MAT 132A Homework assignment #3

Exercises 4.2 and 4.3

	(RRR)	(RRD)	(RDR)	(RDD)	(DRR)	(DRD)	(DDR)	(DDD)
(RRR)	.8	.2	0	0	0	0	0	0
(RRD)			.4	.6				
(RDR)					.6	.4		
(RDD)							.4	.6
(DRR)	.6	.4						
(DRD)			.4	.6				
(DDR)					.6	.4		
(DDD)							.2	.8

Exercise 4.5

No.

Exercise 4.6

It is immediate for $n = 1$, so assume for n. Now:

$$\begin{aligned} P_{11}^{n+1} &= P_{11}\, P_{11}^{n} + P_{12}\, P_{12}^{n} \\ &= P\left[\frac{1}{2} + \frac{1}{2}(2p-1)^n\right] + (1-p)\left[\frac{1}{2} - \frac{1}{2}(2p-1)^n\right] \\ &= \frac{1}{2} + \frac{1}{2}(2p-1)^n[p-(1-p)] \\ &= \frac{1}{2} + \frac{1}{2}(2p-1)^{n+1} \end{aligned}$$

The verification that P_{22}^{n+1} is the same as above is identical, and the other $n+1$ step transition probabilities are determined since the row sums must be equal 1.

Exercise 4.7

$$
\begin{aligned}
P_{30}^2 + P_{31}^2 &= P_{31}\,P_{10} + P_{33}\,P_{11} + P_{33}\,P_{31} \\
&= (.2)(.5) + (.8)(0) + (.2)(0) + (.8)(.2) \\
&= .26
\end{aligned}
$$

Exercise 4.8

Let the state on any day be the number of the coin that is flipped on that day.

$$
P = \begin{pmatrix} .7 & .3 \\ .6 & .4 \end{pmatrix}
$$

and so,

$$
P^2 = \begin{pmatrix} .67 & .33 \\ .66 & .34 \end{pmatrix}
$$

and

$$
P^3 = \begin{pmatrix} .667 & .333 \\ .666 & .334 \end{pmatrix}
$$

hence

$$
\frac{1}{2}(P_{11}^3 + P_{21}^3) = 0.6665
$$