MAT 129 Practice Midterm Exam

The actual midterm exam will be conducted in class on Friday May 7, 2010

Name:____

Student ID #: ____________________________________

- Read each problem carefully.
- Write every step of your reasoning clearly.
- Usually, a better strategy is to solve the easiest problem first.
- This is a closed-book exam. You may not use the textbook, crib sheets, notes, or any other outside material. Do not bring your own scratch paper. Do not bring blue books.
- No calculators/laptop computers/cell phones are allowed for the exam. The exam is to test your basic understanding of the material.
- Everyone works on their own exams. Any suspicions of collaboration, copying, or otherwise violating the Student Code of Conduct will be forwarded to the Student Judicial Board.

Problem #	Score
1 (30 pts)	
2 (20 pts)	
3 (25 pts)	
4 (25 pts)	
Total	

Problem 1 (30 pts)

(a) (10 pts) Compute the Fourier series of $f(\theta) = \theta$ on $[-\pi, \pi]$, which is 2π periodic.

(b) (10 pts) Compute the Fourier *cosine* series of $f(\theta) = \theta$ on $[0, \pi]$. Compare the decay rate of the coefficients with that of Part (a). Which coefficients decay faster?

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(c) (10 pts) Prove

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6},$$

using the result of Part (b).

Problem 2 (20 pts) Let $\{\phi_n(x)\}_{n=1}^{\infty}$ be an orthonormal *set* in $L^2[a,b]$.

(a) (10 pts) For any function $f \in L^2[a,b]$, state *Bessel's inequality* for this function.

(b) (10 pts) Under what condition Bessel's inequality becomes *Parseval's equality*?

Problem 3 (25 pts)

The nth Legendre polynomial is defined as

$$P_n(x) \triangleq \frac{1}{2^n n!}\frac{\mathrm{d}^n}{\mathrm{d}x^n}(x^2-1)^n, \quad n = 0, 1, \ldots$$

The set $\{P_n\}_{n=0}^{\infty}$ form an *orthogonal* basis for $L^2[-1,1]$. Thus any $f \in L^2[-1,1]$ can be written as:

$$f(x) = \sum_{n=0}^{\infty} a_n P_n(x).$$

From the above definition, we also know that $P_n(x)$ is a polynomial of degree n. Thus, a monomial x^M can be always written as $x^M = \sum_{n=0}^{M} a_n P_n(x)$, which is called the *Legendre expansion* of x^M.

(a) (10 pts) Obtain the Legendre expansion of x^2.

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(b) (15 pts) Let $\mathcal{P}_1$ be a set of all possible polynomial of degree 1, i.e., a set of all possible straight lines in $\mathbb{R}^2$. What is the best linear L^2-approximation to x^2 in $\mathcal{P}_1$ over the interval $[-1, 1]$? In other words, what is the least squares line to approximate x^2 over $[-1, 1]$?

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Problem 4 (25 pts) Find the eigenvalues and normalized eigenfunctions for the problem

$$u'' + \lambda u = 0, \quad u'(0) = u'(1) = 0, \quad \text{on } [0, 1].$$

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