MAT 167: Applied Linear Algebra
Lecture 26: Reviews/Study Guide for the Final Exam

Naoki Saito

Department of Mathematics
University of California, Davis

June 7, 2017
Outline

1. Basic Notions
2. SVD and its Relatives
3. Real World Applications
Basic Notions

SVD and its Relatives

Real World Applications
Basic Notions Covered by the Midterm Exam

- Matrix-vector multiplication
- Range, nullspace, basis, rank, dimension, linear independence
- Vector & matrix norms
- Inner product
- Orthogonality and orthonormal basis (orthogonal matrix)
- Projectors (including orthogonal projectors)
- QR factorization (classical & modified Gram–Schmidt orthogonalization, Householder triangularization, and Givens rotations)
- Least Squares Problems (pseudoinverse, normal equations)
Outline

1. Basic Notions
2. SVD and its Relatives
3. Real World Applications
SVD

- Definition and meaning of SVD
- Reduced SVD vs Full SVD
- Relation of the singular values and matrix 2-norm and Frobenius norm
- Similarity and difference between SVD and Eigenvalue Decomposition
- How range and nullspace are represented by some singular vectors
The Best Rank $k$ Approximation

- Why the first $k$ terms of the SVD of $A$ is the best rank $k$ approximation of $A$?
- Condition number and SVD
- Rank and SVD
SVD and the Least Squares Problem

- Solution of the Least Squares Problem using the reduced SVD
- Pseudoinverse and SVD
- Pseudoinverse and Orthogonal Projectors
PCA and SVD

- The meaning of Principal Component Analysis (a.k.a. Karhunen-Loève Transform)
- Centered data matrix and Covariance matrix
- classical vs neoclassical data analysis settings
- PCA as an eigenvalue decomposition of a covariance matrix
- Rank of a covariance matrix
- PCA using SVD of a centered data matrix
NNMF and SVD

- The motivation and definition of NonNegative Matrix Factorization
- Computing the NNMF of a given nonnegative matrix using the Alternating Least Squares method
- Its relationship with SVD
- Application to term-document matrix analysis
Outline

1. Basic Notions
2. SVD and its Relatives
3. Real World Applications
Clustering and Classification

- Difference between clustering (unsupervised learning) and classification (supervised learning)
- The $k$-means algorithm for clustering
- The $k$-nearest neighbor method for classification
Handwritten Digit Classification

- Notion of training and test datasets
- A simple classification based on the distances from the mean (representative) digit images
- The $k$-nearest neighbor classification
- Classification using SVD basis vectors (left singular vectors)
Text Mining

- Preprocessing (stop words elimination; stemming)
- How to construct term-document matrices
- The Vector Space Model
- How to measure the similarity between a query and documents
- Performance modeling: Precision vs Recall
- Latent Semantic Indexing $\Rightarrow$ the best rank $k$ approximation of the term-document matrix
- Using $k$-means clustering for text mining
- Using NNMF for text mining
Web Document Searches

- Link structure of the Web: inlinks and outlinks of webpages
- Idea of an authority score and a hub score of a webpage
- Authority scores and hub scores often mutually reinforce each other
- Web graphs, adjacency matrices
HITS

- The HITS (Hyperlink Induced Topic Search) algorithm
- Neighborhood graphs based on query terms
- Power iteration on authority scores and hub scores
- Basics of power iteration (why it works, etc.)
- Strengths and weaknesses of HITS
PageRank

- PageRank computation is independent from query terms
- PageRank vector and power iteration
- Markov chains/random walks interpretation
- “Google” matrix; row stochastic matrices; dangling nodes
- Irreducible and reducible Markov chains
- How to force irreducibility
- Importance (PageRank value) vs relevancy (query topic)
- Importance of sparsity in matrix computations
- Strengths and weaknesses of PageRank