MAT 167: Homework Assignment #5 (due Monday, May 14)

First of all, do the following:

- Read Chapter 6.

Problem 1. Determine SVDs of the following matrices by hand calculation:

$$\text{(a)} \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix}, \quad \text{(b)} \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}, \quad \text{(c)} \begin{bmatrix} 0 & 2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \text{(d)} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \quad \text{(e)} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}.$$

[Hint: Let $A = U\Sigma V^{\mathsf{T}}$. Then, if you express $A^{\mathsf{T}}A$ using the SVD of A, then you see that σ_j^2, $\boldsymbol{v}_j$ are the jth eigenvalue and the corresponding eigenvector of $A^{\mathsf{T}}A$.]

Problem 2. Let $A \in \mathbb{R}^{m\times n}$, and suppose $B \in \mathbb{R}^{n\times m}$ is obtained by rotating A 90 degree clockwise on paper. More precisely,

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix} \quad B = \begin{bmatrix} a_{m1} & \cdots & a_{21} & a_{11} \\ \vdots & \vdots & \vdots & \vdots \\ a_{mn} & \cdots & a_{2n} & a_{1n} \end{bmatrix}.$$

Do A and B have the same singular values? Prove that the answer is yes or give a counterexample.

[Hint: Express B as a product of A^{T} and a matrix P that permutes the column vectors of A^{T}.]

Problem 3. Write a MATLAB program which, given a real 2×2 matrix A, plots the right singular vectors $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ in the unit circle and also the left singular vectors $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$ in the appropriate ellipse, as in the figure of my Lecture 13. Apply your program to the matrices in Problem 1, and attach the resulting figures to your homework submission.

Problem 4 Two matrices $A, B \in \mathbb{R}^{m\times m}$ are *orthogonally equivalent* if $A = QBQ^{\mathsf{T}}$ for some orthogonal matrix Q. Is it true or false that A and B are orthogonally equivalent if and only if they have the same singular values?