229A: Numerical Methods in Linear Algebra
Homework 3: due 2/25/02

Problem 1: Solve Exercise 9.1

Problem 2: Solve Exercise 9.2

Problem 3: Solve Exercise 10.1

Problem 4: Solve Exercise 10.2

Problem 5: Solve Exercise 10.3

Problem 6: Solve Exercise 11.3

Problem 7: Consider a function \( \chi_{[0,1]}(x) \), which is the characteristic function of the interval \([0, 1] \), and its expansion into the Legendre polynomials \( \{P_n\}_{n=0}^{\infty} \).

(a) Show that

\[
\chi_{[0,1]}(x) \sim \frac{1}{2} + \sum_{k=0}^{\infty} \frac{(-1)^k(4k + 3)(2k)!}{4k+1k!(k + 1)!} P_{2k+1}(x).
\]

(b) Using the matlab program you created in Problem 1 (or its modified version), plot the \( \chi_{[0,1]}(x) \) and its \( N \)-term approximation:

\[
S_N(x) = \frac{1}{2} + \sum_{k=0}^{N} \frac{(-1)^k(4k + 3)(2k)!}{4k+1k!(k + 1)!} P_{2k+1}(x),
\]

for \( N = 1, 2, \ldots, 10 \) and \(-1 \leq x \leq 1\).