2.3.7 a. $O(n^{-13.2}) = O(n^{-(12+1.2)})$; 12 derivatives

b. Expanding $2^{\pm x}$ in Taylor series,

$$2^x = a_0 + a_1 x + a_2 x^2 + \cdots$$

; infinitly many derivatives

c. $\left(\frac{\cos 2^n\theta}{2^n}\right)' = -\sin 2^n\theta$. $\Sigma \sin^2 2^n\theta$ does not converge except at $\theta=0$ since $\sin 2^n\theta \not\to 0$.

No derivatives

2.4.3

$f(\theta) = \sin\theta$.

Fourier sine: $f(\theta) = \sin\theta = \Sigma a_n \sin n\theta$: $a_1 = 1$ $a_n = 0$ $(n \neq 1)$.

: f converges to 0 at $\theta = 0, \pi$

Fourier cosine:

Construct an even extension of f as follows;

$$f_{even}(\theta) = \begin{cases} \sin\theta & 0 \le \theta \le \pi \\ -\sin\theta & -\pi \le \theta \le 0 \end{cases} = |\sin\theta|.$$

$$\sim \frac{2}{\pi} - \frac{4}{\pi}\Sigma \frac{\cos 2n\theta}{4n^2-1} \quad \text{on } [-\pi, \pi].$$

$\therefore$ F. cosine series: $\frac{2}{\pi} - \frac{4}{\pi}\Sigma \frac{\cos 2n\theta}{4n^2-1}$

converges to $\underbrace{\frac{2}{\pi} - \frac{4}{\pi}\Sigma \frac{1}{4n^2-1}}_{(=0)}$ at $\theta = 0, \pi$

2.4.9

First, we take a linear transform to translate the interval $[0,4]$ to $[0,\pi]$. Let $g(x)=f(\frac{4x}{\pi})$.

Second, compute the cosine series of $g(x)$ on $[0,\pi]$.

$$g(x)=\frac{1}{2}a_0+\sum_1^\infty a_n\cos nx$$

where

$$a_0=0$$

$$\begin{aligned} a_n &= \tfrac{2}{\pi}\int_0^\pi g(x)\cos nx dx \\ &= \tfrac{2}{\pi}\int_0^\pi f(\tfrac{4x}{\pi})\cos nx dx \end{aligned}$$

Let $u=\frac{4x}{\pi}$,

$$a_n=\tfrac{2}{\pi}\int_0^4 f(u)\cos\tfrac{n\pi}{4}u du$$

$$=\tfrac{1}{2}[\int_0^2 f(u)\cos\tfrac{n\pi}{4}u du-\int_2^4 f(u)\cos\tfrac{n\pi}{4}u du]=\tfrac{4}{n\pi}\sin\tfrac{n}{2}\pi$$

Hence,

$$g(x)=\sum_1^\infty\frac{4}{n\pi}\sin\frac{n}{2}\pi\cos nx$$

Since $\sin\frac{n}{2}\pi=0$ when n is even. Let $n=2k-1$,where $k=1,2,....$

$$g(x)=\sum_1^\infty\frac{4(-1)^{k+1}}{(2k-1)\pi}\cos(2k-1)x$$

Third,we transform back to get the cosine series of $f(x)$.

$$f(x)=\frac{4}{\pi}\sum_1^\infty\frac{(-1)^{k+1}}{2k-1}\cos\frac{(2k-1)\pi}{4}x$$