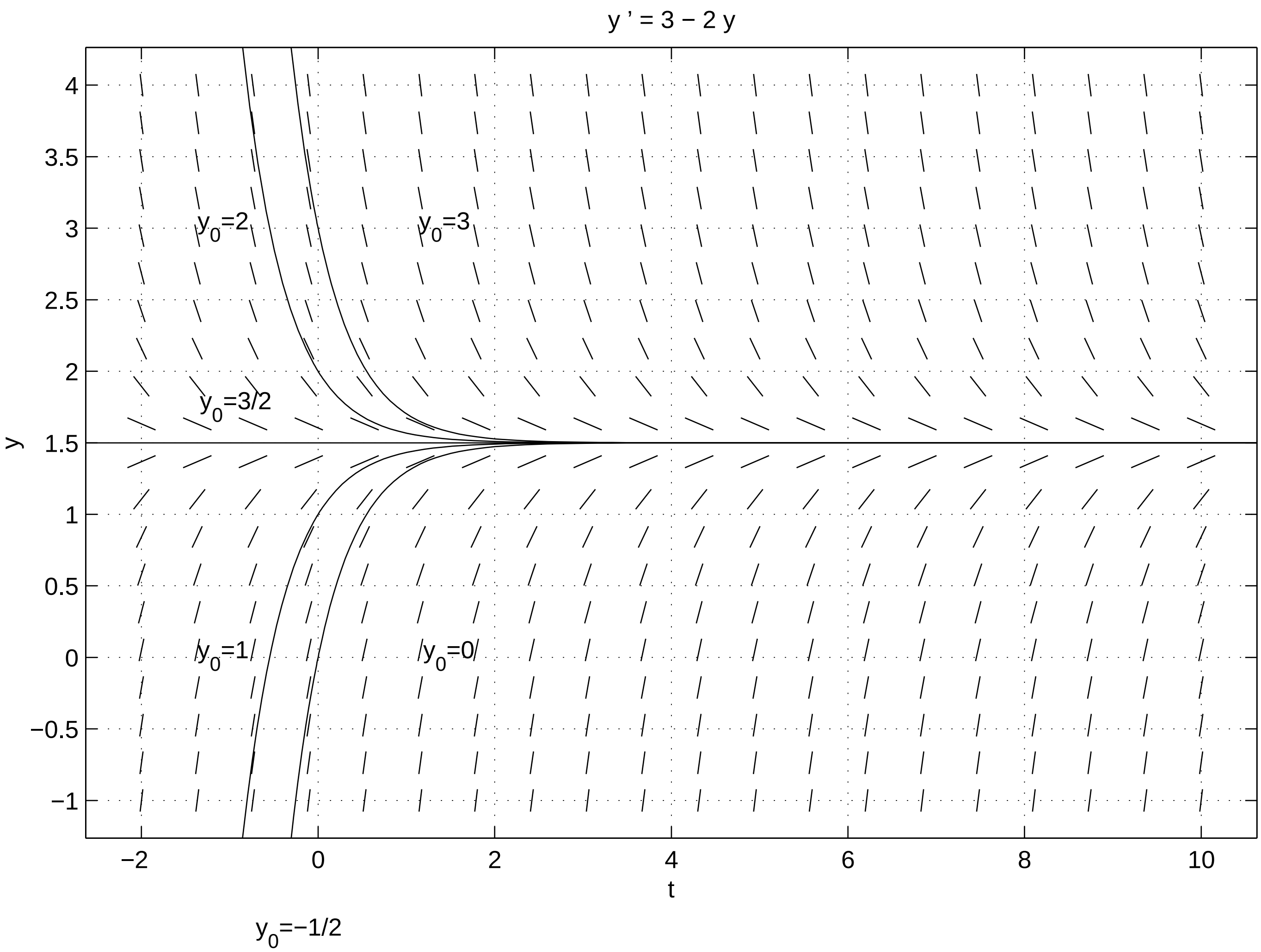
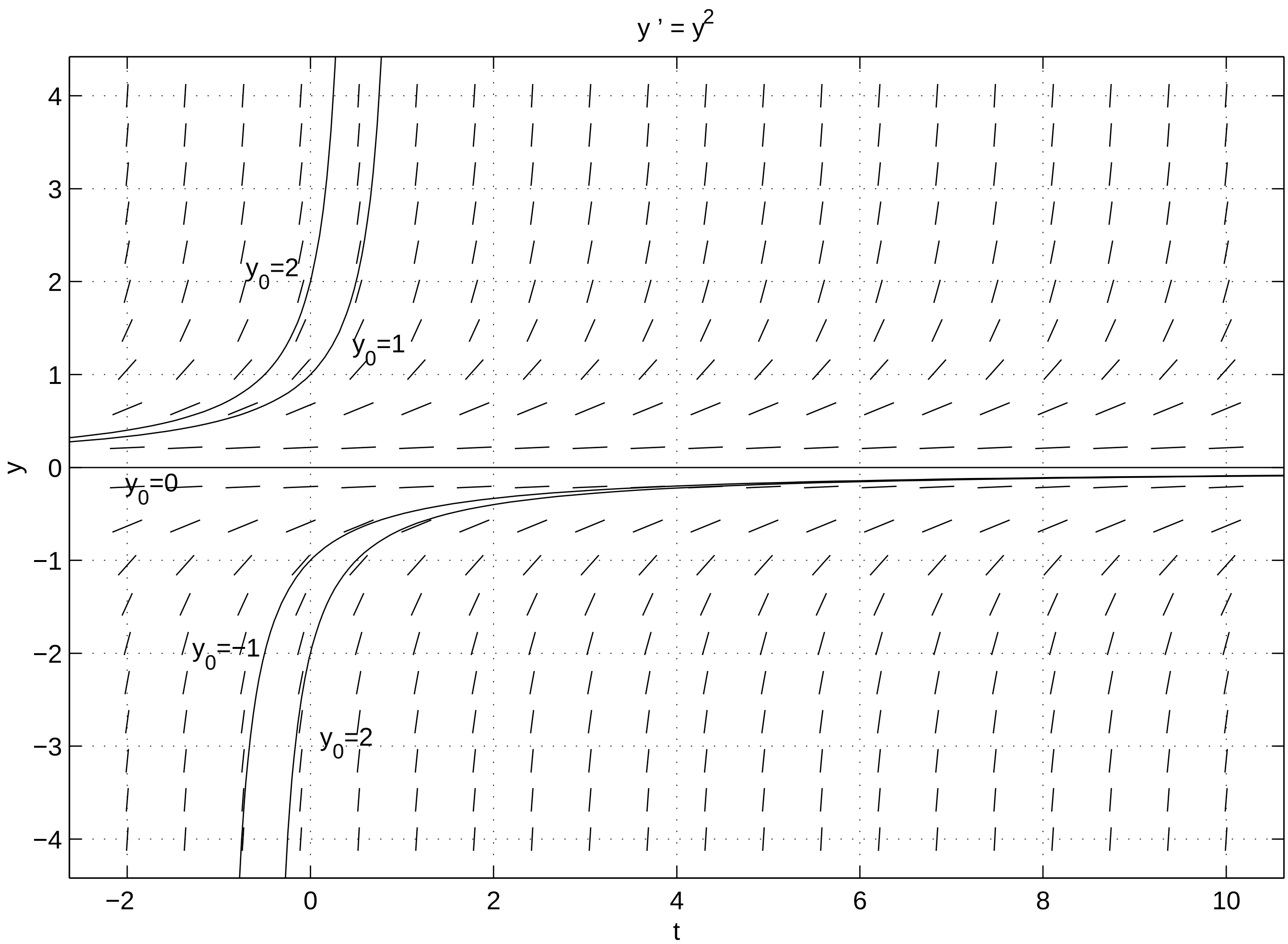
§1.1

<u>#1</u>: Draw a direction field for the equation $y' = 3 = 2y$.



#13: Draw a direction field for $y' = y^2$.



#15: a)To set up the differential equation let $A(t) =$"The amount (in grams) of chemical in the tank at time t". Then we have the following:

$$\frac{dA}{dt} = Rate\ of\ change\ of\ the\ amount\ of\ chemical\ in\ the\ tank.$$

$$Rate\ of\ Change = (Rate\ In) - (Rate\ Out)$$

$$Rate\ In = .01\frac{g}{gal} \times 300\frac{gal}{min}$$

$$Rate\ Out = \frac{A(t)}{10^6}\frac{g}{gal} \times 300\frac{gal}{min}$$

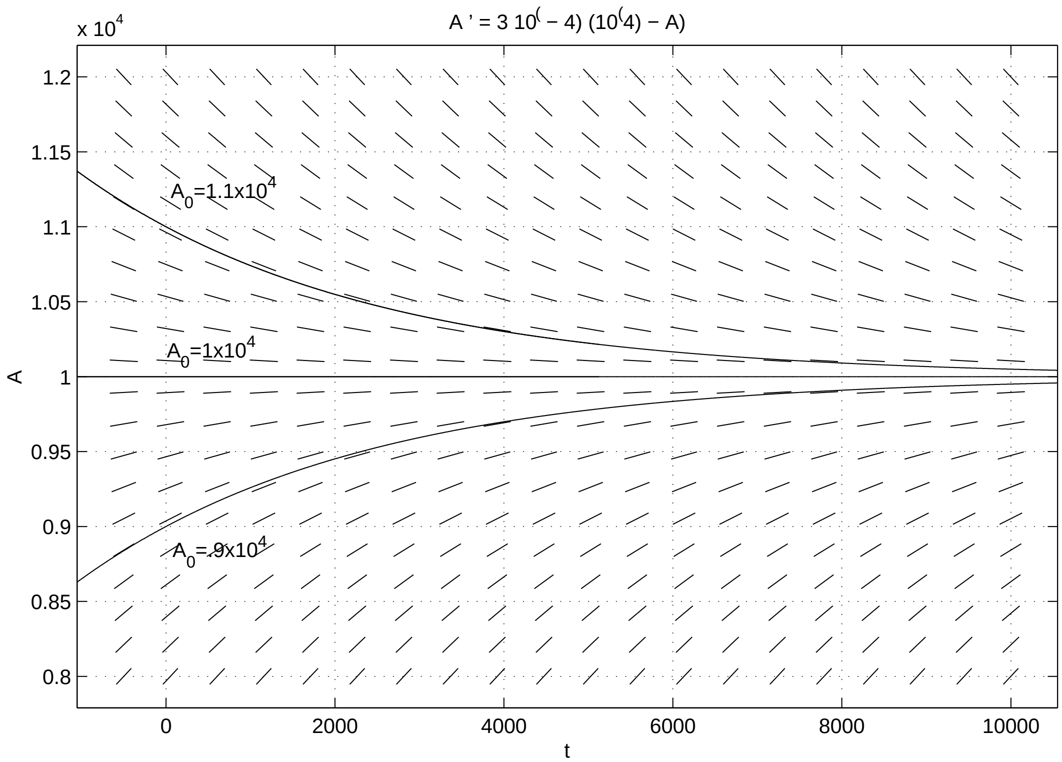
thus, we have the following differential equation:

$$\frac{dA}{dt} = 3(1 - \frac{A(t)}{10^4}) = (3 \times 10^{-4})(10^4 - A(t))$$

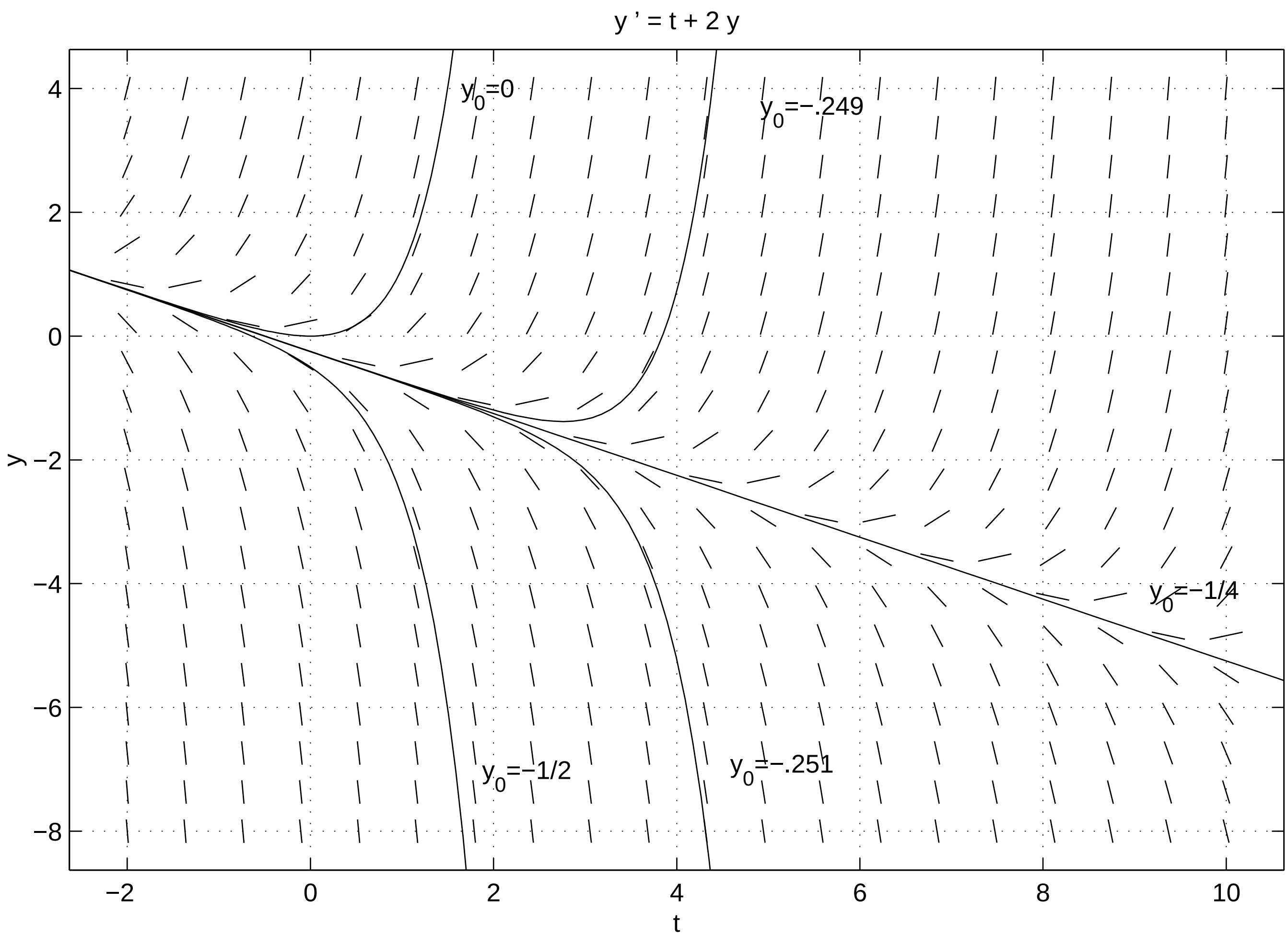
so

$$\frac{dA}{dt} = (3 \times 10^{-4})(10^4 - A(t))$$

b)From the direction field below, we see that after a long time there will be 10 kg of chemmical in the tank, regardless of the I.C.



#22: Draw a direction field for $y' = t + 2y$.



§1.2

#1a: Solve the following:

$$\frac{dy}{dt} = -y + 5; \qquad y(0) = y_0$$

We use separation of variables to write:

$$\frac{dy}{5-y} = dt$$

then substitute with $u = 5 - y$ and integrate:

$$ln(5-y) = -t + C$$

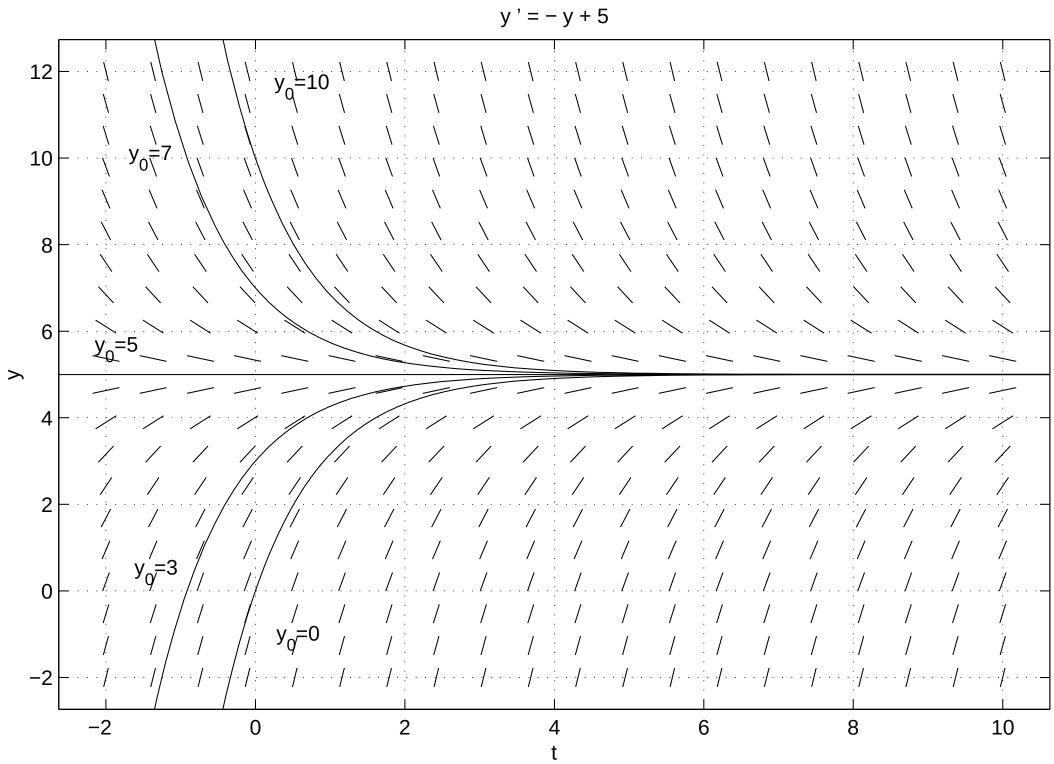
exponentiating yields:

$$5 - y = C_1 e^{-t}$$

thus, solving for $y(t)$ and C_1 we get

$$y(t) = (y_0 - 5)e^{-t} + 5$$

Some sample solutions to the above ODE:



#2b: Solve the following

$$\frac{dy}{dt} = 2y - 5; \qquad y(0) = y_0$$

Again we use separation of variables to solve the problem. however this time the appropriate substitution when integrating is $u = 2y - 5$. So we get:

$$\frac{dy}{2y-5} = dt$$

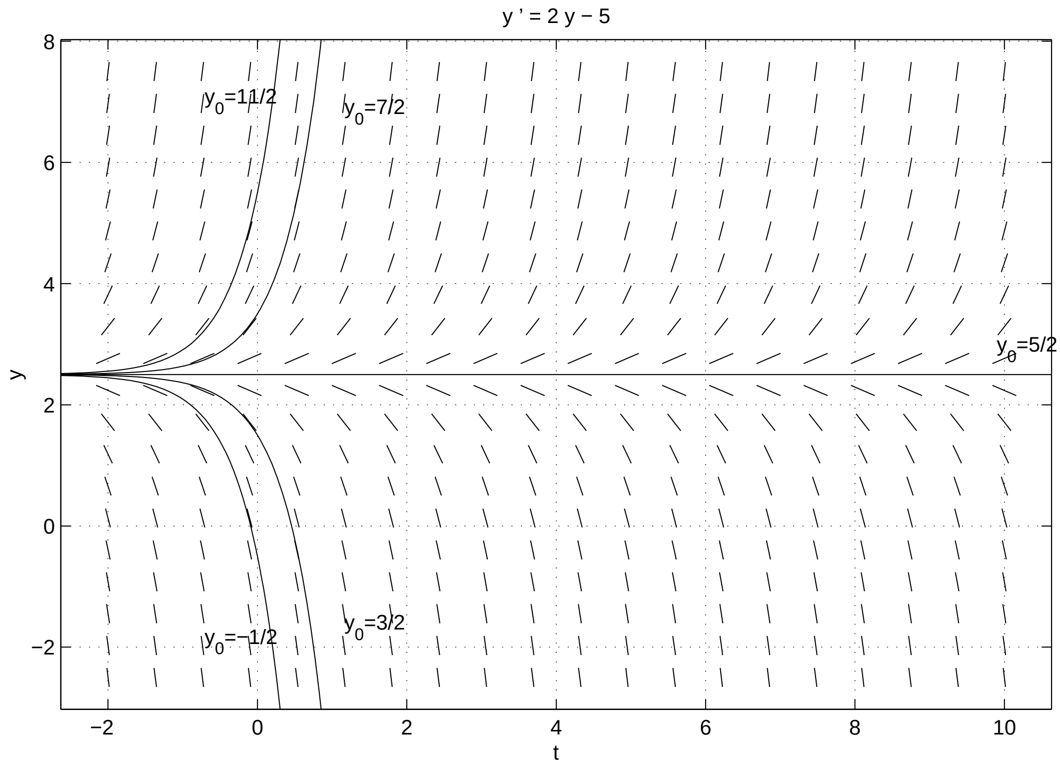
$$ln(2y - 5) = 2t + C$$

$$y = C_2 e^{2t} + \frac{5}{2}$$

thus after solving for C_2 we get

$$y(t) = (y_0 - \frac{5}{2})e^{2t} + \frac{5}{2}$$

Some sample solutions to the above ODE:



#3: Solve the following:

$$\frac{dy}{dt} = -ay + b; \qquad a, b > 0; \;\; y(0) = y_0$$

We use separation of variables to write:

$$\frac{dy}{-ay + b} = dt$$

then substitute with $u = -ay + b$ and integrate:

$$ln(-ay + b) = -at + C$$

exponentiating yields:

$$-ay + b = C_1 e^{-at}$$

thus, solving for $y(t)$ and C_1 we get

$$y(t) = (y_0 - b)e^{-at} + b$$

§1.3

#1: $t^2y'' + ty' + 2y = \sin t$ $\qquad 2^{nd}$ order, linear.
#3: $y'''' + y''' + y'' + y' + y = 1$ $\qquad 1^{st}$ order, linear.
#4: $y' + ty^2$ $\qquad 1^{st}$ order, non-linear.
#4: Show that the following has the given solution:

$$y' - 2ty = 1 \qquad y(t) = e^{t^2} \int_0^t e^{-s^2} ds + e^{t^2}$$

To show that $y(t)$ is a solution to the given equation we first rewrite $y(t)$ then differentiate:

$$y(t) = e^{t^2}\int_0^t e^{-s^2}ds + e^{t^2} = e^{t^2}\left(\int_0^t e^{-s^2}ds + 1\right)$$

thus

$$y'(t) = \frac{d}{dt}\left(e^{t^2}\left(\int_0^t e^{-s^2}ds + 1\right)\right)$$

$$= \frac{d}{dt}(e^{t^2})\left(\int_0^t e^{-s^2}ds + 1\right) + e^{t^2}\frac{d}{dt}\left(\int_0^t e^{-s^2}ds + 1\right)$$

$$= 2te^{t^2}\left(\int_0^t e^{-s^2}ds + 1\right) + (e^{t^2})(e^{-t^2})$$

by the Fundamental Theorem of Calculus. Thus we have that:

$$y'(t) = 2te^{t^2}\left(\int_0^t e^{-s^2}ds + 1\right) + 1 = 2ty(t) + 1$$

Now, substituting into the equation above we get:

$$y' - 2ty = (2ty + 1) - 2ty = 1$$

Thus the given $y(t)$ is a solution.

<u>#18</u>: Determine the values of r such that $y = e^{rt}$ is a solution to:

$$y''' - 3y'' + 2y' = 0$$

Differentiating $y = e^{rt}$ and substituting we get:

$$r^3e^{rt} - 3r^2e^{rt} + 2re^{rt} = (r^3 - 3r^2 + 2r)e^{rt} = 0$$

but $e^{rt} \neq 0$ for all t, thus we have that:

$$(r^3 - 3r^2 + 2r) = r(r-1)(r-2) = 0$$

Thus, if $y = e^{rt}$ is a solution, then $r = 0, 1, 2$.

<u>#20</u>: Determine the values of r such that $y = t^r, t > 0$ is a solution to:

$$t^2y'' - 4ty' + 4y = 0$$

As before we differnetiate and subbstitute to get:

$$r(r-1)t^r - 4rt^r + 4t^r = (r(r-1) - 4r + 4)t^r = 0$$

but again t¿0 so

$$r(r-1) - 4r + 4 = r^2 - 5r + 4 = (r-1)(r-4) = 0$$

Thus $r = 1, 4$.

#21: $u_{xx} + u_{yy} + u_{zz}$ 2^{nd} order, linear.
#24: $u_t + uu_x = 1 + u_{xx}$ 2^{nd} order, non-linear.