229B: Numerical Methods in Linear Algebra
Homework 4: due Monday 03/06/00

Problem 1: Solve Exercise 40.1

Problem 2: Consider the difference equation:

$$-u_{n-1} + 4u_n - u_{n+1} = f_n,$$

where $n = 0, \ldots, N$. Sove this equation with

(a) $f_n = 1$;

(b) $f_n = \delta(n) - \delta(n - N/2)$

subject to the following boundary condition:

(1) $u_0 = u_N = 0$;

(2) $u_1 - u_{-1} = 0,\ u_{N+1} - u_{N-1} = 0$;

(3) $u_0 = u_N,\ u_{-1} = u_{N-1}$.

Note that there are six different boundary value problems. In each case, use the appropriate form of the DFT with either the component perspective or the perational perspective. If necessary, express the solution in terms of the coefficients U_k.

Problem 3: Write a matlab program to solve the discrete Poisson equation on a square using FFTs. The inputs should be the dimension N and a square $N \times N$ matrix of values of f_{jk}. The outputs should be an $N \times N$ matrix of solution u_{jk} and the residual $\|T_{N\times N}u - h^2 f\|_2 \,/\, \|T_{N\times N}\| \cdot \|u\|$. You should also produce three-dimensional plots of f and u. Your program should not have to be more than a few lines long if you use all the features of matlab. Using your matlab program, solve the following inputs with the Dirichlet boundary condition:

(a) $f_{jk} = \sin(\frac{j\pi}{N+1}) \cdot \sin(\frac{k\pi}{N+1})$;

(b) $f_{jk} = \sin(\frac{j\pi}{N+1}) \cdot \sin(\frac{k\pi}{N+1}) + \sin(\frac{3j\pi}{N+1}) \cdot \sin(\frac{5k\pi}{N+1})$;

(c) f has a few sharp spikes (both positive and negative) and is 0 elsewhere.