229B: Numerical Methods in Linear Algebra
Homework 1: due Friday 01/21/00

Problem 0: Familiarize yourself to the matlab environment using the matlab primers. (Only applicable for the people who do not have much matlab experience.)

Problem 1: Get Sparse Matrix chapter of matlab manual. Read pages 9-5 to 9-14 and familiarize yourself to the matlab sparse matrix environment.

Problem 2: Consider an $m \times m$ tridiagonal matrix:

$$T_\alpha = \begin{bmatrix} \alpha & -1 & 0 & 0 & \cdots & \cdots & 0 \\ -1 & \alpha & -1 & 0 & \ddots & & \vdots \\ 0 & -1 & \alpha & -1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ \vdots & & \ddots & \ddots & \ddots & \ddots & 0 \\ \vdots & & & \ddots & -1 & \alpha & -1 \\ 0 & \cdots & \cdots & \cdots & 0 & -1 & \alpha \end{bmatrix}, \qquad (1)$$

where $\alpha \in \mathbb{R}$.

(a) Verify that the eigenvalues of T_α are given by

$$\lambda_j = \alpha - 2\cos(j\theta), \quad j = 1, \dots, m,$$

where $\theta = \frac{\pi}{m+1}$, and that an eigenvector associated with each λ_j is

$$\boldsymbol{q}_j = [\sin(j\theta), \sin(2j\theta), \cdots, \sin(mj\theta)]^T.$$

(b) Under what condition on α does this matrix become positive definite?

- In the following questions, take $\alpha = 2$.

(c) Will the Jacobi iteration converge for this matrix? If so, what will its convergence factor be?

(d) Will the Gauss-Seidel iteration converge for this matrix? If so, what will its convergence factor be?

(e) For which value of ω will the SOR iteration converge?

Problem 3: Solve Exercise 33.2 (in Trefethen & Bau)

Problem 4: Solve Exercise 33.3

Problem 5: Solve Exercise 34.1 (Note: There is a typo. In Equation (34.7), c_{m-1} should be c_{n-1}.)