MAT 167: Homework Assignment #1 (due Monday, April 16, 2012)

First of all, do the following:

- Read Chapter 1 & 2;

- Given the prerequisites for this class (MAT 22A or 67) and the material outlined in the course syllabus, the following concepts are important for you to understand. We will discuss these concepts in my lectures, but do not wait for my lectures. If your memory is vague on these subjects, review them yourself now!

  - Solving linear systems
  - LU decomposition
  - Equivalences of non-singular matrices
  - Rank and span
  - Kernel, domain, range
  - Linear combinations
  - Basis vectors, change of basis
  - Column space and row space
  - Orthogonal vectors, Gram-Schmidt orthogonalization
  - Dot product and outer product
  - Eigenvalues, eigenvectors, diagonalization

Problem 1. This is a simple MATLAB exercise, in fact, a small scale version of Example 1 in my Lecture01.

(a) Download the data file: http://www.math.ucdavis.edu/~saito/courses/167.s12/hw01.mat to your working directory, name it as hw01.mat, and load it into your MATLAB session by:

```
>> load hw01;
```

Then, draw the signal x in the data file using the following commands:

```
>> figure(1);
>> stem(x); hold on; plot(x); grid;
```

Note that this signal x consists of only 8 points, i.e., a very short signal (vector).

(b) In a different figure window, draw the 8 basis vectors stored as column vectors of the matrix U as follows:

```
>> figure(2);
>> for k=1:8
      subplot(8,1,k);
      stem(U(:,k)); axis([0 9 -0.5 0.5]); axis off; hold on;
   end
>> for k=1:8
      subplot(8,1,k);
      plot(U(:,k));
   end
```

You may need to see the details of these 8 plots by enlarging the window to a full screen. Print this figure and attach it to your HW submission.

(c) Compute the expansion coefficients (i.e., the weights of the linear combination) of x with respect to the basis vectors $U(:,1), \ldots, U(:,8)$ via

```
>> a=U'*x;
```

(d) Check the values of the entries of the coefficient vector a and create a new vector $a2$ of length 8 whose only nonzero entries are the two largest entries of a in terms of their absolute values.

(e) Construct an approximation $x2$ of x using $a2$. Then, plot $x2$ over Figure 1 as follows:

```
>> figure(1); stem(x2,'r*'); plot(x2,'r');
```

(f) Now, instead of $a2$, let's construct $a4$ of length 8 whose only nonzero entries are the four largest entries of a in terms of their absolute values. Then,

(g) Construct an approximation $x4$ of x using $a4$. Then, plot $x4$ over Figure 1 as follows (note using the different color from $x2$):

```
>> figure(1); stem(x4,'gx'); plot(x4,'g');
```

Then, print out Figure 1, and attach it to your HW submission.

(h) Consider now $x8$, which is just a full reconstruction without throwing out any coefficients, i.e.,

```
>> x8=U*a;
```

Finally, compute the relative error of $x8$ by

```
>> sqrt(sum((x-x8).^2)/sum(x.^2))
```

and report the result. Similarly compute the relative error of $x4$ and $x2$, and report the results.

Problem 2. Consider the following set of terms (words) and documents (or rather book titles):

Terms		Documents	
T1:	Book (Handbook, BOOK)	D1:	The Princeton Companion to Mathematics
T2:	Equation (Equations)	D2:	NIST Handbook of Mathematical Functions
T3:	Function (Functions)	D3:	Table of Integrals, Series, and Products
T4:	Integral (Integrals)	D4:	Linear Integral Equations
T5:	Linear	D5:	Proofs from THE BOOK
T6:	Mathematics (Mathematical)	D6:	The Book of Numbers
T7:	Number (Numbers)	D7:	Number Theory in Science and Communication
T8:	Series	D8:	Green's Functions and Boundary Value Problems
		D9:	Discourse on Fourier Series
		D10:	Basic Linear Partial Differential Equations
		D11:	Mathematical Physics, An Advanced Course

(a) Construct 8×11 term-document matrix.

(b) Suppose we want to query "Integral Equation." Construct the query vector q.

(c) Find three closest documents for the query in (b).

Problem 3. (This is another review problem.)

At the beginning of 2009, the population of California was 36,453,973, The population living in the United States but outside of California was 271,491,582. During that year, 458,682 people moved to California from another state. Similarly, 545,921 people moved from California to elsewhere in the United States. Set up a matrix vector multiplication problem whose solution shows the population changes in California and in the rest of the United States for 2009.

Problem 4. (This is yet another review problem.)

Use the Gram-Schmidt Process to construct an orthonormal basis for $\mathbb{R}^4$ starting with the following set of vectors.

(a)

$$x_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \qquad x_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \qquad x_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \qquad x_4 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

(b)

$$y_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \qquad y_2 = \begin{bmatrix} 4 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \qquad y_3 = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \qquad y_4 = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

Problem 5. (This is also a review problem.)

Consider the following matrix:

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 5 & 7 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) View this matrix as a linear transformation between two vector spaces. Identify the domain and the range space (codomain) of this linear transformation.

(b) Identify the image of this linear transformation.

(c) Write a basis for the image of this linear transformation.

(d) Identify the kernel of this linear transformation.

(e) Write a basis for the kernel of this linear transformation.

(f) What is the column space of this linear transformation?

(g) What is the row space of this linear transformation?

(h) What is the rank of the matrix A?

Problem 6. Convert the following binary numbers to decimal:

(a) 11111

(b) 1000000

(c) 1001101101

(d) 10101010

(e) 000011110000

And then, convert the following decimal numbers to binary:

(f) 73

(g) 127

(h) 402

(i) 512

(j) 1000