MAT 167: Homework Assignment #4 (due Monday, May 7)

First of all, do the following:

- Read Chapters 4 & 5.

Problem 1. Let $E \in \mathbb{R}^{m \times m}$ that extracts the "even part" of an m-vector: $E\boldsymbol{x} = (\boldsymbol{x} + F\boldsymbol{x})/2$, where $F \in \mathbb{R}^{m \times m}$ flips $\boldsymbol{x} = [x_1, \ldots, x_m]^{\mathsf{T}}$ to $\boldsymbol{x} = [x_m, \ldots, x_1]^{\mathsf{T}}$. Is E an orthogonal projector, an oblique projector, or not a projector at all? What are its entries?

Problem 2. Let $A \in \mathbb{R}^{m \times n}$, $m \geq n$, and let its QR factorization (regardless of reduced or full) be $A = QR$. Show that $\|A\|_F = \|R\|_F$.

Problem 3. Let

$$A = \begin{bmatrix} 1 & 1 \\ \epsilon & 0 \\ 0 & \epsilon \end{bmatrix},$$

where ϵ is a small positive number (e.g., 10^{-7}) so that ϵ^2 can be ignored numerically.

(a) Compute the reduced QR factorization $A = \widehat{Q}\widehat{R}$ using the classical Gram-Schmidt algorithm by hand.

(b) Compute the reduced QR factorization $A = \widehat{Q}\widehat{R}$ using the modified Gram-Schmidt algorithm by hand.

(c) Compute the full QR factorization $A = QR$ using the Householder triangularization by hand.

(d) Check the quality of these results by computing the Frobenius norm of $\|\widehat{Q}^{\mathsf{T}}\widehat{Q} - I\|_F$ for the results obtained by the CGS and MGS algorithms and $\|Q^{\mathsf{T}}Q - I\|_F$ for the result obtained by the Householder triangularization.

Problem 4. Take $m = 50$, $n = 12$. Using MATLAB's `linspace`, define t to be the m-vector corresponding to linearly spaced grid points from 0 to 1. Using MATLAB's `vander` and `fliplr`, define A to be the $m \times n$ matrix associated with least squares fitting on this grid by a polynomial of order $n-1$. Take $\boldsymbol{b}$ to be the function $\cos(4t)$ evaluated on the grid. Now, calculate and print (to 16 digit precision) the least squares coefficient vector $\boldsymbol{x}$ by the following three methods.

(a) Solving the normal equation explicitly computing $(A^{\mathsf{T}}A)^{-1}$.

(b) Using the MATLAB implementation of the classical Gram-Schmidt algorithm `cgs`, which can be downloaded from http://www.math.ucdavis.edu/~saito/courses/167.s12/cgs.m .

(c) Using the MATLAB implementation of the modified Gram-Schmidt algorithm `mgs`, which can be downloaded from http://www.math.ucdavis.edu/~saito/courses/167.s12/mgs.m .

(d) QR factorization using MATLAB's `qr`, which is based on the Householder triangularization.

(e) `x = A \ b` in MATLAB, which is also based on QR factorization.

(f) The calculations above will produce five lists of twelve coefficients. In each list, shade with red pen (or highlighter/marker) the digits appear to be wrong (affected by rounding error). Comment on what differences you observe. Do the normal equations exhibit instability? You do not have to explain your observations.