229B: Numerical Methods in Linear Algebra Final Project/Report: due Friday 03/24/00

For our final project/report, please select one from the following two choices:

Choice 1: Describe your thesis (in fact any) project and how to apply the methods we learned in this course to solve your problem. If you already applied some of the methods to your own problem, that would be great, but you don't have to solve your problem completely. You can just write your plan of actions to solve your problem.

Please write by wordprocessing software. It should be less than 10 pages.

Choice 2: Solve **all** of the following problems.

Problem 1: Which of CG, GMRES, CGN, or BCG would you expect to be most effective for the following $m \times m$ problems $Ax = b$, and why?

(a) A dense nonhermitian matrix with $m = 10^4$, all but three of whose eigenvalues are approximately equal to -1.

(b) The same, but with all but three of the eigenvalues scattered about the region $-10 \leq \mathrm{Real}(\lambda) \leq 10, -1 \leq \mathrm{Imag}(\lambda) \leq 1$.

(c) A sparse nonhermitian matrix with $m = 10^6$ but only 10^7 nonzero entries, with eigenvalues as in (a).

(d) A sparse hermitian matrix with $m = 10^5$ whose eigenvalues are scattered through the interval $[1, 100]$.

(e) The same, except for outlying eigenvalues at 0.01 and $10,000$.

(f) The same, but with additional outliers at -1, -10, and -100.

(g) A sparse, normal matrix with $m = 10^5$ whose eigenvalues are complex numbers scattered about the annulus $1 \leq |\lambda| \leq 2$.

Problem 2: A Matlab program implementing multigrid to solve the discrete model problem on a square is available via anonymous ftp at `ftp://math.ucdavis.edu/pub/users/saito/mg/`, by courtesy of Prof. Jim Demmel at UC Berkeley. Start by running the demonstration (type `mgdemo` and `testfmgv`). Then, try running `testfmg` for different right-hand sides (input array `b`), different numbers of weighted Jacobi iterations before and after each recursive call to the multigrid solver (inputs `jac1` and `jac2`), and different numbers of iterations (input `iter`). The software will plot the convergence rate (ratio of consecutive residuals); does this depend on the size of b, the frequencies in b, the values of `jac1` and `jac2`? For which values of `jac1` and `jac2` is the solution most efficient?

Problem 3: Analyze computational complexity of the FFT-based method (in fact Fast Sine Transform) and the multigrid method for the 1D Poisson equation with N discretized points with the Dirichlet boundary condition. Describe pros and cons of these two methods.