7.2.3 (a) $\mathcal{F}[f(x-a)] = \int e^{-i\xi x} f(x-a)\, dx = \int e^{-i\xi x} e^{-i\xi a} f(x)\, dx$

$$= e^{-i\xi a} \int e^{-i\xi x} f(x)\, dx = e^{-i\xi a} \hat{f}(\xi).$$

$$\mathcal{F}[e^{iax} f(x)] = \int e^{-i\xi x} \cdot e^{iax} f(x)\, dx = \int e^{ix(a-\xi)} f(x)\, dx = \hat{f}(\xi-a)$$

(b) $\hat{f}_\delta(\xi) = \int e^{-i\xi x} f_\delta(x)\, dx = \int e^{-i\xi x} \frac{1}{\delta} f\left(\frac{x}{\delta}\right) dx$

Let. $z = x/\delta$ $\quad x = z\delta \quad dz = \frac{1}{\delta} dx.$

$$= \int e^{-i\xi\delta z} f(z)\, dz = \hat{f}(\delta\xi)$$

$\mathcal{F}[f(\delta x)] = \int e^{-i\xi x} f(\delta x)\, dx \qquad$ let $\quad z = \delta x. \quad dz = \delta dx.$

$$= \int e^{-i\xi\frac{z}{\delta}} \frac{1}{\delta} f(z)\, dz = \frac{1}{\delta} \int e^{-i\xi z\left(\frac{1}{\delta}\right)} f(z)\, dz.$$

$$= \hat{f}_{\delta}(\xi)_{\,\shortparallel}$$

7.2.4. $f(x) = \begin{cases} 1 & -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$

$$\hat{f}(\xi) = \int_{-1}^{1} e^{-i\xi x}\, dx = \frac{e^{-i\xi} - e^{i\xi}}{-i\xi} = \frac{2\sin\xi}{\xi}\ _{\shortparallel}$$

$$(f * f)(x) = \begin{cases} x+2 & -2 \le x \le 0 \\ 2-x & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

$$(f*f)^{\wedge}(\xi) = \int_{-2}^{0} e^{-i\xi x} (x+2)\, dx + \int_{0}^{2} e^{-i\xi x} (2-x)\, dx$$

$$= \frac{1 + 2i\xi - e^{2i\xi}}{\xi^2} + \frac{1 - 2i\xi - e^{-2i\xi}}{\xi^2} = \frac{4\sin^2\xi}{\xi^2}$$

$$\therefore\ (\hat{f})^2 = (f*f)^{\wedge}\ _{\shortparallel}$$

7.26 $\displaystyle\int_{(n-1)\pi}^{n\pi} \frac{1}{x}|\sin x|\cdot dx = \pi\int_0^1 \frac{|\sin\pi((n-1)+t)|}{\pi((n-1)+t)}\,dt$ by $x = \pi((n-1)+t)$

$$= \pi\int_0^1 \frac{\sin\pi t}{\pi((n-1)+t)}\cdot dt \geq \int_0^1 \frac{\sin\pi t}{(n-1)+t}\,dt \geq \int_0^1 \frac{\sin\pi t}{n}\,dt = \frac{2}{\pi n}$$

$$\int_0^\infty x^{-1}|\sin x|\cdot dx = \sum_{n=1}^{\infty}\int_{(n-1)\pi}^{n\pi} x^{-1}|\sin x|\cdot dx \geq \sum_{n=1}^{\infty}\frac{2}{\pi n}$$

Since $\sum \frac{1}{n}$ diverges, $\int_0^\infty x^{-1}\cdot|\sin x|\cdot dx = \infty$