2.2.1. a. At $\theta=0$, $f(\theta)$ doesn't have the left-hand limit and the right-hand limit. $\Rightarrow$ Not p.w. continuous (You don't have check "Continuity" and "p.w smoothness")

b. $f'(\theta) = \frac{1}{3}(\sin\theta)^{-\frac{2}{3}} \cdot \cos\theta$.

$f'(\theta)$ doesn't have the left-hand and the right hand limit at $\theta=0$.

Not p.w smooth.

$\Rightarrow$ Continuous (then automatically p.w continuous)

c. Continuous and p.w smooth.

d. 

$$f'(\theta) = \begin{cases} -\sin\theta & \theta>0 \\ \sin\theta & \theta<0 \end{cases}$$

$\Rightarrow$ p.w smooth (automatically p.w conti. ~~and conti.~~)

e. Conti. and p.w smooth.

f. p.w. conti. ; $f'(\theta) = \frac{1}{3}(\sin\theta)^{\frac{4}{5}}$ $\theta < \frac{1}{2}\pi$

$f'(\theta)$ doesn't have the left-hand limit and the right-hand limit at $\theta=0$.

2.2.2.

6. The series converges to 0 at discontinuities

7. The series converges to $\frac{1}{2}$ at discontinuities

12. The series converges to $\frac{1}{4a}$ at discontinuities.

18. The series converges to $\frac{e^{b\pi}+e^{-b\pi}}{2} = \cosh(b\pi)$ at discontinuities.

2.2.4.

Use $\theta^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos n\theta$

Let $\theta = \pi$. $\quad \pi^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos n\pi$

Since $\cos n\pi = (-1)^n$ $\quad \pi^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n(-1)^n}{n^2}$.

$$\Rightarrow \frac{2}{3}\pi^2 = 4\sum_{n=1}^{\infty} \frac{1}{n^2}$$

$$\Rightarrow \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}.$$

Let $\theta = 0$. $\quad 0 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos 0$.

Since $\cos 0 = 1$ $\quad \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} = -\frac{\pi^2}{12}$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{\pi^2}{12}$$