MAT 207B Methods of Applied Mathematics
Homework 2: due Friday, 01/26/18

Problem 1: A flexible rope of length $2a$ made of uniform material is hanged between the two points $(x, y) = (0, 0)$ and $(x, y) = (2b, 0)$ with $b < a$. The potential energy of this rope is:

$$V = \int_0^{2b} \sqrt{1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\, y \,\mathrm{d}x$$

Find a curve of $y = y(x)$ of this rope that minimizes this potential energy (which is called a *catenary*). Note that we have the following relationship:

$$\int_0^{2b} \sqrt{1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x = 2a.$$

Problem 2: Do Problem 23 on page 27.

Problem 3: Do Problem 28 on page 27. Note that in this problem, p, q, r are functions of x.

Problem 4: Do Problem 31 on page 27.

Problem 5: Do Problem 32 on page 33.

Problem 6: Do Problem 33 on page 33.

Problem 7: Find the partial differential equation (with an appropriate boundary condition) whose solution $u = u(x, y)$ minimizes the following functional

$$I = \iint\limits_D \left(u_{xx} + u_{yy}\right)^2 \mathrm{d}x\,\mathrm{d}y = \iint\limits_D \|\Delta u\|^2 \,\mathrm{d}x\,\mathrm{d}y$$

subject to the natural boundary condition where $D \in \mathbb{R}^2$ is a simply-connected domain. Note that the solution of such PDE leads to the natural spline functions in 2D corresponding to the cubic spline functions in 1D.