# MAT 21C Practice Final Exam

**The actual final exam will take place on Tuesday, March 15, 2011 from 3:30pm to 5:30pm at YOUNG 198.**

Last Name:________________________________

First Name:________________________________

Student ID #: ________________________________

Discussion Section Time: (Circle 3pm, 4pm, 5pm, 6pm, or 7pm)

Name of Left Neighbor: ________________________________

Name of Right Neighbor: ________________________________

If you are next to the aisle or wall, then please write “aisle” or “wall” appropriately as your left or right neighbor.

- Read each problem carefully.
- **Write every step of your reasoning clearly.**
- Usually, a better strategy is to solve the easiest problem first.
- This is a closed-book exam. You may not use the textbook, crib sheets, notes, or any other outside material. Do not bring your own scratch paper. Do not bring blue books.
- No calculators/laptop computers/cell phones are allowed for the exam. The exam is to test your basic understanding of the material.
- Everyone works on their own exams. Any suspicions of collaboration, copying, or otherwise violating the Student Code of Conduct will be forwarded to the Student Judicial Board.

| | Problem # | Score |
|---|---|---|
| 1 | (10 pts) | |
| 2 | (10 pts) | |
| 3 | (10 pts) | |
| 4 | (10 pts) | |
| 5 | (10 pts) | |
| 6 | (10 pts) | |
| 7 | (10 pts) | |
| 8 | (10 pts) | |
| 9 | (10 pts) | |
| 10 | (10 pts) | |
| 11 | (10 pts) | |
| 12 | (10 pts) | |
| Total | (120 pts) | |

**Problem 1** (10 pts) Find the radius of convergence and interval of convergence of the series

$$\sum_{n=0}^{\infty} \frac{n(x+2)^n}{5^{n-1}}.$$

Score of this page:________________

**Problem 2** (10 pts) Consider the function $f(x) = \ln x$.

**(a)** (5 pts) Approximate $f(x)$ by a Taylor polynomial of degree 2 at $x = 2$.

**(b)** (5 pts) How accurate is this approximation when $1 \le x \le 3$?

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**Problem 3** (10 pts) State and prove the *Cauchy-Schwarz Inequality*. Note that you also need to state and prove the condition for the equality to hold.

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**Problem 4** (10 pts) Let $P(1,4,6)$, $Q(-2,5,-1)$, $R(1,-1,1)$.

**(a)** (5 pts) Find the area of the triangle $\triangle PQR$.

**(b)** (5 pts) Find the distance from $P$ to the line $QR$.

**Problem 5** (10 pts) Find the limit if it exists, or show that the limit does not exist.

**(a)** (5 pts)

$$\lim_{(x,y)\to(0,0)} \frac{y^4}{x^4+3y^4}.$$

Hint: Consider $(x, y) \to (0,0)$ along the line $y = mx$.

**(b)** (5 pts)

$$\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}}.$$

Hint: Consider the limit in the polar coordinates $(r,\theta)$.

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**Problem 6** (10 pts) Verify that the function of two variables

$$u(x,t) = \mathrm{e}^{-\alpha^2 k^2 t} \sin kx,$$

where $k$ is an arbitrary positive integer is a solution of the heat conduction equation over the interval $x \in [0, \pi]$

$$u_t = \alpha^2 u_{xx},$$

with the boundary condition

$$u(0,t) = u(\pi,t) = 0 \quad \text{for all } t \geq 0.$$

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**Problem 7** (10 pts) Assuming that the equation

$$\sin x + \cos y = \sin x \cos y.$$

defines $y$ as a differentiable function of $x$, use the Implicit Differentiation Theorem to find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

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**Problem 8** (10 pts) Find the direction in which $f(x, y, z) = \ln(xy^2z^3)$

**(a)** (3 pts) Increases most rapidly at the point $(1, 2, 3)$,

**(b)** (3 pts) Decreases most rapidly at the point $(1, 2, 3)$.

**(c)** (4 pts) Does not change (i.e., is flat) at the point $(1, 2, 3)$.

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**Problem 9** (10 pts)

**(a)** (5 pts) Show the equation of the tangent plane at the point $P_0(x_0, y_0, z_0)$ on the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ is

$$\frac{x_0\,x}{a^2} + \frac{y_0\,y}{b^2} + \frac{z_0\,z}{c^2} = 1.$$

**(b)** (5 pts) Compute the normal line to the same ellipsoid at the same point $P_0$. Furthermore, compute the point where this normal line intersects with the $xy$-plane. Assume that $z_0 \neq 0$.

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**Problem 10** (10 pts) Find the linearization of $f(x,y,z) = \sqrt{x^2+y^2+z^2}$ at the point $(3,2,6)$. Then use it to approximate $f(3.1,1.9,6.2)$ by the three decimal point number. Note that the true value of $f(3.1,1.9,6.2)$ is 7.188 in the three decimal point number.

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**Problem 11** (10 pts) Find the absolute maximum and minimum values of

$$f(x,y) = x^2 - xy + y^2 + 1,$$

on the closed triangular domain bounded by the lines $x = 0,\ y = 2,\ y = 2x$, i.e.,

$$\Omega = \{(x,y) \mid x \geq 0, y \leq 2, y \geq 2x\}.$$

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**Problem 12** (10 pts) Use Lagrange multipliers to find the maximum and minimum values of the function

$$f(x,y) = e^{xy} \quad \text{subject to} \quad x^2 + y^2 = 1.$$

Note that we only consider the real values for $x$ and $y$, not the complex values.

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