MAT 21C Practice Midterm Exam
The actual midterm will take place on Monday, February 7, 2011 in class.

Name: ________________________________
Student ID #: ________________________________
Discussion Section Time: (Circle 3pm, 4pm, 5pm, 6pm, or 7pm)
Name of Left Neighbor: ________________________________
Name of Right Neighbor: ________________________________
If you are next to the aisle or wall, then please write “aisle” or “wall” appropriately as your left or right neighbor.

• Read each problem carefully.
• Write every step of your reasoning clearly.
• Usually, a better strategy is to solve the easiest problem first.
• This is a closed-book exam. You may not use the textbook, crib sheets, notes, or any other outside material. Do not bring your own scratch paper. Do not bring blue books.
• No calculators/laptop computers/cell phones are allowed for the exam. The exam is to test your basic understanding of the material.
• Everyone works on their own exams. Any suspicions of collaboration, copying, or otherwise violating the Student Code of Conduct will be forwarded to the Student Judicial Board.

<table>
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<th>Problem #</th>
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<tr>
<td>1 (15 pts)</td>
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<td>2 (20 pts)</td>
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<td>3 (20 pts)</td>
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<td>4 (20 pts)</td>
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<td>5 (25 pts)</td>
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<td>Total (100 pts)</td>
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Problem 1  (15 pts) Does the following sequence converge or diverge as \( n \to \infty \)? Give reasons for your answer. If it converges, find the limit.

(a)  (7 pts)
\[
\frac{1}{n^2 e^{-n}}.
\]

(b)  (8 pts)
\[
\frac{1}{n} - \cos \left( \frac{1}{n} \right).
\]

[ Hint: Consider how the graph of \( \cos x \) behave near \( x = 0 \). You may also want to use the fact: \( \cos 1 \approx 0.54 \). ]
Problem 2 (20 pts) Does the following series absolutely converge, conditionally converge, or diverge? Give reasons for your answer.

$$\sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^{n^2}$$

[Hint: You may want to use the following formula for a particular value of \(x\):

$$\lim_{n \to \infty} \left(1 + \frac{x}{n}\right)^n = e^x \quad \forall x \in \mathbb{R}.$$ 

Also, you may want to use the fact: \(e^{-1} \approx 0.36788\).]
Problem 3  (20 pts) Does the following series absolutely converge, conditionally converge, or diverge? Give reasons for your answer.

\[ \sum_{n=1}^{\infty} \frac{\ln n}{n} \]

[ Hint: Use either the Comparison Test or the Integral Test. ]
**Problem 4** (20 pts) Determine the radius and the interval of convergence of the power series:

\[ f(x) = \sum_{n=1}^{\infty} (-1)^{n-1} n(x - 1)^n. \]

Justify your answers.
Problem 5  (25 pts) Let \( f(x) = \cos x \).

(a)  (10 pts) Find the Maclaurin series for \( f(x) \).

(b)  (10 pts) Suppose we want to approximate \( \cos x \) using \( P_2(x) \), i.e., the Taylor polynomial of order 2, centered at \( x = 0 \). Use the Remainder Estimation Theorem to determine the range of \( x \) if we want to keep the magnitude of error between \( \cos x \) and \( P_2(x) \) less than 0.0001, i.e., \( |\cos x - P_2(x)| < 0.0001 \).

[ Hint: You may want to use the fact: \( (0.0006)^{1/3} \approx 0.0843 \). ]

(c)  (5 pts) Prove \( \cos 2\theta = \cos^2 \theta - \sin^2 \theta \) using Euler's Identity.

Score of this page:__________________