HW9.

3.1.5. $\forall \vec{u_i} \in \{ \vec{u_1}, \cdots, \vec{u_m} \}$; $\vec{a} - \sum_1^m c_j \vec{u_j}$ is orthogonal with $\vec{u_i}$

which means.

$\forall i = 1, 2 \cdots m$

$$< \vec{a} - \sum_1^m c_j \vec{u_j} \, , \, \vec{u_i} > = 0$$

$$< \vec{a}, \vec{u_i} > - \sum_1^m c_j < \vec{u_j}, \vec{u_i} > = 0 .$$

Since $\{ \vec{u_1}, \vec{u_2} \cdots \vec{u_m} \}$ is an orthogonal set.

$$< \vec{u_j} \, , \vec{u_i} > = \begin{cases} 1 & \text{if } j = i \\ 0 & \text{if } j \neq i \end{cases}$$

$$\Rightarrow < \vec{a}, \vec{u_i} > - c_i < \vec{u_i}, \vec{u_i} > = 0$$

$$\Rightarrow c_i = < \vec{a}, \vec{u_i} > . \quad \forall i = 1 \cdots m$$

3.2.2.

$$\int_0^l \frac{2}{l} \cdot \cos(n-\tfrac{1}{2}) \frac{\pi x}{l} \cdot \cos(m-\tfrac{1}{2}) \frac{\pi x}{l} \, dx .$$

$$= \frac{1}{2} \frac{2}{l} \int_0^l \left(\cos(n+m-1) \frac{\pi}{l} x + \cos(n-m) \frac{\pi}{l} x \right) dx$$

$$= \frac{1}{2} \frac{2}{l} \left[\frac{1}{n+m-1} \cdot \frac{l}{\pi} \sin(n+m-1) \frac{\pi}{l} x \Big|_0^l \begin{cases} + \frac{1}{n-m} \cdot \frac{l}{\pi} \sin(n-m) \frac{\pi}{l} \Big|_0^n \Big] & \text{if } n \neq m \\ l & \text{if } n = m \end{cases} \right.$$

$$= \delta_{nm} .$$

3.2.3.

$$\int_{-1}^1 (x^2 + ax + b) \cdot dx = 0 \qquad \frac{1}{3} x^3 \Big|_{-1}^1 + \frac{1}{2} a x^2 \Big|_{-1}^1 + bx \Big|_{-1}^1 = 0$$

$$\Rightarrow \frac{2}{3} + 2b = 0 \qquad b = -\frac{1}{3} .$$

$$\int_{-1}^1 x (x^2 + ax - \tfrac{1}{3}) \, dx = 0 \quad \Rightarrow \frac{1}{4} x^4 + \frac{1}{3} a x^3 - \frac{1}{6} x^2 \Big|_{-1}^1 = 0$$

$$\frac{2}{3} a = 0 \quad . \; a = 0$$

$$\therefore f_2(x) = x^2 - \frac{1}{3}$$