors as functions of time. What do you think is the order of this method?
Problem 2:
Solve the above boundary value problem using the finite-difference method (take advantage of the special structure of the matrix when solving the linear system of equations) with $h = 0.1$ and $h = 0.01$. Compare the accuracy and the computational effort (for the later you may use the *flops* routine) of this method to the linear shooting method of Problem 1.
Problem 3:
The Van der Pol equation
$$y'' - \mu(y^2 - 1)y' + y = 0$$

governs the flow of current in a vacuum tube. Let $\mu = \frac{1}{2}, y(0) = 0, y(2) = 1$. Approximate the solution $y(t)$ for $0 \le t \le 2$ with $h = 0.1$ and $h = 0.01$ using the Non-linear Shooting method with Newton's method.

Note: Your programs should be written such that they can handle general initial value problems, not only the ones given above. Make use of graphics to illustrate your results.

Format for Computation Problems

Your task in each of the programming assignments is to write a brief paper which answers the given questions and illustrates your ideas in clear and concise prose. The report should separate the required tasks and document each in the appropriate section: *Analysis, Computer Program, Results.*

Analysis (30%): Brief statement of the problem. Mathematical derivations necessary to solve the problem. Brief description of all algorithms you plan to use in your code. Discussion of numerical considerations (if applicable)

Computer Program (30%): The source code should be readable and printed with margins. Internal comments should describe algorithms and variables, relating them to those described in your Analysis section. Briefly describe input and output to and from your code. Do not expect bugs to be found during the grading process.

Results (40%): Output of your program and explanation of the results. Answers on qualitative questions. Discussion (why it worked, why it did not work, comparison to the predictions, error bounds)

Computer assignments may be done individually or in groups of up to three students (but not more!).