Problem 0: Familiarize yourself to the matlab environment using the matlab primers. (Only applicable for the people who do not have much matlab experience.)

Problem 1: Get Sparse Matrix chapter of matlab manual. Read pages 9-5 to 9-14 and familiarize yourself to the matlab sparse matrix environment.

Problem 2: Consider an $m \times m$ tridiagonal matrix:

\[
T_\alpha = \begin{bmatrix}
\alpha & -1 & 0 & 0 & \cdots & \cdots & 0 \\
-1 & \alpha & -1 & 0 & \ddots & \vdots & \vdots \\
0 & -1 & \alpha & -1 & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\
\vdots & & \ddots & \ddots & \ddots & \ddots & 0 \\
0 & \cdots & \cdots & -1 & \alpha & -1 \\
0 & \cdots & \cdots & \cdots & 0 & -1 & \alpha
\end{bmatrix},
\]  

where $\alpha \in \mathbb{R}$.

(a) Verify that the eigenvalues of $T_\alpha$ are given by

\[
\lambda_j = \alpha - 2 \cos(j\theta), \quad j = 1, \ldots, m,
\]

where $\theta = \frac{\pi}{m+1}$, and that an eigenvector associated with each $\lambda_j$ is

\[
q_j = [\sin(j\theta), \sin(2j\theta), \cdots, \sin(mj\theta)]^T.
\]

(b) Under what condition on $\alpha$ does this matrix become positive definite?

- In the following questions, take $\alpha = 2$.

(c) Will the Jacobi iteration converge for this matrix? If so, what will its convergence factor be?

(d) Will the Gauss-Seidel iteration converge for this matrix? If so, what will its convergence factor be?

(e) For which value of $\omega$ will the SOR iteration converge?

Problem 3: Solve Exercise 33.2 (in Trefethen & Bau)

Problem 4: Solve Exercise 33.3

Problem 5: Solve Exercise 34.1 (Note: There is a typo. In Equation (34.7), $c_{m-1}$ should be $c_{n-1}$.)