# MAT 207B Methods of Applied Mathematics
## Homework 3: due Friday, 01/31/25

**Problem 1:** Find the Euler-Lagrange equation for the vibrating elastic rod (for potential energy see HW 1 Problem 8).

**Problem 2:** Consider only such deflections of the vibrating rod for which $\partial u/\partial x$ is small of first order, and neglect all terms small of second and higher order. What is the Euler-Lagrange equation of the vibrating elastic rod under these simplified conditions?

**Problem 3:** Reduce the following partial differential equations to ordinary differential equations by Bernoulli's separation method:

**(a)**

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0,$$

**(b)**

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2},$$

**(c)**

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + \frac{1}{x}\frac{\partial u}{\partial x} + \frac{1}{x^2}\frac{\partial^2 u}{\partial y^2},$$

**(d)**

$$x\frac{\partial^2}{\partial x^2}(xu) + \frac{1}{\sin(y)}\frac{\partial}{\partial y}\left(\sin(y)\frac{\partial u}{\partial y}\right) = 0.$$

**Problem 4:** Find all values of $\lambda$ for which the following boundary value problems have non-trivial solutions:

(a) $y'' + \lambda y = 0,\ y(0) = 0 = y(1)$

(b) $y'' + \lambda y = 0,\ y(0) = 0 = y\left(\frac{3\pi}{2}\right)$.

**Problem 5:** Find constants $a_0, a_1, b_1$ such that

$$I(a_0, a_1, b_1) = \int_0^{2\pi} \left(\phi(x) - \frac{a_0}{2} - a_1\cos(x) - b_1\sin(x)\right)^2 \mathrm{d}x$$

is a minimum where $\phi(x)$ is a given function.

**Problem 6:** Show that $u(x, y) = f(x + iy)$, where $f$ is a twice-differentiable function and $i = \sqrt{-1}$, is a solution of

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$$

**Problem 7:** Show that the functions

$$u(x,y) = \log\sqrt{x^2 + y^2},$$
$$u(x,y) = x^2 - y^2,$$
$$u(x,y) = 2xy$$

are solutions of the partial differential equation in problem 6.

**Problem 8:** Formulate the boundary and initial value problem for a stretched string of length $\pi$ which is initially plucked up at the midpoint; i.e., the point $x = \pi/2$ is raised through a distance $h$ above the rest position and released.

**Problem 9:** Apply Bernoulli's separation method to Problem 8 and state all boundary and initial conditions for $X(x)$ and $T(t)$.