SOLUTION TO 129 HW1

1.1.1 $u(x,t) = t^{-\frac{1}{2}} e^{-\frac{x^2}{4kt}}$. We take derivatives and get

$$u_t = -\frac{1}{2} t^{-\frac{3}{2}} e^{-\frac{x^2}{4kt}} + \frac{x^2}{4k} t^{-\frac{5}{2}} e^{-\frac{x^2}{4kt}}$$
$$u_x = -\frac{x}{2k} t^{-\frac{3}{2}} e^{-\frac{x^2}{4kt}}$$
$$u_{xx} = -\frac{1}{2k} t^{-\frac{3}{2}} e^{-\frac{x^2}{4kt}} + \frac{x^2}{4k^2} t^{-\frac{5}{2}} e^{-\frac{x^2}{4kt}}$$

It is easy to see that $u_t = k u_{xx}$.

1.1.3 $u(x,y) = \log(x^2 + y^2)$. We take partial derivatives and get

$$u_x = \frac{2x}{x^2+y^2}$$
$$u_{xx} = \frac{2(y^2-x^2)}{(x^2+y^2)^2}$$
$$u_y = \frac{2y}{x^2+y^2}$$
$$u_{yy} = \frac{2(x^2-y^2)}{(x^2+y^2)^2}$$

It is easy to see that $u_{xx} + u_{yy} = 0$.

1.1.4 $u(x,y,z) = (x^2+y^2+z^2)^{-\frac{1}{2}}$. We take partial derivatives and get

$$u_x = -\frac{1}{2}(x^2+y^2+z^2)^{-\frac{3}{2}} \cdot 2x = -\frac{x}{(x^2+y^2+z^2)^{\frac{3}{2}}} = -x(x^2+y^2+z^2)^{-\frac{3}{2}}$$
$$u_{xx} = -(x^2+y^2+z^2)^{-\frac{3}{2}} - x \cdot (-\frac{3}{2})(x^2+y^2+z^2)^{-\frac{5}{2}} \cdot 2x = \frac{2x^2-y^2-z^2}{(x^2+y^2+z^2)^{\frac{5}{2}}}$$

Because of symmetry,

$$u_{yy} = \frac{2y^2-x^2-z^2}{(x^2+y^2+z^2)^{\frac{5}{2}}}$$
$$u_{zz} = \frac{2z^2-x^2-y^2}{(x^2+y^2+z^2)^{\frac{5}{2}}}$$

It is easy to see that $u_{xx} + u_{yy} + u_{zz} = 0$.