SOLUTION TO 129 HW6

2.3.3 Given $\phi(\pi-|\phi|)=\frac{8}{\pi}\sum_1^\infty \frac{\sin(2n-1)\phi}{(2n-1)^3}$.Integrate on each side,$\int_0^\theta \phi(\pi-|\phi|)d\phi=\frac{8}{\pi}\sum_1^\infty \int_0^\theta \frac{\sin(2n-1)\phi}{(2n-1)^3}d\phi+C$

$$\text{left}=\begin{cases}\int_0^\theta \phi(\pi-\phi)d\phi=\frac{\pi}{2}\theta^2-\frac{\theta^3}{3} & \text{if } \theta\geq 0\\ \int_0^\theta \phi(\pi+\phi)d\phi=\frac{\pi}{2}\theta^2+\frac{\theta^3}{3} & \text{if } \theta<0\end{cases}=\frac{\pi}{2}\theta^2-\frac{1}{3}|\theta|^3$$

$$\text{right}=-\frac{8}{\pi}\sum_1^\infty \frac{\cos(2n-1)\theta}{(2n-1)^4}+C$$

Hence, $\frac{\pi}{2}\theta^2-\frac{1}{3}|\theta|^3=-\frac{8}{\pi}\sum_1^\infty \frac{\cos(2n-1)\theta}{(2n-1)^4}+C$, where $C=\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{\pi}{2}\theta^2-\frac{1}{3}|\theta|^3 d\theta=\frac{1}{\pi}\int_0^\pi \frac{\pi}{2}\theta^2-\frac{1}{3}\theta^3 d\theta=\frac{\pi^3}{12}$
$\Rightarrow \sum_1^\infty \frac{\cos(2n-1)\theta}{(2n-1)^4}=-\frac{\pi}{8}(\frac{\pi}{2}\theta^2-\frac{1}{3}|\theta|^3-\frac{\pi^3}{12})=-\frac{\pi^2}{16}\theta^2+\frac{\pi}{24}|\theta|^3+\frac{\pi^4}{96}$.

2.3.6 In the solution, f_{11} and f_{22} mean the function in 11 and 12 respectively, and f'_{11} means the derivative of the function in 11. Let's first derive the relation between f'_{11} and f_{12} by Fourier series.

$$f_{11}=\frac{2}{\pi-a}\sum_1^\infty \frac{\sin na}{n^2}\sin n\theta$$

$$f'_{11}=\frac{2}{\pi-a}\sum_1^\infty \frac{\sin na}{n}\cos n\theta$$

$$\Rightarrow \sum_1^\infty \frac{\sin na}{n}\cos n\theta=\frac{\pi-a}{2}f'_{11}$$

On the other hand,

$$f_{12}=\frac{1}{2\pi}+\frac{1}{\pi}\sum_1^\infty \frac{\sin na}{na}\cos n\theta=\frac{1}{2\pi}+\frac{1}{\pi a}\sum_1^\infty \frac{\sin na}{n}\cos n\theta$$

$$\Rightarrow \sum_1^\infty \frac{\sin na}{n}\cos n\theta=a\pi f_{12}-\frac{a}{2}$$

Hence,

$$\frac{\pi-a}{2}f'_{11}=a\pi f_{12}-\frac{a}{2}$$

This is the relation between the derivative of the function in 11 and the function in 12.We can verify this relation by examining functions. The derivative of the function in 11 is:

$$f'_{11}=\begin{cases}1 & \text{if } |\theta|<a\\ \frac{a}{a-\pi} & \text{if } |\theta|>a\end{cases}$$

After plugging f'_{11} and f_{12} to the relation, you will find the relation is satisfied.