Applications of Fourier Transforms

Part B. Fourier Optics

8.4. Fraunhofer Diffraction

Fraunhofer diffraction is a physical phenomenon in which light waves are diffracted by apertures or obstacles. The diffraction pattern is observed in the far field, where the wavefronts are planar and the diffraction is independent of the shape of the aperture.

The Fraunhofer diffraction pattern can be described by the Fourier transform of the aperture function.

\[ \mathcal{F} \{ a(x) \} = \frac{1}{\lambda d} \int_{-\infty}^{\infty} a(x) e^{i \frac{2\pi}{\lambda d} x y} dx \]

where \( a(x) \) is the aperture function and \( \lambda \) is the wavelength of the light.

The Fourier transform of the aperture function gives the distribution of light intensity in the far field. The diffraction pattern is characterized by the formation of bright and dark rings, which are due to the constructive and destructive interference of the diffracted waves.

The Fraunhofer diffraction pattern is often studied in the context of optical systems, such as lenses and gratings. The first-order diffraction pattern is typically dominant in these systems.

In the limit of large angles, the diffraction pattern approaches the sinc function.

\[ I(\theta) \propto \left| \mathcal{F} \{ a(x) \} \right|^2 \approx \frac{\sin^2 (\pi d \sin \theta / \lambda)}{(\pi d \sin \theta / \lambda)^2} \]

where \( \theta \) is the angle of diffraction.

The Fraunhofer diffraction pattern is a fundamental concept in optical science and has numerous applications in fields such as astronomy, microscopy, and telecommunications.
Exponentially, it is found that when the wave amplitude \( A \) is recorded,
\[
(a' \cdot n) Y_{\phi} / (y / x) = A
\]
we can write (in the form)
\[
y / w = a \quad y / x = n
\]

where the amplitude of the interference pattern is
\[
A \left( x + x_{0} \right) / (x_{y} / x) = A
\]
and the waves, {equation}\( a \cdot n \), arrive at \( \phi \) with amplitudes
\[
\left( x + x_{0} \right) / (x_{y} / x) = A
\]

Comparing (6') with (4') we obtain
\[
A \left( x + x_{0} \right) / (x_{y} / x) = A
\]

since where

\[
A = \left| x_{y} / x \right|
\]

and these are the coordinates (of the two rays) since

\[
(a' \cdot n) Y_{\phi} / (y / x) = I
\]

because of the (4') (\( a \cdot n \)) we see that the focal plane of the lens is

\[
\left| x_{y} / x \right| = I
\]

the intensity. Hence, from (4') we have

Opposite a & a precaution is taken to avoid the receiving screen, we record only
The function $f(y) = \frac{1}{\pi} \frac{\sin u}{u}$ is defined by

$$f(y) = \begin{cases} \frac{1}{\pi} \frac{\sin u}{u} & \text{if } u \neq 0 \\ 1 & \text{if } u = 0 \end{cases}$$

Then, by Example 7.2, Chapter 6, we have

$$f(y) > 0 \quad \text{and} \quad f(y) > |y|$$

for all $y$.

**5. Rectangular Apertures**

More realistic situations in Figure 7 are expected to occur in Figure 7, so that

$$\phi = \frac{1}{\pi} \frac{\sin u}{u}$$

for $u \neq 0$. The classical result of the rectangular aperture is shown in Figure 7.

Example why high light is not a section of a section of the rectangular aperture in Figure 7.

**Exercises**

For more details, see the references above or feel free to ask.
Exercises

Figure 7.9. Direction pattern for a vertically distributed rectangular aperture. The pattern is shown in (a). (b) An actual direction pattern produced by a rectangular aperture similar to the one shown in Figure 7.9.

Figure 7.10. A graph of the intensity distribution for a rectangular aperture. The graph is obtained by integrating the distribution over a rectangular aperture. The pattern is shown in Figure 7.8(a). The rectangular aperture is shown in Figure 7.8(b). The rectangular aperture is shown in Figure 7.8(c).
6. Circular Apertures

Consider a circular aperture of radius $a$. The aperture function in this case is

$$f(x, y) = \begin{cases} 
1 & \text{if } \sqrt{x^2 + y^2} < a \\
0 & \text{otherwise}
\end{cases}$$

where $(x, y)$ are the coordinates of the point in the aperture.

Figure 7.12: Aperture and Aperture Function (Unit Circle)

The function $f(x, y)$ is a characteristic function, which is equal to 1 inside the circle of radius $a$ and 0 outside.

Figure 7.13: Aperture and Aperture Function (Unit Circle)

The aperture function can be expressed in terms of polar coordinates as

$$f(r, \theta) = \begin{cases} 
1 & \text{if } r < a \\
0 & \text{otherwise}
\end{cases}$$

where $(r, \theta)$ are the polar coordinates of the point.

Figure 7.14: Aperture and Aperture Function (Unit Circle)

Using Theorem 7.15, Chapter 6, we have

$$S[I] = G^* \ast I$$

where $S$ is the operator that convolves the function $I$ with $G^*$.

Figure 7.15: Aperture and Aperture Function (Unit Circle)

The convolution theorem states that the Fourier transform of the convolution of two functions is equal to the product of their Fourier transforms.

Figure 7.16: Aperture and Aperture Function (Unit Circle)

Using the convolution theorem, we can express the aperture function in the frequency domain as

$$F[f(r, \theta)] = F[G^*] \ast F[I]$$

where $F$ denotes the Fourier transform.

Figure 7.17: Aperture and Aperture Function (Unit Circle)

In the frequency domain, the aperture function is a low-pass filter, allowing low-frequency components to pass through but attenuating high-frequency components.

Figure 7.18: Aperture and Aperture Function (Unit Circle)

The aperture function can be used to analyze the effect of the aperture on the propagated field.

Figure 7.19: Aperture and Aperture Function (Unit Circle)

The aperture function is a key component in many optical systems, such as telescopes and microscopes, where it determines the amount of light that enters the system.

Figure 7.20: Aperture and Aperture Function (Unit Circle)

The aperture function can be used to design optical systems with desired properties, such as increased resolution or reduced aberrations.
(9.6) Let the aperture consist of an annular ring with aperture function $A(x) = \begin{cases} 0 & \text{if } x < a \\
 1 & \text{otherwise} \end{cases}$.

Describe the resulting diffraction patterns for $a > b$ and for $a < b$ close to $b$.

**Exercises**

The aperture is the collecting dish. The aperture is a section problem encountered with radio telescopes (where diffraction is a severe problem encountered with radio telescopes), where this effect is predicted but not seen. Because of the large wavelengths used, this effect is predicted but not seen. Because of this effect, the aperture is a section problem encountered with radio telescopes. The aperture is a section problem encountered with radio telescopes. The aperture is a section problem encountered with radio telescopes. The aperture is a section problem encountered with radio telescopes.
are an essential tool of modern science.

8. Diffraction Gratings

The resultant diffraction pattern, where $c^\prime$, $c^\prime\prime$, and $d^\prime$ are positive, describes the arrangement that is, the positions are assigned to a particular point on the grid. Suppose we have four positions for Eq. (6.7) arranged in a square grid. Now do four equally spaced vertical slits.

Describe the diffraction pattern of the equal-slit pattern similar to that of the two vertical slits. [Hints: Add the transform of the occlar slit to that of the two vertical slits.]

Describes the diffraction pattern of the equal-slit pattern. If the characteristic of the pattern is proportional to the distance between the slits, show that the number of interfering fringes per unit length in both directions is proportional to the number of vertical slits.

Exercises

Figure 7.17 shows the actual diffraction pattern that results in this case. Figure 7.17 shows the actual diffraction pattern that connects the center of the two apertures. The center of the diffraction pattern forms a single maximum, while the positions of the central maximum are shown in Figure 7.14. The central maximum is at the center of the distribution for a single central aperture. Where is the intensity distribution for a single central aperture?

\[
\begin{align*}
\phi_d' &= \sin c' \cos \theta \cos \psi - d' \\
\phi_d'' &= \sin c'' \cos \theta \cos \psi - d''
\end{align*}
\]

Here for the intensity, we have

\[
\phi_d' \phi_d'' = \sin c' \cos \theta \cos \psi - d' \sin c'' \cos \theta \cos \psi - d''
\]

In the previous section, we considered the shift property and hence we obtain

\[
\left(\sin c' \cos \theta \cos \psi - d'\right) \cos \theta \cos \psi = (a'' \alpha')
\]

The $x$ axis. In this case, our aperture function will be the product of two partially separated circular apertures.

\[
(\begin{array}{cc}
\sin c' & \cos \theta \\
\sin c'' & \cos \theta
\end{array})
\]
The height of the distribution of each vertical line is the weight of each column in the table. The weight of each column is no more than \( \frac{1}{2} \), where \( a \) is the height of the column.

**Figure 7.8** The intensity distribution of each vertical line is the height of each column.

\[
\begin{align*}
\phi(a) &= n \cdot \phi(a) \quad \forall \phi \in C
\end{align*}
\]

We are assuming here that the light is a linear process, and that the light of each vertical line is the height of the distribution of each vertical line.

\[
\begin{align*}
\phi(\gamma) &= n \cdot \psi \quad \forall \psi \in C
\end{align*}
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**Figure 7.9** The height of the distribution of each vertical line is the height of each column.

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\begin{align*}
\phi(\gamma) &= n \cdot \psi \quad \forall \psi \in C
\end{align*}
\]

Hence, from Figure 7.8, we see that the light between dots is \( \psi = n \cdot \phi \). We will obtain a special decomposition of the light where the light of the distribution of each vertical line is the height of the distribution of each vertical line.

**Figure 7.9** The height of the distribution of each vertical line is the height of each column.

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\[
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\end{align*}
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Above, projecting our light onto the last quantity we get

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Any pattern for one particle in a square array with \( a = b = 0.334 \) square units from (a) 0.20 \( \text{cm} \) to (b) 0.40 \( \text{cm} \) is formed by selecting the appropriate dimension of the pattern. Suppose that a square array of 11 x 11 circular particles is formed by selecting 10 of the 11 square units.

### Example (6.7)

The intensity distribution is calculated by applying the Lorentz-Mie theory to each particle in the array. The intensity at a point in the array is given by

\[
I = \frac{n^2}{\pi d^2} \int |F(r)|^2 \, dA
\]

where \( n \) is the refractive index of the medium, \( d \) is the particle diameter, and \( F(r) \) is the electric field distribution in the vicinity of the particle.

The intensity at the point \( x = y = 0 \) is calculated as

\[
I(0,0) = \frac{n^2}{\pi d^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |F(r)|^2 \, dx \, dy
\]

where \( F(r) \) is the electric field distribution in the vicinity of the particle. The intensity is then given by the integral of the intensity over the area of the particle.

### Section 7.2

The array is formed by constructing a square array of circular particles. The intensity distribution in the vicinity of each particle is calculated using the Lorentz-Mie theory. The intensity distribution in the entire array is then calculated by integrating the intensity distribution in the vicinity of each particle over the area of the array.

### Section 7.3

The array is then used to calculate the intensity distribution in the vicinity of a point in the array. The intensity is calculated by integrating the intensity distribution in the vicinity of each particle over the area of the point.

### Section 7.4

The intensity is then calculated by integrating the intensity distribution in the vicinity of each particle over the area of the point. The intensity is then given by the integral of the intensity over the area of the point.

### Section 7.5

The intensity distribution in the vicinity of a point in the array is calculated using the Lorentz-Mie theory. The intensity is then given by the integral of the intensity over the area of the point.

### Section 7.6

The intensity distribution in the vicinity of a point in the array is calculated using the Lorentz-Mie theory. The intensity is then given by the integral of the intensity over the area of the point.
4.3 Fourier Theory

In this section we shall briefly describe the theory of lens imaging via Fourier analysis. The theory is due to Abbe and Zernike.

**4.3.1 Imaging Theory**

In the study of spatial frequencies, the point-spread function is defined as the Fourier transform of the aperture function. The point-spread function at a point $p$ is given by $F(p) = \mathcal{F}[f(x)](p) = \int f(x) e^{-2\pi i px} dx$.

**Remark.** The area of dots in Figure 7.25(b) is called the reciprocal lattice.

**Theorem.** The array of dots in Figure 7.25(b) is called the reciprocal lattice.

**Exercise.** Describe the diffraction pattern of an array of $15 \times 15$ squares of aperture $1.0 \times 1.0 \text{ mm}$ formed by letting $c = 0.2 \text{ mm}$.

See also Legendre (1967), p. 401-431.

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Weber and Lippmann's (1975) method of diffraction is based on the idea that the intensity of the light scattered from a point source is proportional to the square of the amplitude of the electric field at that point. This is known as the Huygens principle. The intensity distribution is given by

$$I = \left| E(x) \right|^2 = \left| \frac{2\pi}{\lambda} \int f(x') \delta(x - x') dx' \right|^2$$

where $\delta(x)$ is the Dirac delta function.

---

![Figure 7.25](image)
\[ W(l \cdot a + \beta) - |\mathbf{G}| = \frac{W}{(l \cdot a + \beta)} \cdot (0 \cdot \mathbf{x}) - |\mathbf{G}| = |\mathbf{G}| \]

Next, we obtain from (10'), using a well-known formula for the cross product of two vectors, which represents the area of the parallelogram spanned by these vectors. Since \( \mathbf{G} \) is the cross product of the unit vector \( u \) with \( \mathbf{G} \), we have:

\[ W = \frac{W}{\sin(\theta + \gamma)} - |\mathbf{G}| = |\mathbf{G}| \]

where \( \theta \) is a positive constant. Using (20'), the formula for the angle between \( \mathbf{G} \) and \( \mathbf{G} \) should satisfy the equation:

\[ \sin(\theta + \gamma) - |\mathbf{G}| = |\mathbf{G}| \]

After the angle condition is satisfied, we now invoke the Parallelogram Law of Vectors, which states that the sum of the squares of the sides of a parallelogram is equal to the sum of the squares of the diagonals. This approximation is valid when the angles are small.

---

**Figure 7.26**

Each point in the image plane emanates a wave, with amplitude \( W \). These waves interfere to give the final image.
where \( p \) is a function for which \( p(x) \) is large enough.

\( P \) and \( Q \) are functions for minimizing spatial frequencies. We replace \( a \) in (10) by \( \log \). The result may be expressed as

\[
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Phi(x,y) \, dx \, dy = \left( \frac{\alpha}{\beta} \right) \Phi(0,0)
\]

This result allows us to rewrite (10), using (10)'s results (and 10)'s usefulness in the analysis of pattern recognition.
\[ \int_{\mathbb{R}^n} f(x) \, dx = (x)f \]  

\[ \text{Proof:} \]  

Because \( f(x) \) is integrable for all \( x \), provided \( f(x) \) is bounded on \( x \).  

\[ (\vec{\mathbf{a}} - \vec{\mathbf{a}}) \int_{\mathbb{R}^n} f(x) \, dx = (x)f \]  

\[ \text{Theorem (11.1):} \]  

Suppose that \( f(x) \) is continuous and absolutely integrable over \( \mathbb{R}^n \) and is limited. Then \( f(x) \) is a continuous function of \( \vec{\mathbf{a}} \), and its Fourier transform \( \hat{f}(\vec{\mathbf{k}}) \) is unique, up to a constant of proportionality.  

The inverse Fourier transform of \( \hat{f}(\vec{\mathbf{k}}) \) is \( f(x) \), or, conversely, \( f(x) \) is the Fourier transform of \( \hat{f}(\vec{\mathbf{k}}) \).  

\[ \text{Exercises} \]  


For further discussion of inversions, see Chapter 7 of Goodman (1974).  

Note that the transformation of the partial order is \( \mathcal{O}(f \cdot g) \) and the transformation of the partial order is \( \mathcal{O}(f + g) \).  

Formulas (11.1, 11.2, 11.3) are given in Izuka (1972).  


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