MAT 207B Methods of Applied Mathematics
Homework 5: due Friday, 02/16/18

Problem 1: Do Problems 13, 14, 15 on pages 104 and 105. Note that approximating that function f by a trigonometric polynomial of five terms turns out to be the same as truncating the Fourier series with $n = 2$, i.e., terms involving a_0, a_1, b_1, a_2, b_2 if they are all nonzeros. If some of them are zeros, you should find the first five nonzero such coefficients.

Problem 2: Do Problems 20 and 21 on page 113.

Problem 3: Do Problem 26 on page 119.

Problem 4: Do Problem 31 on page 125.

Problem 5: Let $f(\theta)$ be 2π-periodic and $f(\theta) = \theta^2$ for $-\pi < \theta < \pi$.

(a) Find its Fourier series.

(b) Using the result of Part (a) and the theorem of the Fourier coefficients of an integral of f *twice*, prove

$$\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}.$$

Problem 6: Let $f(\theta)$ be 2π-periodic and $f(\theta) = \mathrm{e}^{\theta}$ for $-\pi < \theta \le \pi$. Let $\sum_{-\infty}^{\infty} c_n \mathrm{e}^{\mathrm{i}n\theta}$ be its Fourier series; thus $\mathrm{e}^{\theta} = \sum_{-\infty}^{\infty} c_n \mathrm{e}^{\mathrm{i}n\theta}$ for $|\theta| < \pi$. If we formally differentiate this equation, we obtain $\mathrm{e}^{\theta} = \sum_{-\infty}^{\infty} \mathrm{i}nc_n \mathrm{e}^{\mathrm{i}n\theta}$. But then $c_n = \mathrm{i}nc_n$, or $(1 - \mathrm{i}n)c_n = 0$, i.e., $c_n = 0$ for all $n \in \mathbb{Z}$. This is obviously wrong! Where is the mistake? Explain in details.