Topics in Optimal Transportation, by C. Villani,
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ERRATA (as of July 31, 2005)

p. 1, l.-1: Instead of $\mathbb{R} \cup \{+\infty\}$, read $\mathbb{R}_+ \cup \{+\infty\}$.

p. 15: What I call "Krein-Milman theorem" here would be more properly called "Choquet's theorem". The Krein-Milman theorem only states that a convex compact K is the closed convex envelope of its extremal points; it is Choquet who proved the more constructive version used in the text. Moreover, **the hint for 3.(iii) is false:** $\mathcal{E}(K)$ may be not closed, even in finite dimension. Thanks to Michel Valadier for pointing out.

p. 28, l.3: Ξ is not identically 0: it attributes the value 0 to the null function, and $+\infty$ to all other functions (so it is indeed "trivial", but not in the sense that I was suggesting). The same on p. 39, l. -7. Thanks to Scott Armstrong for pointing out.

p. 35: In the second displayed formula (1.18), the right-hand side should be $\mathcal{T}_d([\mu - \nu]_+, [\nu - \mu]_+)$.

p. 54: In the first proposition, assume that x is not a minimum of φ. In the right-hand side of the formula on **l. -11**, read $\nabla\varphi$ instead of ∇f.

p. 58, l. -6: Instead of $\int \zeta\, D^2\varphi$, read $\int \varphi\, D^2\zeta$.
p. 63: In the first paragraph, read $\overline{\varphi_k}$ and $\overline{\psi_k}$ in place of φ_k and ψ_k.

p. 69, l. -4: In place of Exercise 2.18, read just (2.18).

p. 72, l. 6: In place of Proposition 2.9, read Theorem 2.9.

p. 72, l. 10: $\varphi^* + th$ might not be convex (which does not matter for the argument).

p. 86, l. -3: In place of $|x - y|^2$, read $|x - y|^2/2$.

p. 90, l. -1 and **p. 91, l. 1:** Replace φ by c.

p. 103, (2.78): Replace φ by c (twice).

p. 104, Exercise 2.57: The stated conclusion is **false** for a nonquadratic cost (counterexamples can be constructed). I apologize for this serious mistake, which has minor consequences on pp. 166 and 179 (see below).

p. 105, l. 8: Instead of subgradient, read supergradient.

p. 120, l. 8: Instead of u, read h. In **Remark 3.16 (i)**, instead of $d\lambda$-almost everywhere on Ω, read $d\nu$-almost everywhere on Y.

p. 124, l. 9: Instead of "v satisfies..." read "w satisfies..."

p. 134, l.-3 and -2: Instead of A, read $D^2\varphi(x)$ on l.-3 and $D^2\varphi$ on l.-2.

p. 149, Eq. 5.12: Instead of μ_t, read ρ_t.

p. 153, l. -17: On the very right, instead of W, read $\mathcal{W}$.

p. 166, l.5-8: It is said that (5.36) is all that one can expect in general; in fact, even (5.36) may fail to be true for nonquadratic cost functions.

p. 178, l.-1 and p. 179, l.1-3: Because of Exercise 2.57 being false, the proof here is correct only for quadratic cost (which is the most important case of application). It is however true that there are no shocks for a strictly convex cost function (see e.g. Dario Cordero-Erausquin, *Contemp. Math. 353*, Proposition 5.1 on pp. 68-69).

Theorem 6.21 (iii), p. 201: Instead of $W^{1,p(\mathbb{R}^n)}$, read $W^{1,p}(\mathbb{R}^n)$.

Proposition 7.16 and 7.17, pp. 220-221: In the assumptions of these two propositions, add: "U_1 is independent from U_2, and V_1 from V_2." Similarly, in **Remark 7.18 (i)**, add: "X_1 is independent from X_2". Otherwise the convolution formula given in Remark 7.18 (i) would obviously be false in general. The proofs of Propositions 7.16 and 7.17 which are given in the text make implicit use of this independence assumption (as far as I can judge, they are otherwise correct), in that the law of $\alpha_1 U_1 + \alpha_2 U_2$, say, is assumed to depend only on the respective laws of U_1 and U_2. It is easy to find counterexamples if this assumption is not made. Thanks to Patrick Cattiaux for pointing out.

p. 217: Instead of "Remark 1.1.7" read "Remark 4 on p. 26"

p. 239: The exponent 2 at the very end of the last line should not be there.

p. 248, Exercise 8.5: In step (i) I state that $\mathrm{Lip}_1(\mathbb{R}^n)$ is separable, which is obviously false. Instead, consider elements of Lip_1 that are supported in the ball $\overline{B(0,R)}$; they form a compact set. Then in the end take a sequence $R = R_n \to \infty$.

p. 253: In the last displayed formula, instead of $\overline{V} = V *_x \rho$, read $\overline{V} = \phi *_x \rho$.

p. 264, Theorem 8.13: In the statement of the theorem I forgot to define $\nabla\varphi$; see the second step of the proof for the definition.

p. 264, Remark 8.14: $\mathrm{Id} - \nabla\varphi^*$ is not the optimal velocity field to transport ρ_0 back to σ, but the *opposite* of that velocity field.

Formula (9.28), p. 281: My statement that condition (9.28) of Djellout, Guillin and Wu is stronger than condition (iv) of Theorem 9.9 is, strictly speaking, true, but actually both conditions are equivalent. If (X,d) is an abstract metric space and μ a probability measure on X, the three conditions

$$\exists \alpha > 0; \qquad \int e^{\alpha\, d(x,y)^2}\, d\mu(x)\, d\mu(y) < +\infty,$$

$$\exists \alpha > 0, \quad \forall y \in X; \qquad \int e^{\alpha\, d(x,y)^2}\, d\mu(x) < +\infty,$$

$$\exists \alpha > 0, \quad \exists y \in X; \qquad \int e^{\alpha\, d(x,y)^2}\, d\mu(x) < +\infty$$

seem at first sight to be weaker and weaker, but in fact are all equivalent (simple exercise).