MAT 207B Methods of Applied Mathematics
Homework 1: due Friday, 01/19/18

Problem 1: Show that a moving particle with mass m obeys Newton's equation of motion: $m\ddot{\boldsymbol{r}} = -\nabla V(\boldsymbol{r})$, then it is a conservative field, i.e., $E = T + V$ is a constant independent of time t.

[Hint: Take the time derivative of E.]

Problem 2: Let $\boldsymbol{r}(t) \in \mathbb{R}^3$ be the position of a particle of mass m at time t under the conservative potential field $V(\boldsymbol{r})$ such that the usual Newton's equation of motion

$$m\ddot{\boldsymbol{r}} = -\nabla V \tag{1}$$

holds. Consider the polar coordinate system (r, θ, φ) as follows:

$$\begin{cases} x = r\sin\theta\cos\varphi; \\ y = r\sin\theta\sin\varphi; \quad 0 \le \theta \le \pi,\, 0 \le \varphi < 2\pi; \\ z = r\cos\theta. \end{cases}$$

Convert (1) from the (x, y, z)-coordinate system to the polar coordinate system by computing the first and second time derivatives of the variables (r, θ, φ) and write out three equations in those variables. Note that the computations may be tedious.

[Hint: You may need to use the following relationships:

$$\begin{cases} r^2 = x^2 + y^2 + z^2; \\ \tan\varphi = y/x; \\ \tan^2\theta = (x^2 + y^2)/z^2. \end{cases}$$

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Problem 3: Consider the motion of the same particle as Problem 1.

(a) Represent the Lagrangian $L = T - V = \frac{1}{2}m|\dot{\boldsymbol{r}}|^2 - V$ using the polar coordinate variables (r, θ, φ).

(b) Derive the Euler-Lagrange Equation for each variable in (r, θ, φ), and confirm that these agree with the equations of motion you derived in Problem 2.

Problem 4: Do Problems 2 and 3 on page 7 of the textbook.

Problem 5: Do Problem 8 on page 13.

Problem 6: Do Problem 13 on page 14.

Problem 7: Do Problem 16 on page 14.

Problem 8: Do Problem 18 on page 15.

Problem 9: Do Problem 20 on page 21.