A hand is shown holding a white, three-dimensional cube. The cube is positioned in the center of the frame, and the hand is visible from the top and sides, gripping the top edges of the cube. The background is a soft, out-of-focus light blue and white. The text is overlaid on the image, centered horizontally and vertically.

# The Extremes of Interpretability: Sparse Decision Trees and Scoring Systems

Cynthia Rudin  
Duke University

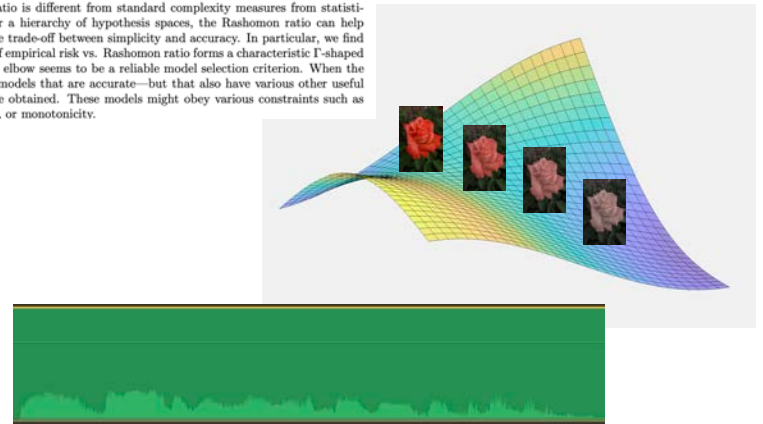
# Problem spectrum

age 45  
congestive heart failure? yes  
takes aspirin  
smoking? no  
gender M  
exercise? yes  
allergies? no  
number of past strokes 2  
diabetes? yes

**Tabular**: All features are interpretable

- many problems in criminal justice, healthcare, social sciences, equipment reliability & maintenance, etc.
- features include counts, categorical data

The *Rashomon effect* occurs when many different explanations exist for the same phenomenon. In machine learning, Leo Breiman used this term to characterize problems where many accurate-but-different models exist to describe the same data. In this work, we study how the Rashomon effect can be useful for understanding the relationship between training and test performance, and the possibility that simple-yet-accurate models exist for many problems. We consider the *Rashomon set*—the set of almost-equally-accurate models for a given problem—and study its properties and the types of models it could contain. We present the *Rashomon ratio* as a new measure related to simplicity of model classes, which is the ratio of the volume of the set of accurate models to the volume of the hypothesis space; the Rashomon ratio is different from standard complexity measures from statistical learning theory. For a hierarchy of hypothesis spaces, the Rashomon ratio can help modelers to navigate the trade-off between simplicity and accuracy. In particular, we find empirically that a plot of empirical risk vs. Rashomon ratio forms a characteristic I-shaped *Rashomon curve*, whose elbow seems to be a reliable model selection criterion. When the Rashomon set is large, models that are accurate—but that also have various other useful properties—can often be obtained. These models might obey various constraints such as interpretability, fairness, or monotonicity.



**Raw**: Features are individually uninterpretable

- pixels/voxels, words, a bit of a sound wave

## Problem spectrum

Very sparse models (trees, scoring systems)

Neural networks

With minor pre-processing, all methods have similar performance

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- ...But don't they lose accuracy?
  - Explainable Machine Challenge (credit scoring data from FICO)
  - Florida COMPAS data (criminal recidivism)

OP-ED CONTRIBUTOR

# When a Computer Program Keeps You in Jail

By Rebecca Wexler

June 13, 2017



Glenn Rodriguez was denied parole because of a miscalculated “COMPAS” score.



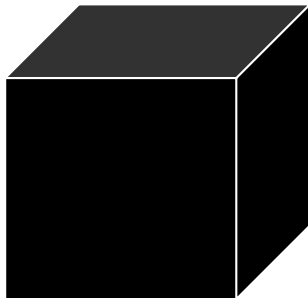
A typographical error in a COMPAS score can lead to years of extra prison time.

How accurate is COMPAS?

# COMPAS vs. CORELS



COMPAS: (Correctional Offender  
Management Profiling for  
Alternative Sanctions)

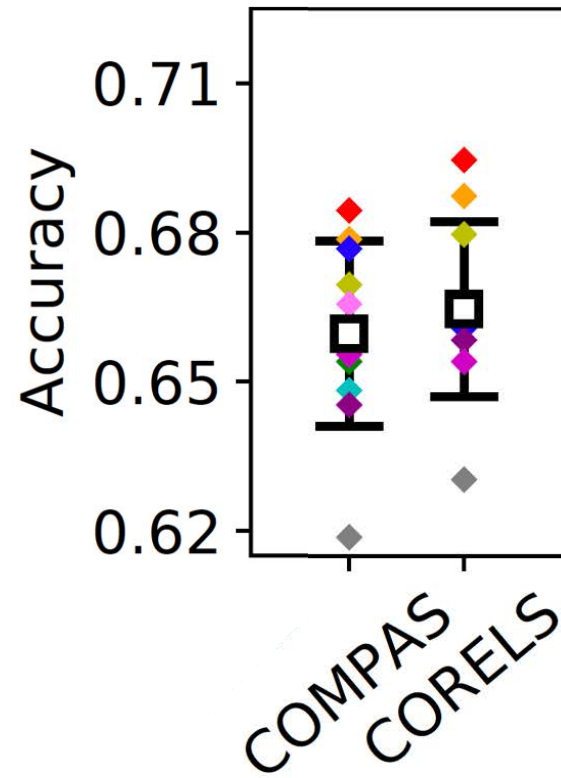


CORELS: (Certifiably Optimal Rule Lists, with  
Elaine Angelino, Nicholas Larus-Stone, Daniel  
Alabi, and Margo Seltzer, KDD 2017 & JMLR 2018)

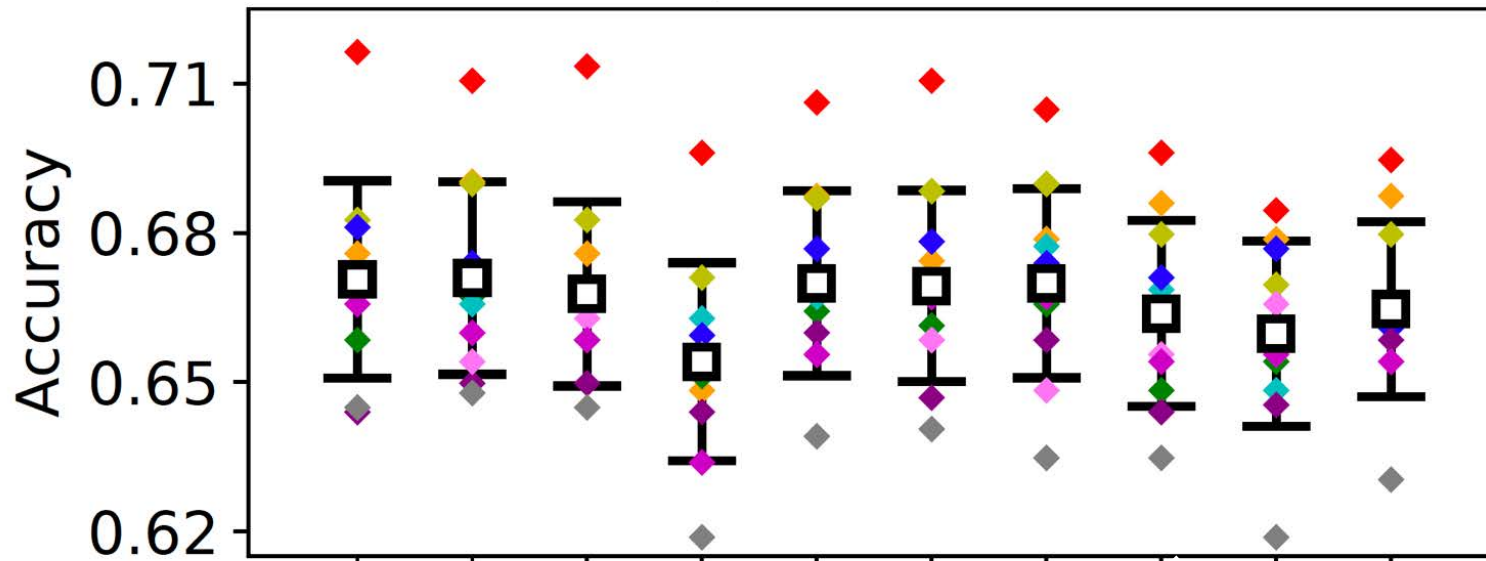
Here is the machine learning model:

If age=19-20 and sex=male, then predict arrest  
else if age=21-22 and priors=2-3 then predict arrest  
else if priors >3 then predict arrest  
else predict no arrest

# Prediction of re-arrest within 2 years



# Prediction of re-arrest within 2 years



COMPAS  
CORELS



If age=19-20 and sex=male, then predict arrest  
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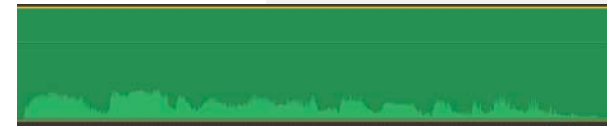
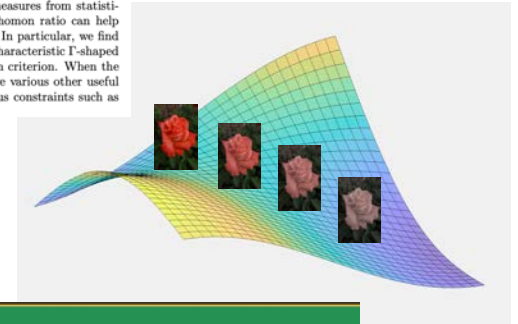
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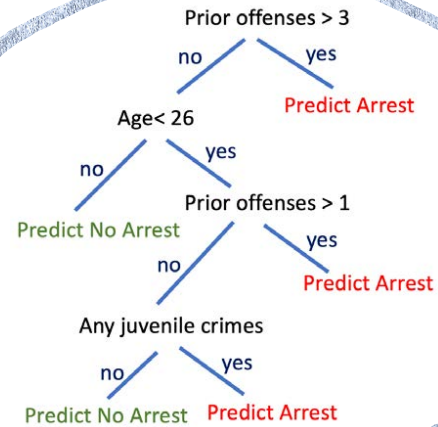


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# The Extremes of Interpretability:

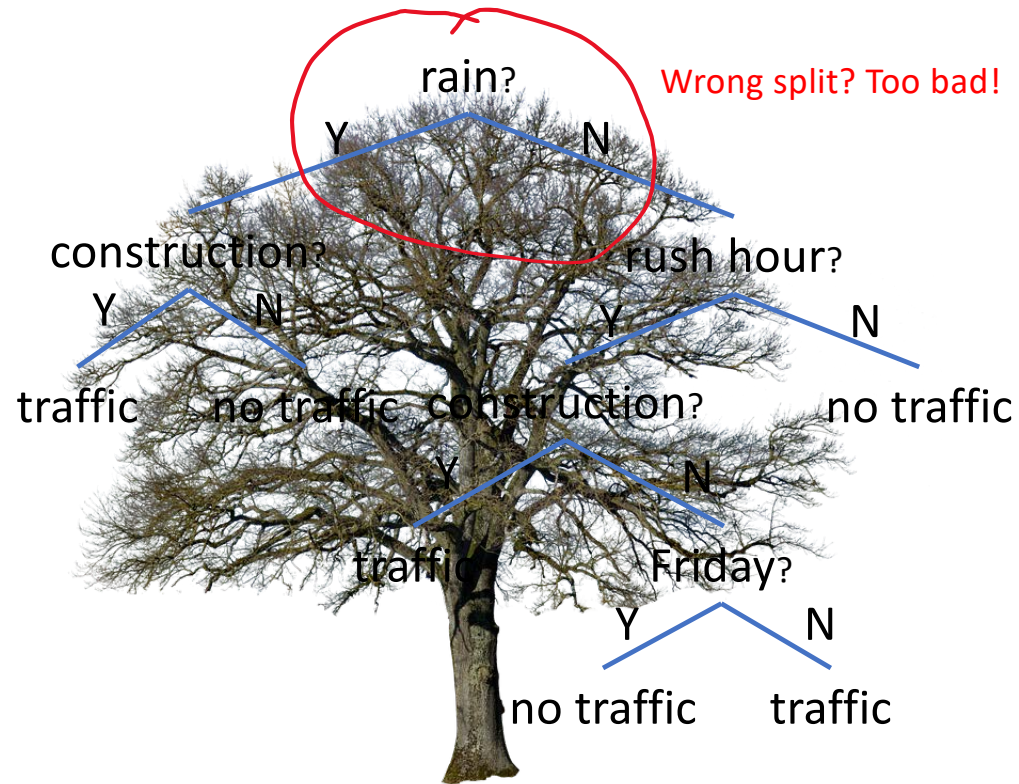
- Optimal decision trees
- Scoring systems

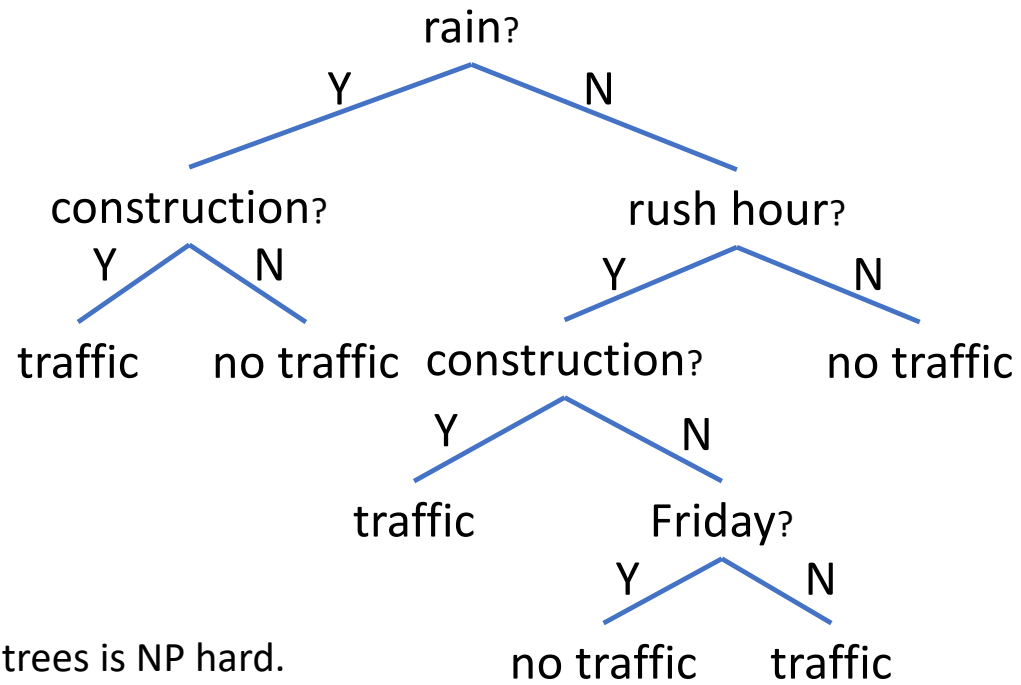


1.	Any cEEG Pattern with Frequency 2 Hz	1 point	...
2.	Epileptiform Discharges	1 point	+ ...
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6.	Brief Rhythmic Discharges	2 points	+ ...
		<b>SCORE</b>	= ...

<b>SCORE</b>	0	1	2	3	4	5	6+
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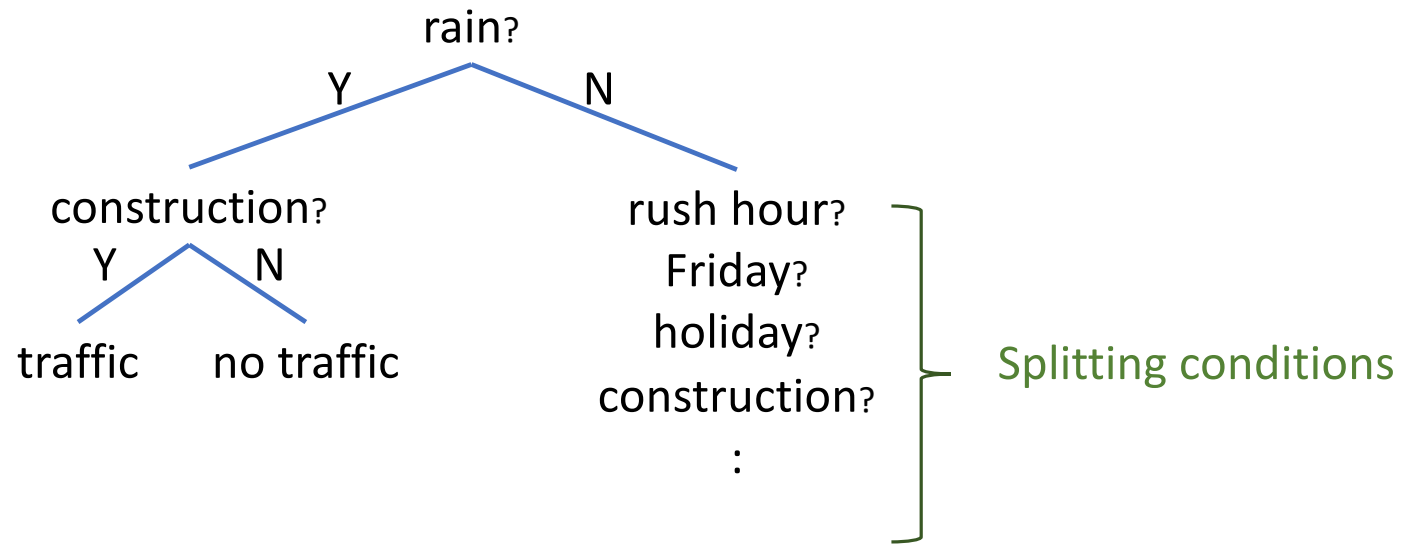
# Optimal Sparse Decision Trees





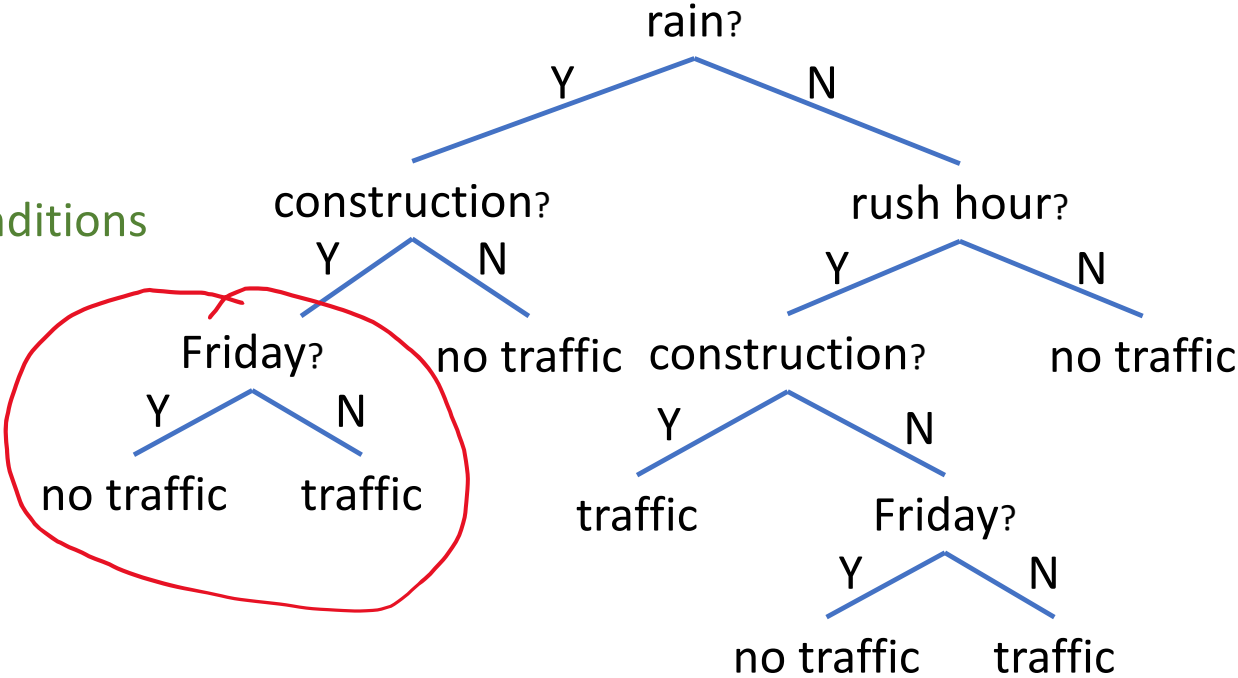
Optimal sparse decision trees is NP hard.  
Factorial in the number of variables.

Greedy construction: both the splitting and pruning conditions are based on statistical testing.



Greedy construction: both the splitting and pruning conditions are based on statistical testing.

Pruning conditions



Automatic Interaction Detection (AID) (Morgan & Sonquist, 1963) regression trees



THeta Automatic Interaction Detection (THAID) (Messenger & Mandell, 1972), classification trees



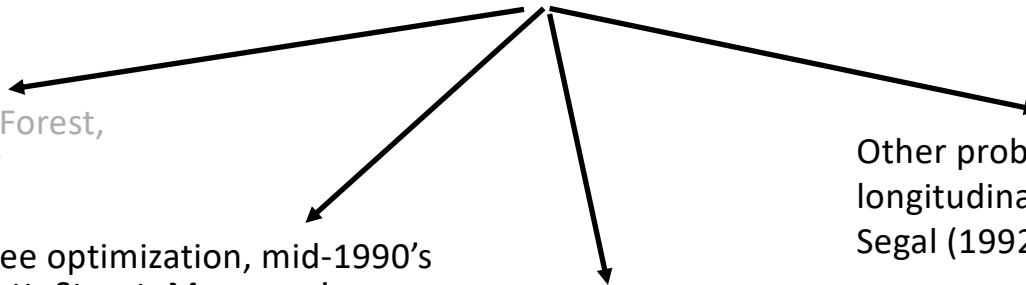
CHi-squared Automatic Interaction Detector (CHAID) (Kass, 1980)



Classification And Regression Trees (CART) (Breiman *et al.*, 1984)



ID3 (Quinlan, 1986), C4.5 (Quinlan, 1993)



Ensemble methods: Random Forest,  
Boosted Decision Trees, BART

Global tree optimization, mid-1990's  
Bennett, Street, Mangasarian

**Global Tree Optimization:  
A Non-greedy Decision Tree Algorithm**

1994

Kristin P. Bennett  
Email [bennek@rpi.edu](mailto:bennek@rpi.edu)  
Department of Mathematical Sciences  
Rensselaer Polytechnic Institute  
Troy, NY 12180 \*

Other problems:  
longitudinal data, survival curves:  
Segal (1992), Simonoff (several papers)

Improvements in splitting criteria for classification and regression  
Hypothesis tests, de-biasing (Strobl), missing variables

Tutorials (Murthy 1998, Loh 2014, L. Rokach & O. Maimon 2004 - beware)



```
graph TD; A(( )) --> B[Ensemble methods: Random Forest, Boosted Decision Trees, BART]; A --> C[Global tree optimization, mid-1990's Bennett, Street, Mangasarian]; A --> D[Improvements in splitting criteria for classification and regression Hypothesis tests, de-biasing (Strobl), missing variables]; A --> E[Other problems: longitudinal data, survival curves: Segal (1992), Simonoff (several papers)];
```

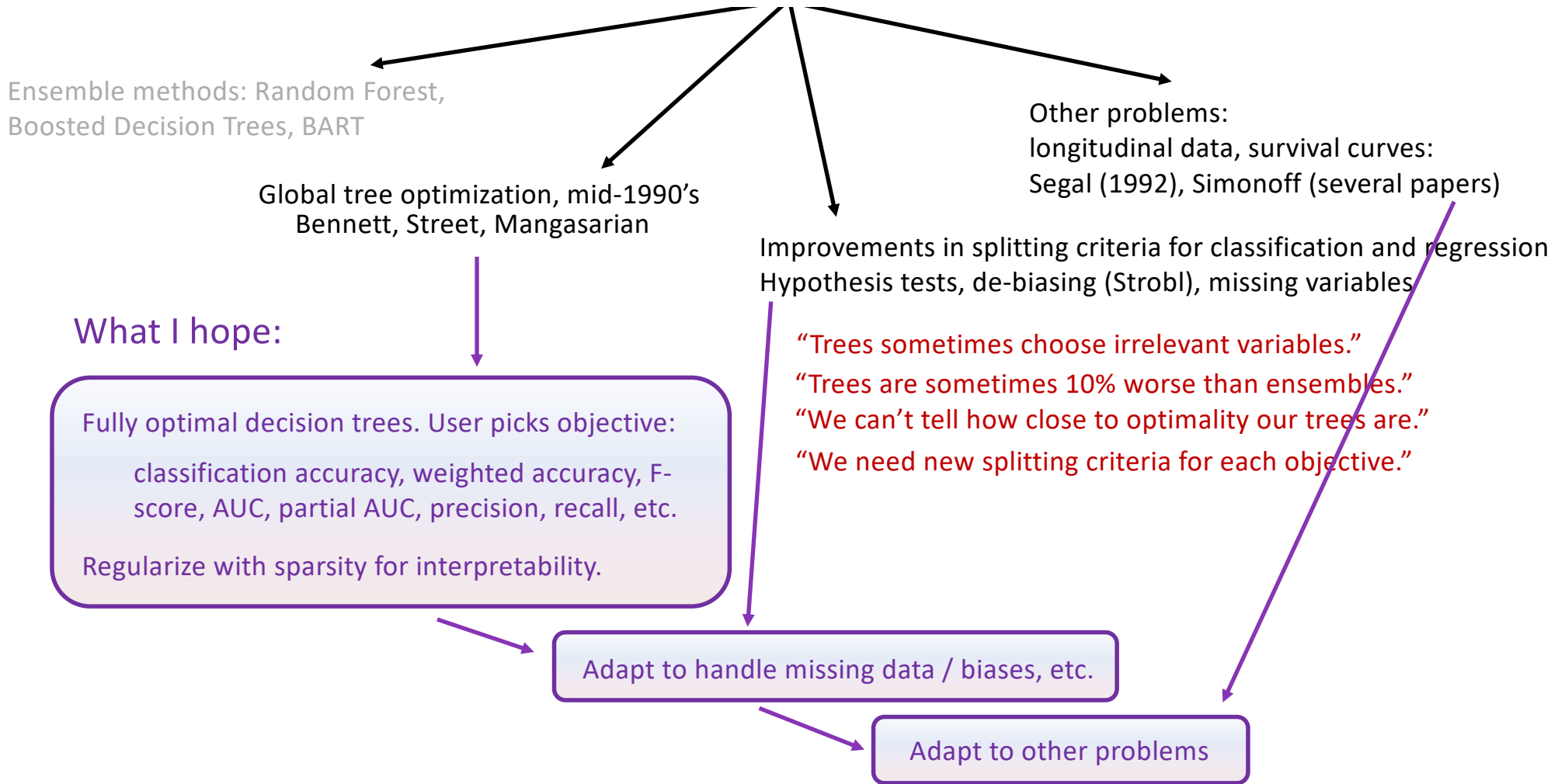
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Improvements in splitting criteria for classification and regression  
Hypothesis tests, de-biasing (Strobl), missing variables

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Fully optimal decision trees. User picks objective:  
classification accuracy, weighted accuracy, F-  
score, AUC, partial AUC, precision, recall, etc.

Regularize with sparsity for interpretability.

Fully optimal decision trees. User picks objective:

classification accuracy, weighted accuracy, F-score, AUC, partial AUC, precision, recall, etc.

Regularize with sparsity for interpretability.

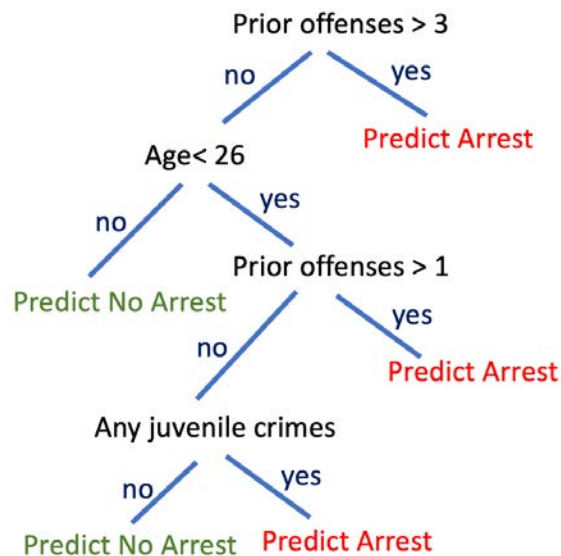
Approaches:

- **Genetic programming** (e.g., Fan & Gray, 2005, Janikow & Malatkar, 2011), or neural networks
  - no optimality gap
- For classification data that is able to be perfectly separated: **SAT solvers** (Narodytska et al., 2018, Janota 2020)
- **Mathematical programming solvers** (Bennett mid-1990's, Blanquero et al., 2018, Menickelly et al., 2018; Vilas Boas et al., 2019, Verwer & Zhang, **BinOCT** 2019)
- **Dynamic programming / Branch and Bound**
  - Garofalakis et al., DTC, 2003 (less relevant since it just finds subtrees of greedy-grown trees)
  - Nijssen & Fromont, DL8, 2007, Nijssen et al., **DL8.5**, 2020
  - Angelino et al, CORELS, 2018, Hu et al., OSDT 2019, Lin et al., **GOSDT**, 2020

with Jimmy Lin, Chudi Zhong, Diane Hu, Margo Seltzer

$\min_{\text{tree}} \hat{L}(\text{tree}, \{(x_i, y_i)\}_i)$  where

$$\hat{L}(\text{tree}, \{(x_i, y_i)\}_i) = \underbrace{\frac{1}{n} \sum_{i=1}^n 1_{[\text{tree}(x_i) \neq y_i]}}_{\text{Misclassification error}} + \underbrace{C(\# \text{ leaves in tree})}_{\text{Sparsity}}$$



An example of an optimal tree on the Broward County Florida re-arrest data

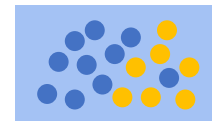
## Dynamic programming / Branch and Bound

Start with the full dataset and a naive label

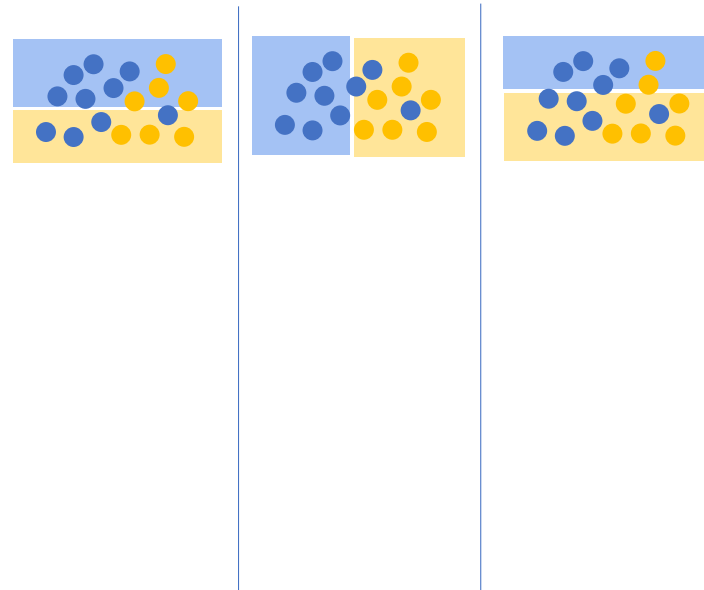


## Dynamic programming / Branch and Bound

Start with the full dataset and a naive label

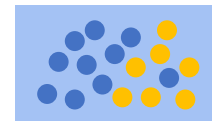


Split it into subsets using each feature

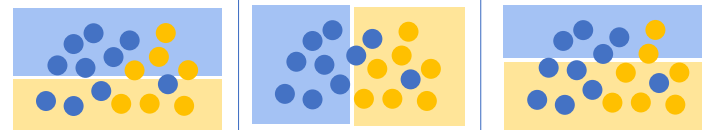


## Dynamic programming / Branch and Bound

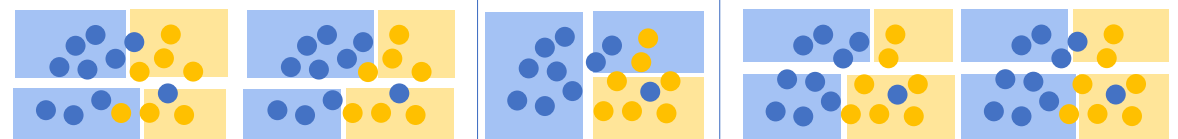
Start with the full dataset and a naive label



Split it into subsets using each feature



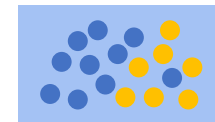
Keep splitting (if permitted)



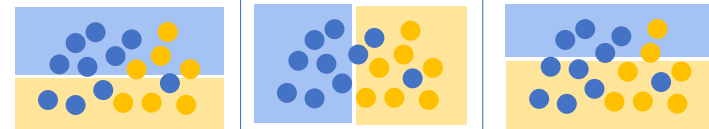
Can't  
split  
anymore

# Dynamic programming / Branch and Bound

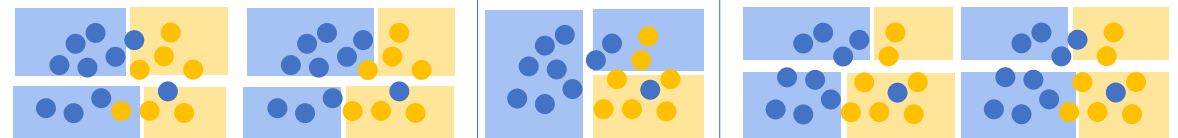
Start with the full dataset and a naive label



Split it into subsets using each feature



Keep splitting (if permitted)



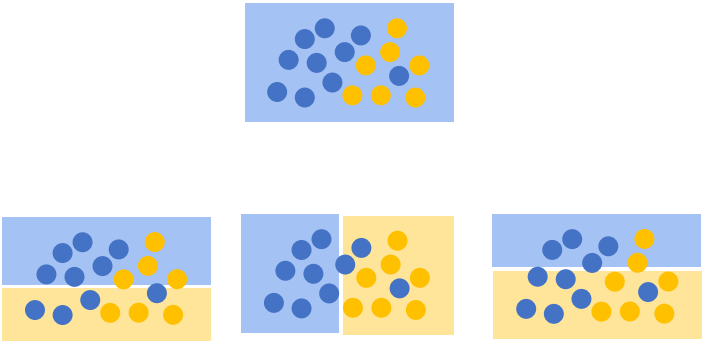
Consolidate any duplication found.

Can't split anymore

Identical subproblems

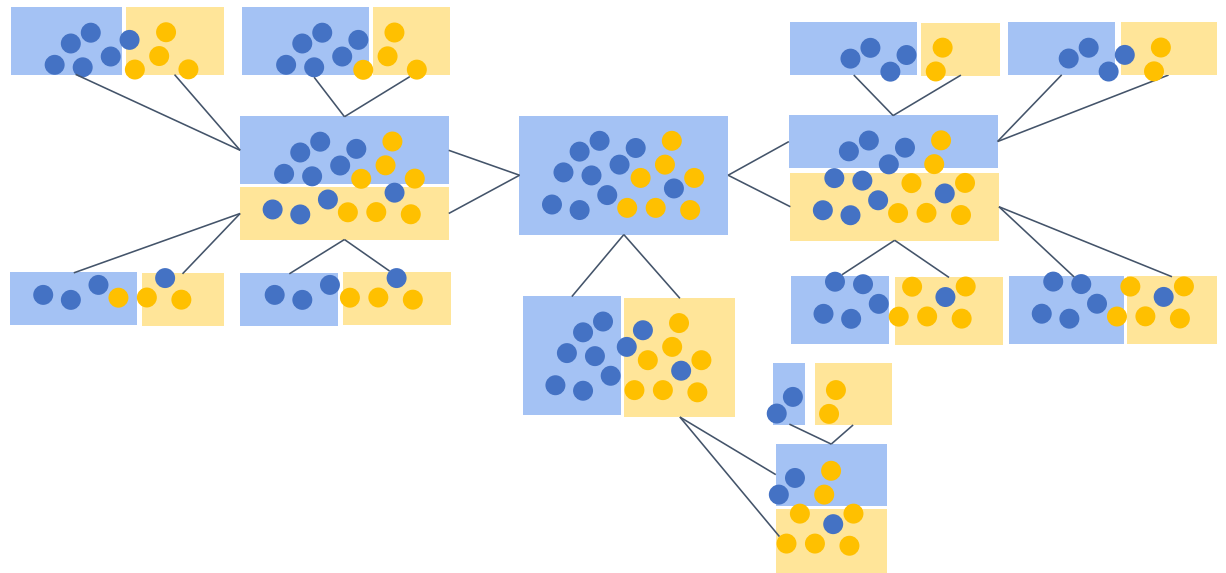


# Dynamic programming / Branch and Bound



## Dynamic programming / Branch and Bound

The solution to each subproblem yields the best feature to split on.

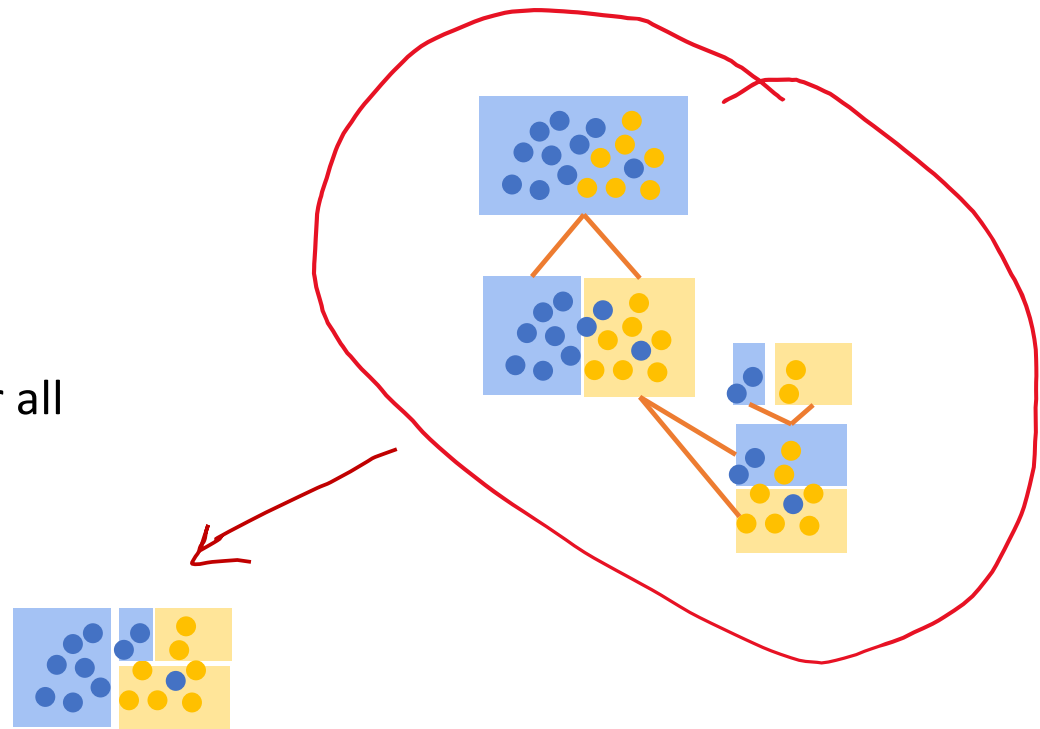


## Dynamic programming / Branch and Bound

The solution to each subproblem yields the best feature to split on.

The optimal solution is found after all subproblems are “completed”

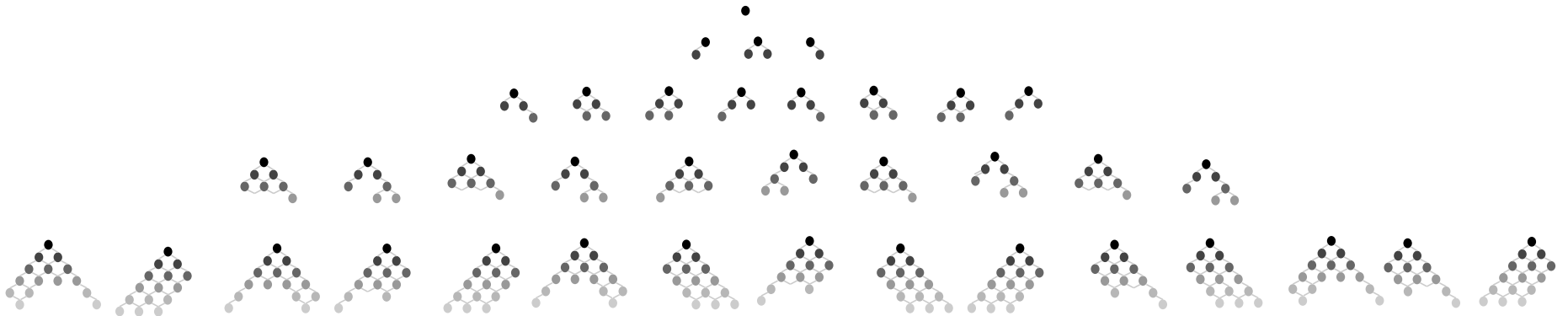
Some subproblems can be proven to yield non-optimal solutions



## Dynamic programming / Branch and Bound

### Analytical Bounds Reduce the Search Space

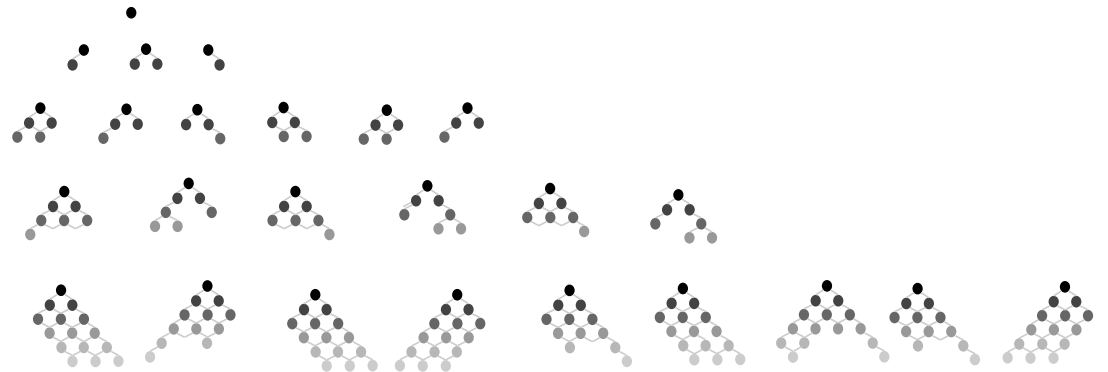
Theorems show that some partial trees can never be extended to form optimal trees.



## Dynamic programming / Branch and Bound

### Analytical Bounds Reduce the Search Space

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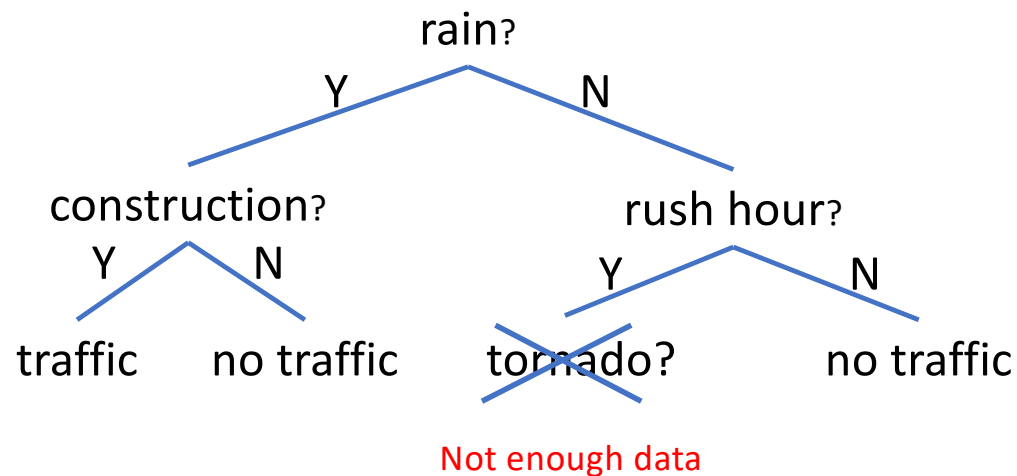




## GOSDT - Generalized and Scalable Optimal Sparse Decision Trees (Lin et al., ICML 2020)

### Analytical Bounds Reduce the Search Space

Theorems show that some partial trees can never be extended to form optimal trees.



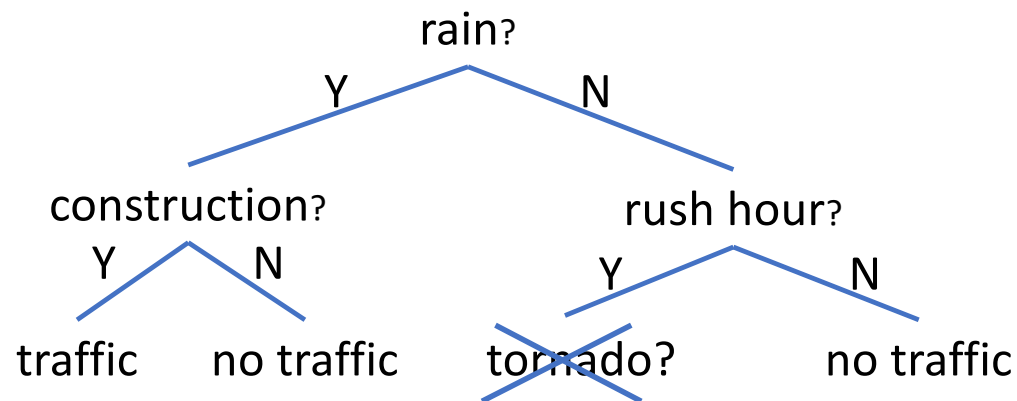
“Theorem”:

If the amount of data traveling through an internal node is  $< 2C$  (where  $C$  is the regularization parameter), the tree cannot achieve the minimum of the objective.

## GOSDT - Generalized and Scalable Optimal Sparse Decision Trees (Lin et al., ICML 2020)

### Analytical Bounds Reduce the Search Space

Theorems show that some partial trees can never be extended to form optimal trees.



Not accurate data

“Theorem”:

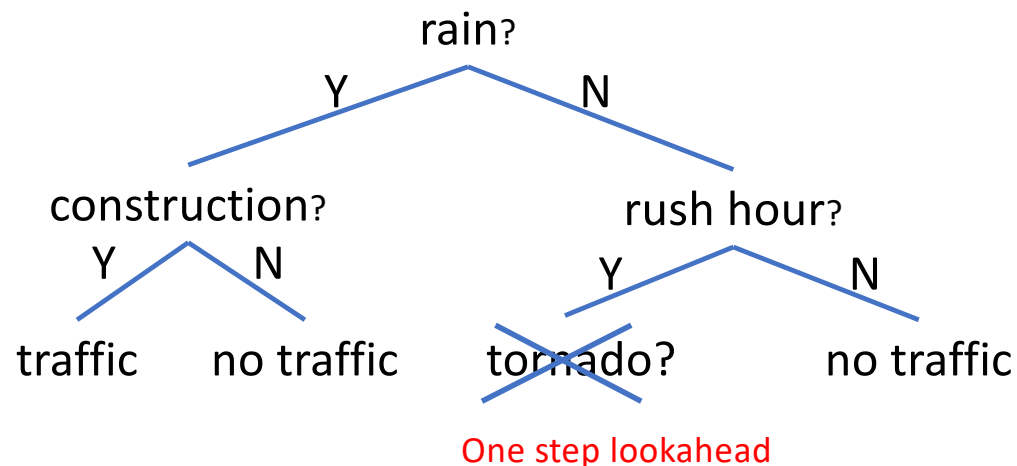
If a proposed split leads to  $< C$  correctly classified data going to either side of the split, then this split can be excluded, and we can exclude that feature anywhere further down the tree extending that leaf.



## GOSDT - Generalized and Scalable Optimal Sparse Decision Trees (Lin et al., ICML 2020)

### Analytical Bounds Reduce the Search Space

Theorems show that some partial trees can never be extended to form optimal trees.



“Theorem”: Consider a tree with lower bound  $b \leq R_{\text{current best}}$ .  
If  $b + C \geq R_{\text{current best}}$ , we can prune all of its child trees.

# GOSDT - Generalized and Scalable Optimal Sparse Decision Trees (Lin et al., ICML 2020)

## Represent a tree by its leaves

rain & construction & traffic

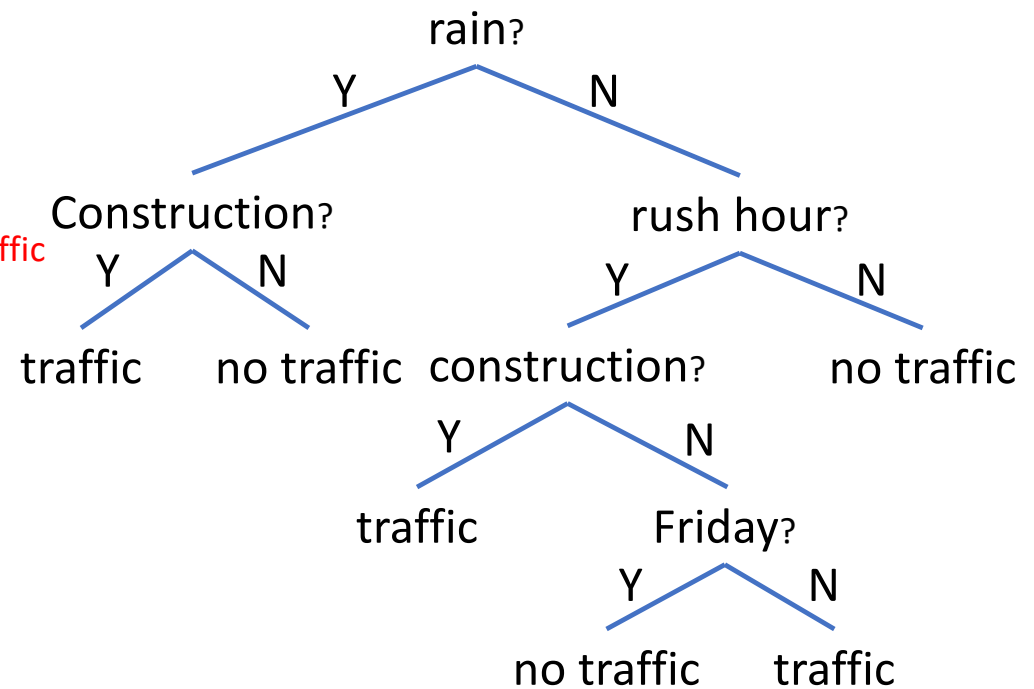
rain & no construction & no traffic

no rain & rush hour & construction & traffic

no rain & rush hour & no construction & Friday and no traffic

no rain & rush hour & no construction & Friday and traffic

no rain & no rush hour & no traffic



# GOSDT - Generalized and Scalable Optimal Sparse Decision Trees (Lin et al., ICML 2020)

## Permutation map: Discover identical trees already evaluated

rain & construction & traffic

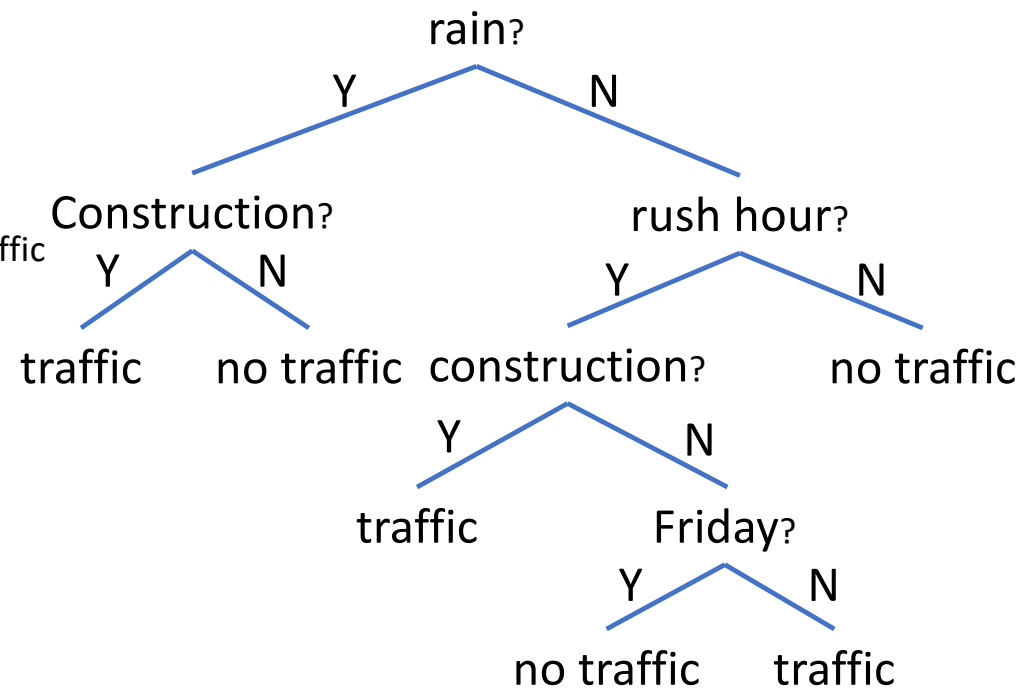
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# GOSDT - Generalized and Scalable Optimal Sparse Decision Trees (Lin et al., ICML 2020)

Bit-vectors describe data represented by each leaf

rain & construction & traffic

[1000010001001110000.....0]

rain & no construction & no traffic

[0110001000000000110.....1]

no rain & rush hour & construction & traffic

[0001000100000001000.....0]

no rain & rush hour & no construction & Friday and no traffic

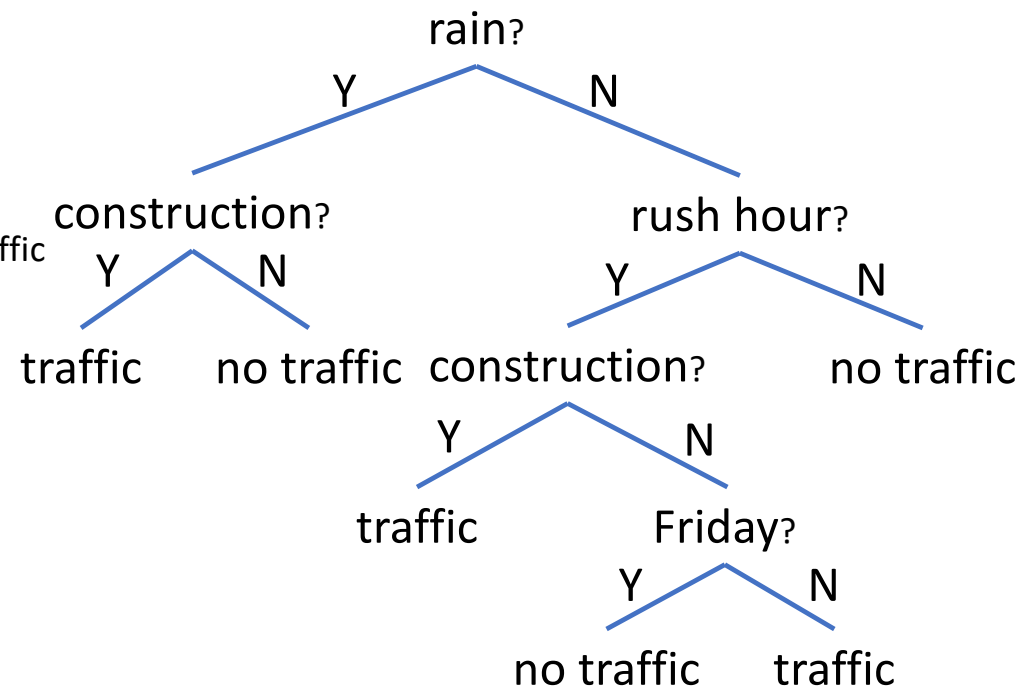
[0000100000000000001.....0]

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[0000000010000000000.....0]

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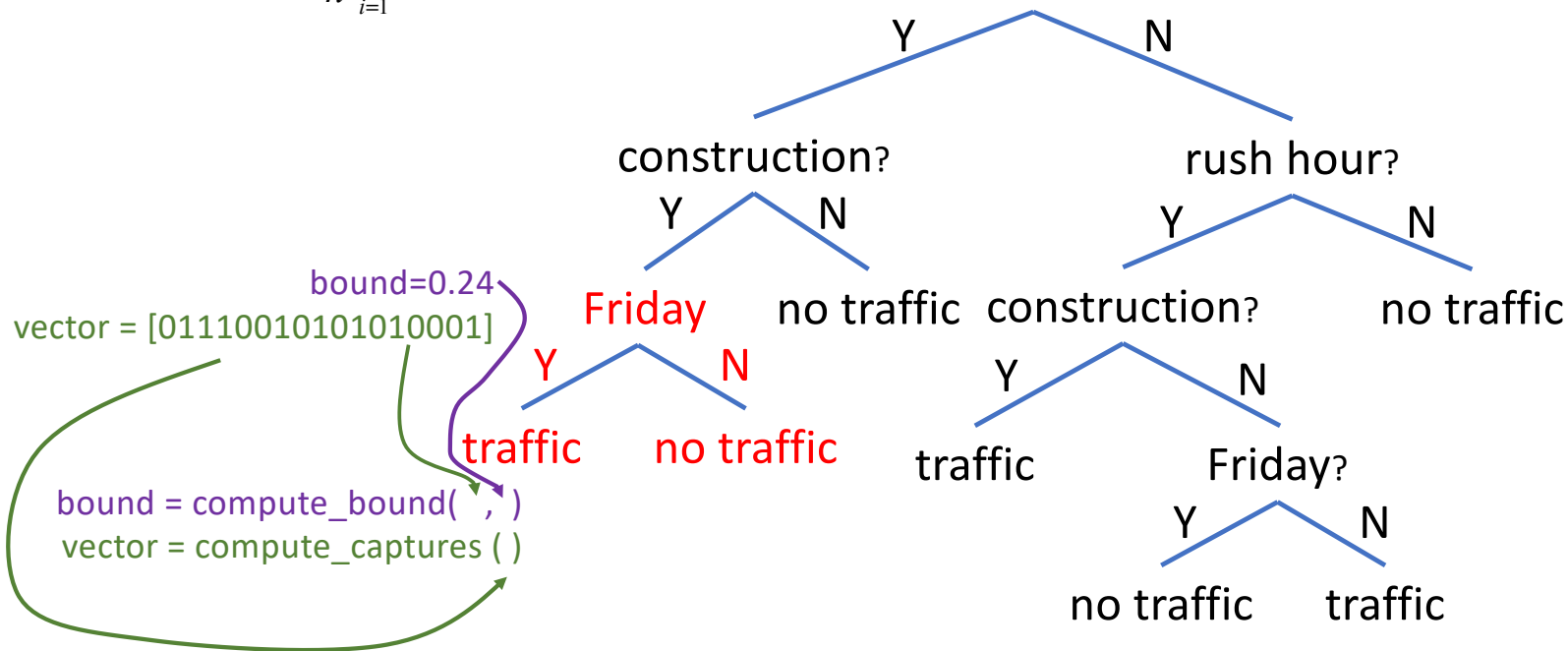
[0000000000011000000.....0]



# GOSDT - Generalized and Scalable Optimal Sparse Decision Trees (Lin et al., ICML 2020)

## Incremental computation of objective and bounds

$$\hat{L}(\text{tree}, \{(x_i, y_i)\}_i) = \frac{1}{n} \sum_{i=1}^n 1_{[\text{tree}(x_i) \neq y_i]} + C(\#\text{leaves in tree})$$



GOSDT - Generalized and Scalable Optimal Sparse Decision Trees  
(Lin et al., ICML 2020)

Strong analytical bounds

Leaf-based representation

Permutation map



Fast Implementation

Caching of intermediate results

Incremental computation

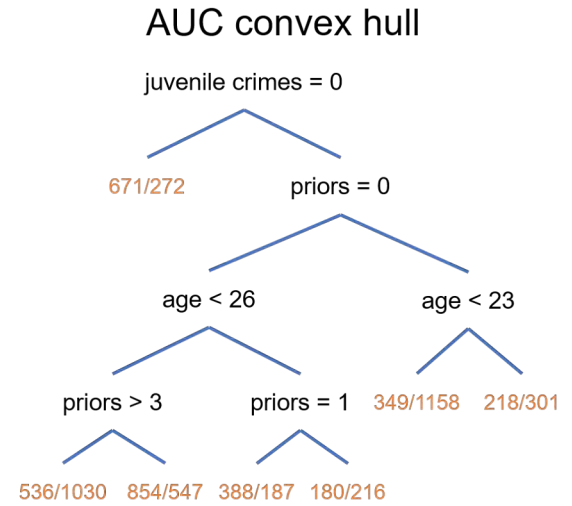
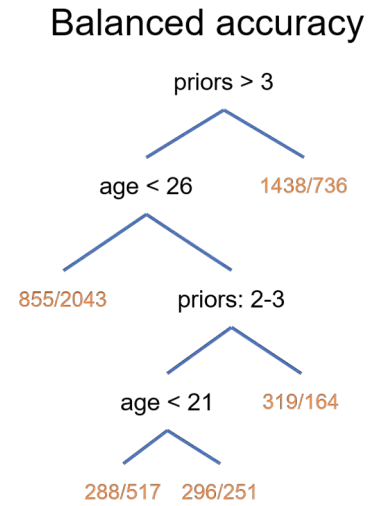
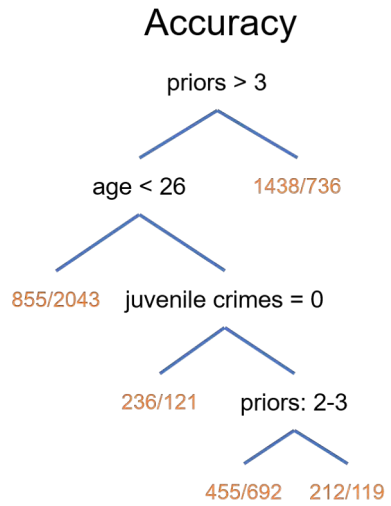
Consolidation of repeated subproblems

# GOSDT - Generalized and Scalable Optimal Sparse Decision Trees (Lin et al., ICML 2020)

$\min_{\text{tree}} \hat{L}(\text{tree}, \{(x_i, y_i)\}_i)$  where

$$\hat{L}(\text{tree}, \{(x_i, y_i)\}_i) = \ell(\text{tree}, \{(x_i, y_i)\}_i) + \lambda(\# \text{ leaves in tree})$$

- Can optimize any loss function monotonically increasing in FP and FN (Balanced accuracy, weighted accuracy, F-1, precision, ...)
- Can optimize rank statistics (AUC and partial AUC under the ROC convex hull)



## GOSDT - Generalized and Scalable Optimal Sparse Decision Trees (Lin et al., ICML 2020)

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- Can optimize any loss function monotonically increasing in FP and FN (Balanced accuracy, weighted accuracy, F-1, precision, ...)
- Can optimize rank statistics (AUC and partial AUC under the ROC convex hull)

### Main experimental results:

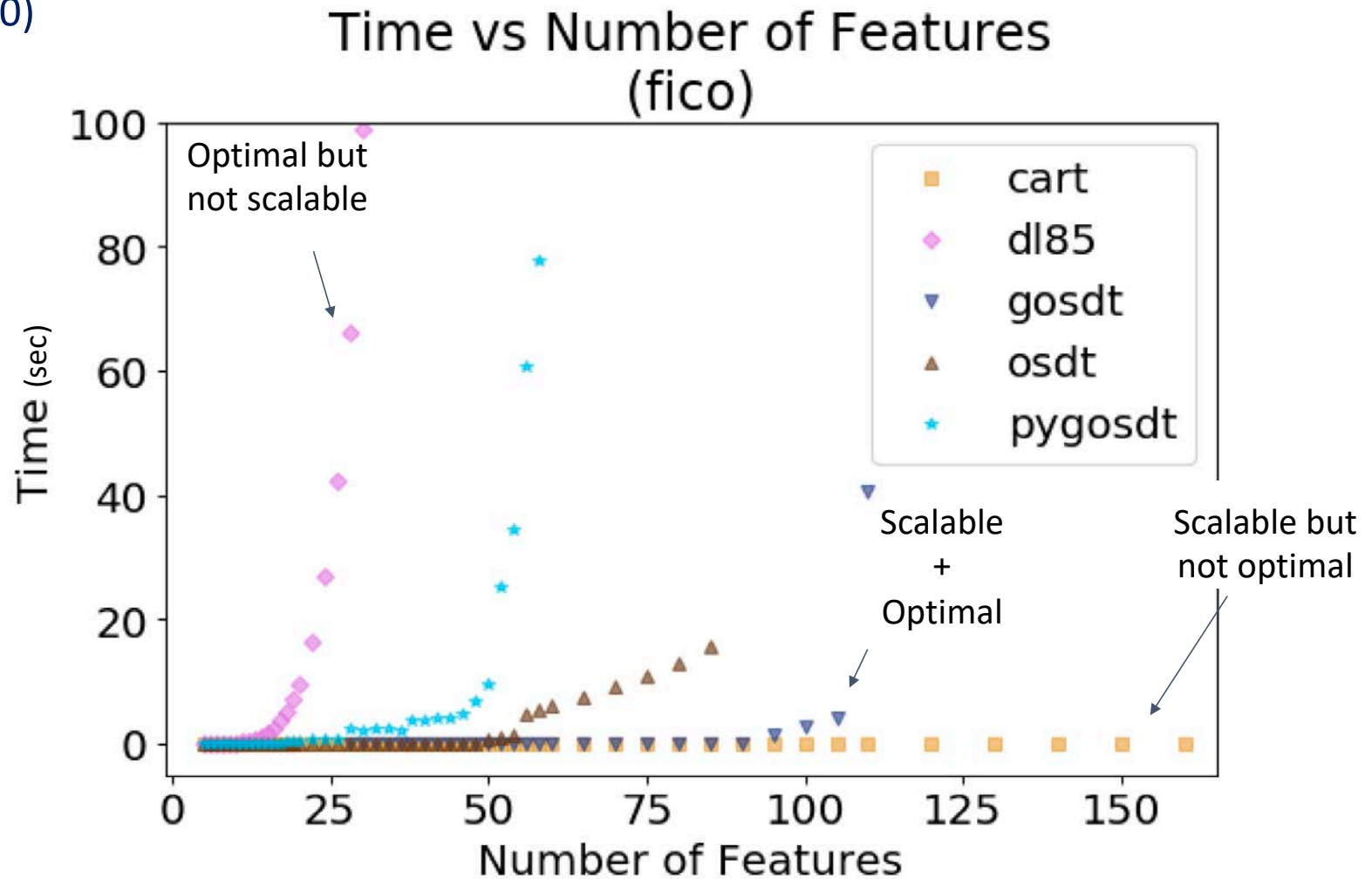
- Similar classification error to black box methods.
- For custom losses, much better loss values than greedy decision trees.
- Sparser than all heuristic methods
- Orders of magnitude faster than the next best method.



# GOSDT - Generalized and Scalable Optimal Sparse Decision Trees (Lin et al., ICML 2020)

## Scalability

Improvements in orders of magnitude



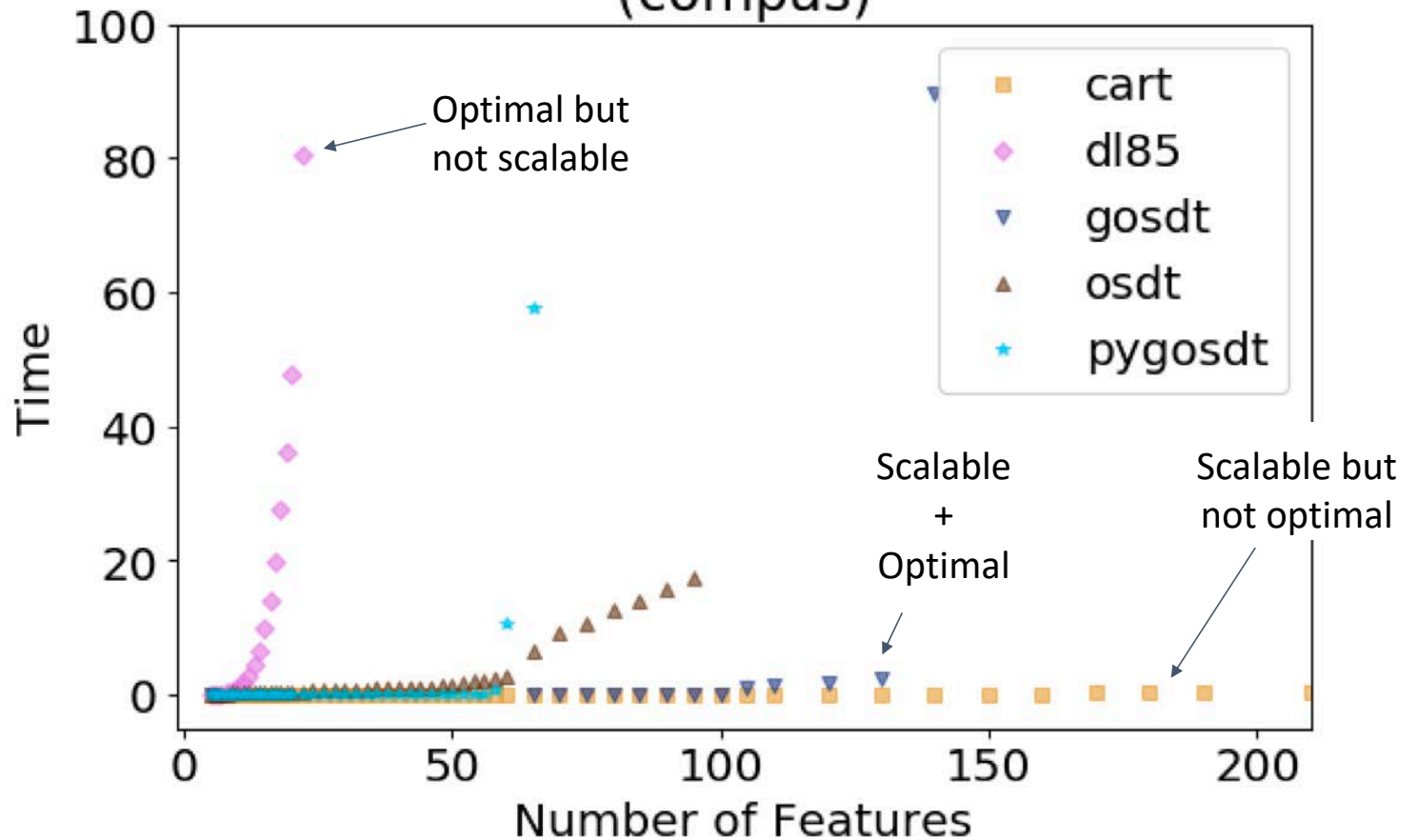
Note: BinOCT too slow to include.

# GOSDT - Generalized and Scalable Optimal Sparse Decision Trees (Lin et al., ICML 2020)

## Time vs Number of Features (compas)

### Scalability

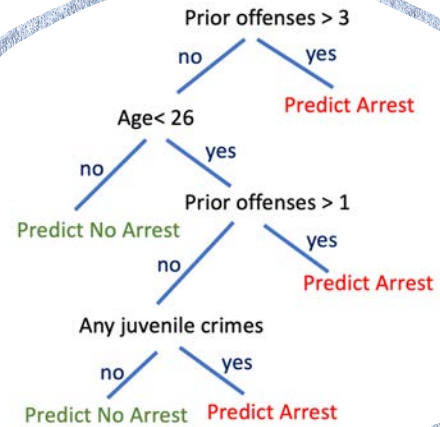
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In this talk

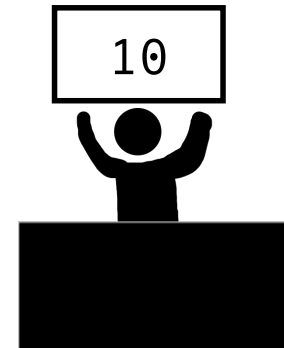
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<b>RISK</b>	<5%	11.9%	26.9%	50.0%	73.1%	88.1%	95.3%

# Scoring systems



1 point if person has social type with below average parole violation rate

SOCIAL TYPE	VIOLATION RATE
All persons.....	26.5%
Ne'er-do-well.....	25.6
Mean citizen.....	30.0
Drunkard.....	38.9
Gangster.....	23.2
Recent immigrant.....	16.7
Farm boy.....	10.2
Drug addict.....	66.7

total score over all 21 significant factors predicts success at parole

POINTS FOR NUMBER OF FACTORS	Per Cent Non-violators of Parole
16-21	98.5
14-15	97.8
13	91.2
12	84.9
11	77.3
10	65.9
7-9	56.1
5-6	32.9
2-4	24.0

Burgess. Factors determining success or failure on parole. 1928

<b>FACTOR</b>	<b>Score *</b>
<b>Gender</b>	
Female	0
Male	1
<b>Age</b>	
Less than 24	3
24-29	2
30-49	1
50+	0
<b>County</b>	
Rural counties	0
Smaller, urban count	1
Allegheny and Philadelphia Counties	2
<b>Total number of prior arrests</b>	
0	0
1	1
2 to 4	2
5 to 12	3
13+	4
<b>Prior property arrests</b>	
No	0
Yes	1
<b>Prior drug arrests</b>	
No	0
Yes	1
<b>Property offender</b>	
No	0
Yes	1
<b>Offense gravity score (OGS)</b>	
4+	0

<b>Risk score</b>	<b>N</b>	<b>% Arrested</b>
0	3	0.0
1	47	17.0
2	181	9.9
3	436	23.6
4	737	24.8
5	1,036	32.4
6	1,067	40.7
7	1,434	47.2
8	1,934	55.5
9	2,103	62.3
10	1,829	69.9
11	1,098	72.2
12	278	79.1
13	25	80.0
14	3	66.7

1. Lived with both biological parents to age 16 (except for death of parent):

Yes ..... -2  
 No ..... +3  
 Evidence:

2. Elementary School Maladjustment:

No Problems..... -1  
 Slight (Minor discipline or attendance) or Moderate Problems..... +2  
 Severe Problems (Frequent disruptive behavior and/or attendance or behavior resulting in expulsion or serious suspensions) ..... +5  
 (Same as CATS Item)

3. History of alcohol problems (Check if present):

\* Parental Alcoholism      \* Teenage Alcohol Problem  
 \* Adult Alcohol Problem    \* Alcohol involved in prior offense  
 \* Alcohol involved in index offense  
 No boxes checked..... -1  
 1 or 2 boxes checked ..... 0  
 3 boxes checked ..... +1  
 4 or 5 boxes checked ..... +2  
 Evidence:

4. Marital status (at the time of or prior to index offense):

Ever married (or lived common law in the same home for at least six months) ..... -2  
 Never married..... +1  
 Evidence:

5. Criminal history score for nonviolent offenses prior to the index offense

Score 0 ..... -2  
 Score 1 or 2..... 0  
 Score 3 or above ..... +3  
 (from the Cormier-Lang system, see below)

6. Failure on prior conditional release (includes parole or probation violation or revocation, failure to comply, bail violation, and any new arrest while on conditional release):

No.....0  
 Yes ..... +3  
 Evidence:

7. Age at index offense

Enter Date of Index Offense: \_\_\_/\_\_\_/\_\_\_  
 Enter Date of Birth: \_\_\_/\_\_\_/\_\_\_  
 Subtract to get Age:  
 39 or over ..... -5  
 34 - 38 ..... -2  
 28 - 33 ..... -1  
 27 ..... 0  
 26 or less..... +2

8. Victim Injury (for index offense; the most serious is scored):

Death..... -2  
 Hospitalized..... -2  
 Treated and released..... +1  
 None or slight (includes no victim)..... +2  
 Note: admission for the gathering of forensic evidence only is NOT considered as either treated or hospitalized; ratings should be made based on the degree of injury.  
 Evidence:

9. Any female victim (for index offense)

Yes ..... -1  
 No (includes no victim)..... +1  
 Evidence:

10. Meets DSM criteria for any personality disorder (must be made by appropriately licensed or certified professional)

No..... -2  
 Yes ..... +3  
 Evidence:

11. Meets DSM criteria for schizophrenia (must be made by appropriately licensed or certified professional)

Yes ..... -3  
 No ..... +1  
 Evidence:

12. a. Psychopathy Checklist score (if available, otherwise use item 12.b. CATS score).....

4 or under ..... -3  
 5 - 9..... -3  
 10-14 ..... -1  
 15-24 ..... 0  
 25-34 ..... +4  
 35 or higher ..... +12  
 Note: If there are two or more PCL scores, average the scores.  
 Evidence:

12. b. CATS score (from the CATS worksheet)

0 or 1 ..... -3  
 2 or 3 ..... 0  
 4 ..... +2  
 5 or higher ..... +3

12. WEIGHT (Use the highest circled weight from 12 a. or 12 b.) ..... \_\_\_\_\_

**TOTAL VRAG SCORE (SUM CIRCLED SCORES FOR ITEMS 1 - 11 PLUS THE WEIGHT FOR ITEM 12):** \_\_\_\_\_

VRAG Score	Category of Risk
-24	Low
-23	Low
-22	Low
-20	Low
-19	Low
-18	Low
-17	Low
-16	Low
-15	Low
-14	Low
-13	Low
-12	Low
-11	Low
-10	Low
-9	Low
-8	Low
-7	Medium
-6	Medium
-5	Medium
-4	Medium
-3	Medium
-2	Medium
-1	Medium
0	Medium
1	Medium
2	Medium
3	Medium
4	Medium
5	Medium
6	Medium
7	Medium
8	Medium
9	Medium
10	Medium
11	Medium
2	Medium
13	Medium
14	High
15	High
16	High
17	High
18	High
19	High
20	High
21	High
22	High
23	High
24	High
25	High
26	High
28	High
32	High

## Violence Risk Appraisal Guide (Quinsey et al, 2006)



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## Calculators

**By Category** [Alphabetically](#)

**Addiction Medicine**



**Anesthesiology**



**Cardiac Surgery**



**Cardiology**



**COVID-19**



**Critical Care**



**Emergency**





---

> Intracerebral Hemorrhage

---

> Ischemic Stroke

---

> Movement Disorder

---

> Multiple Sclerosis & Demyelinating Disease

---

> Neurophysiology

---

^ Seizure

[2HELPS2B Score](#)

[Phenytoin Adjustment in Renal Failure](#)

[Seizure vs Syncope](#)

---

> Subarachnoid Hemorrhage

---

**Obstetrics & Gynecology**



---

**Oncology**



---

**Orthopedics**



---

**Otolaryngology (ENT)**



---

**Pathology & Lab Medicine**



## 2HELPS2B Score

Estimate duration of EEG monitoring needed to detect 95% of seizures



SI

US

### Calculator

### References/About

- 1. Frequency of any periodic or rhythmic pattern of more than 2 Hz except generalized rhythmic delta activity? >
- 2. Independent sporadic epileptiform discharges? >
- 3. Lateralized Periodic Discharges (LPDs), Bilateral Independent Periodic Discharges (BIPDs), or Lateralized Rhythmic Delta Activity (LRDA)? >
- 4. "Plus" features: superimposed rhythmic, fast, or sharp activity only on LRDA, LPDs, or BIPDs? >
- 5. Prior seizure: a history of epilepsy or recent events suspicious for clinical seizures? >
- 6. BIRD: Brief potentially Ictal Rhythmic Discharges? >

1. Frequency of any periodic or rhythmic pattern of more than 2 Hz except generalized rhythmic delta activity?

Yes

No

Next Question →

Created by QxMD



0/6 completed



## Key challenges:

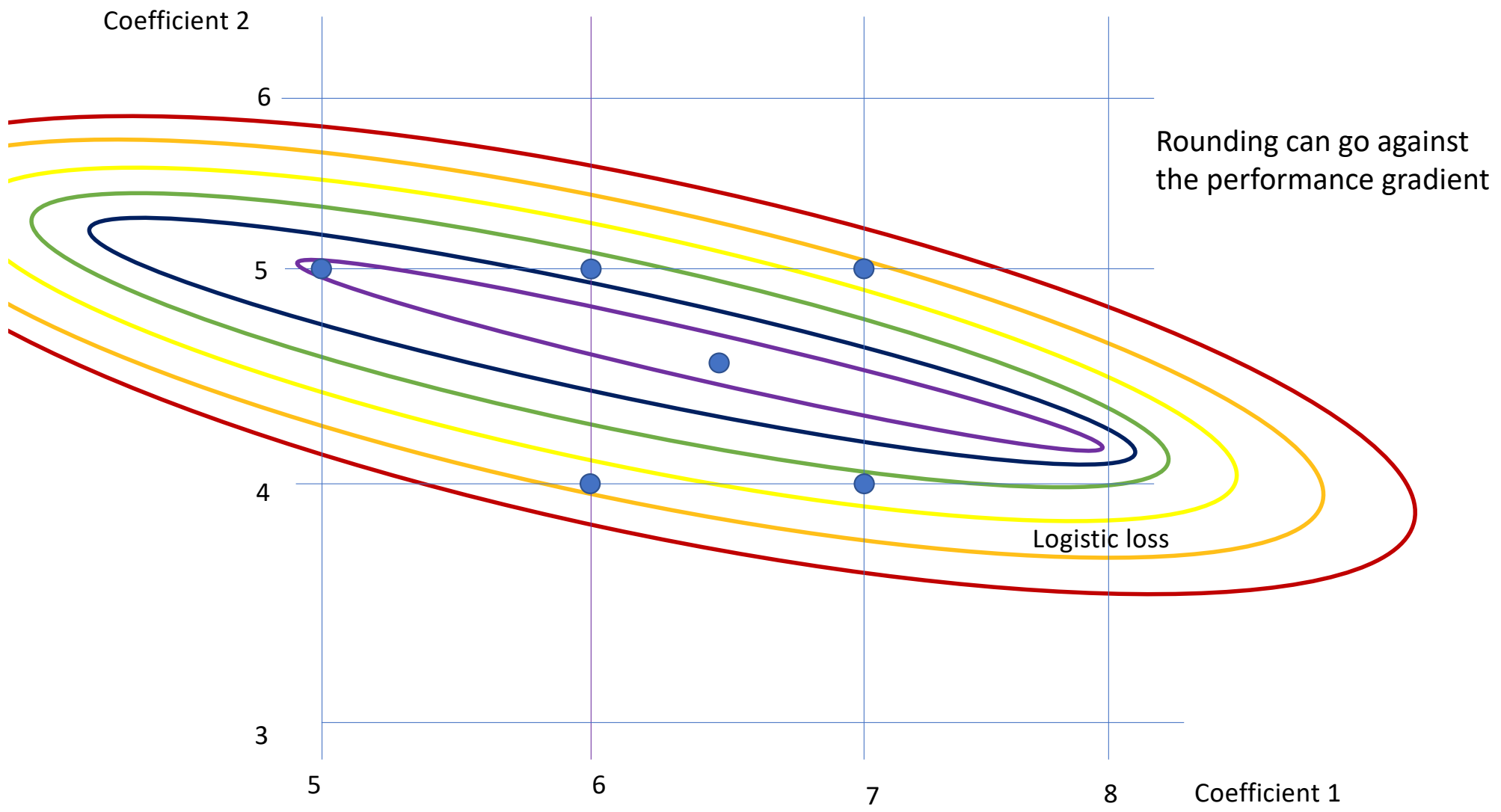
- Constraints (e.g., FP<20%, fairness, etc.)
- Integrality

## Typical approach:

1. <i>Congestive Heart Failure</i>	1 point	...					
2. <i>Hypertension</i>	1 point	+ ...					
3. <i>Age ≥ 75</i>	1 point	+ ...					
4. <i>Diabetes Mellitus</i>	1 point	+ ...					
5. <i>Prior Stroke or Transient Ischemic Attack</i>	2 points	+ ...					
<b>ADD POINTS FROM ROWS 1-5</b>	<b>SCORE</b>	<b>= ...</b>					
<b>SCORE</b>	0	1	2	3	4	5	6
<b>STROKE RISK</b>	1.9%	2.8%	4.0%	5.9%	8.5%	12.5%	18.2%

(Gage et al., 2001), CHADS2 score for stroke prediction: **panel of experts**

(Antman et al., 2000), TIMI risk score for unstable angina/non-ST elevation MI: **preliminary feature selection, followed by logistic regression with the chosen features, scaling, and rounding**



## Elastic Net

**SCORE** =    **1.42**    Rhythmic Patterns Include [BiPD, LRDA, LPD]  
              + **0.31**    Prior Seizure  
              + **0.21**    Epileptiform Discharges  
              + **0.26**    Patterns Superimposed with Fast or Sharp Activity  
              + **0.25**    Brief Rhythmic Discharges  
              - **2.54**

## Elastic Net + Rounding

**SCORE** = 1 Rhythmic Patterns Include [BiPD, LRDA, LPD]  
+ 0 ~~Prior Seizure~~  
+ 0 ~~Epileptiform Discharges~~  
+ 0 ~~Patterns Superimposed with Fast or Sharp Activity~~  
+ 0 ~~Brief Rhythmic Discharges~~  
- 3

## Elastic Net

**SCORE** =    **1.42**    Rhythmic Patterns Include [BiPD, LRDA, LPD]  
              + **0.31**    Prior Seizure  
              + **0.21**    Epileptiform Discharges  
              + **0.26**    Patterns Superimposed with Fast or Sharp Activity  
              + **0.25**    Brief Rhythmic Discharges  
              – **2.54**

## Elastic Net + Scaling + Rounding

<b>SCORE</b>	=	6	Rhythmic Patterns Include [BiPD, LRDA, LPD]
		+ 1	Prior Seizure
		+ 1	Epileptiform Discharges
		+ 1	Patterns Superimposed with Fast or Sharp Activity
		+ 1	Brief Rhythmic Discharges
		- 10	



## Elastic Net

**SCORE** =    **1.42**    Rhythmic Patterns Include [BiPD, LRDA, LPD]  
          + **0.31**    Prior Seizure  
          + **0.21**    Epileptiform Discharges  
          + **0.26**    Patterns Superimposed with Fast or Sharp Activity  
          + **0.25**    Brief Rhythmic Discharges  
          – **2.54**

RiskSLIM model (optimized)

1.	BriefRhythmicDischarge	2 points		...
2.	PatternsInclude LPD	2 points	+	...
3.	PriorSeizure	1 point	+	...
4.	EpiletiformDischarge	1 point	+	
<b>SCORE</b>			=	

<b>SCORE</b>	0	1	2	3	4	5	6
<b>RISK</b>	4.7%	11.9%	26.9%	50.0%	73.1%	88.1%	95.3%

(This one is better calibrated and has large AUC.)

# Risk-Calibrated Supersparse Linear Integer Models (Risk-SLIM)

(Ustun, R, 2019)

$$\min_{\lambda \in L} \underbrace{\sum_{i=1}^n \log(1 + e^{-y_i x_i^\top \lambda})}_{\text{Logistic Loss}} + \underbrace{C \|\lambda\|_0}_{\text{Model Size}}$$

*MINLP – really hard...*

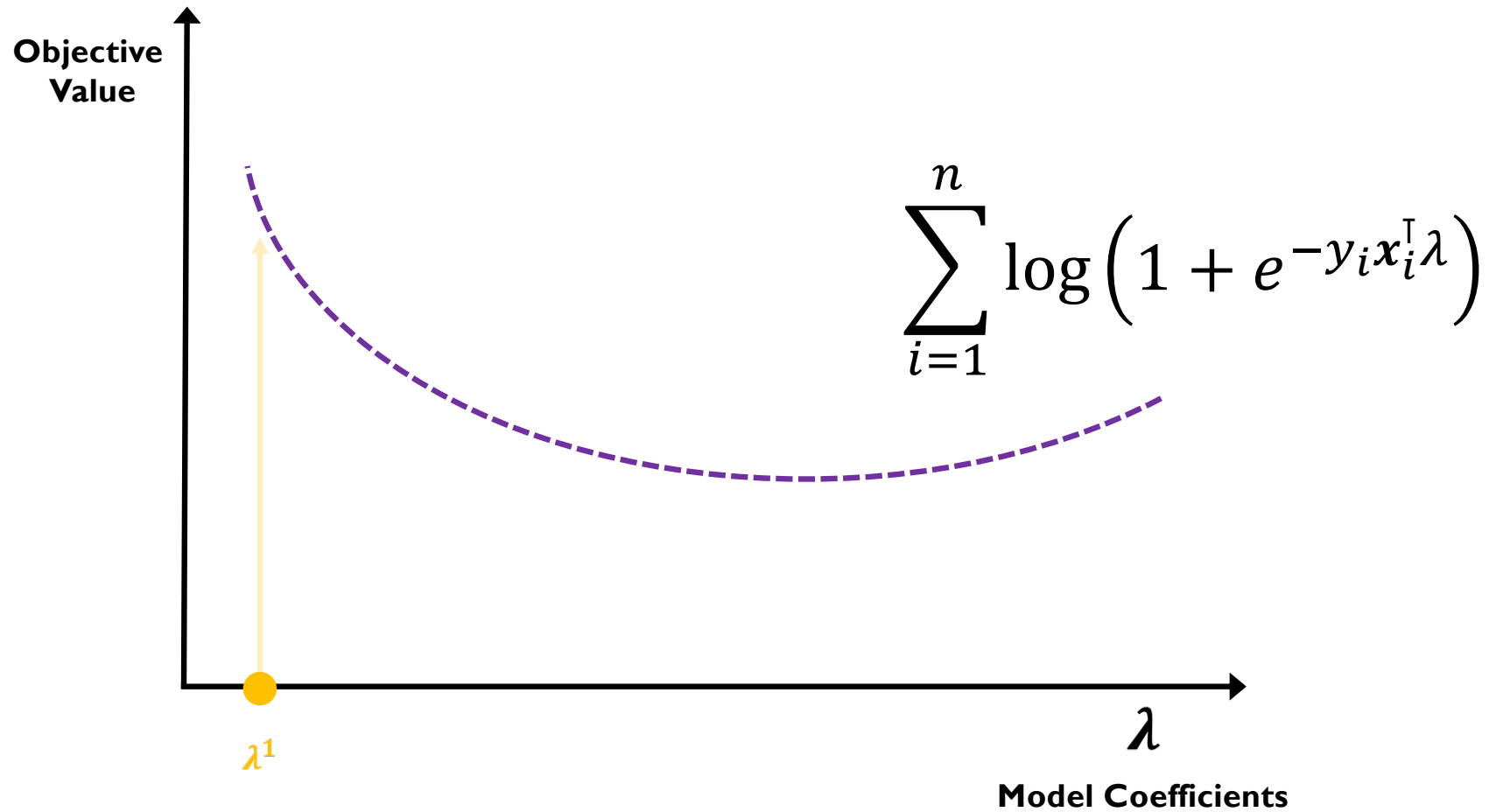
$\lambda \in L$  means that  $\forall j, \lambda_j \in \underbrace{\{-10, -9, \dots, 0, \dots, 9, 10\}}_{\text{Small Integer Coefficients}}$

(optional: additional constraints)

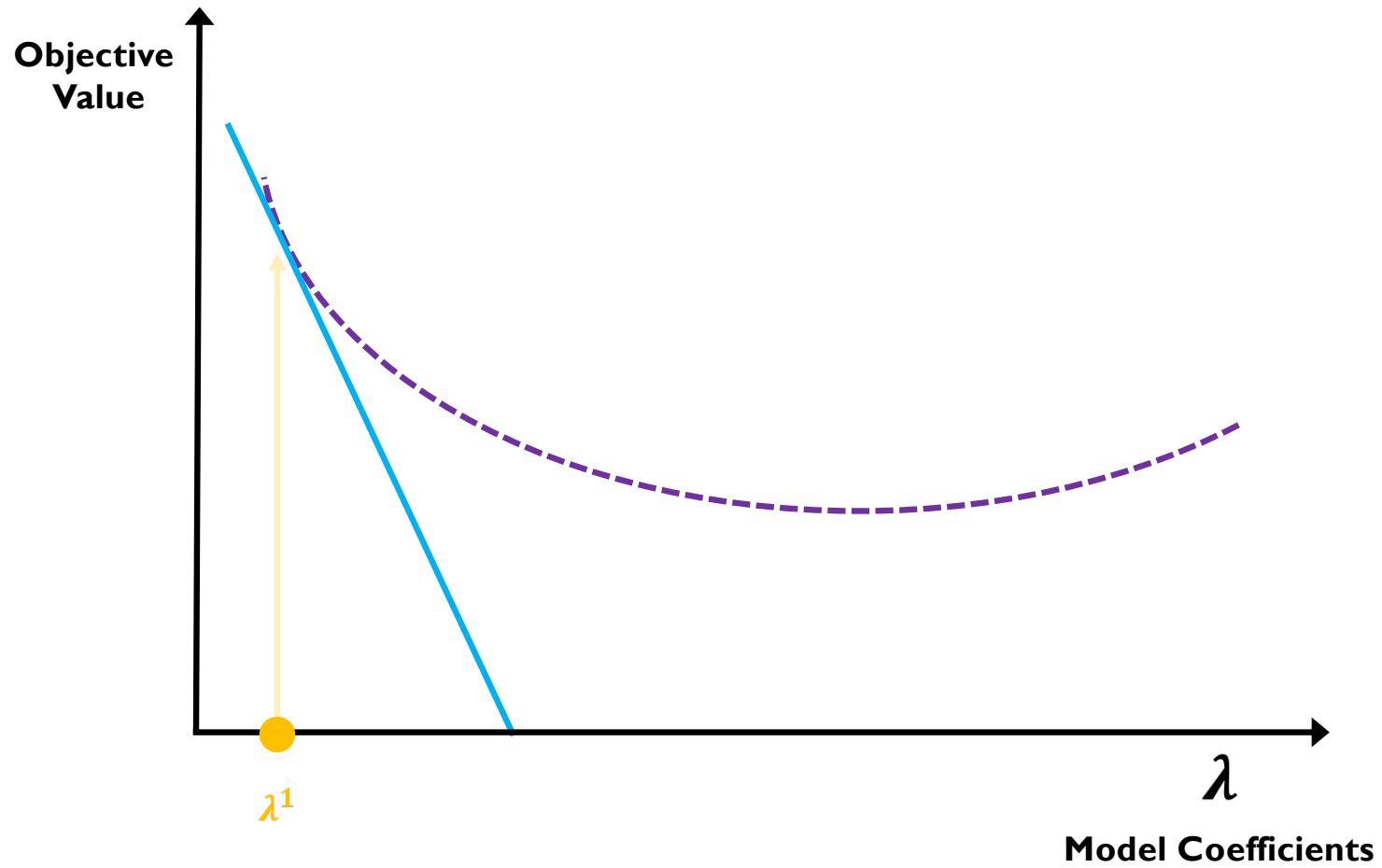
Cutting Planes (Traditional)

$$\min_{\lambda} \sum_{i=1}^n \log \left( 1 + e^{-y_i x_i^T \lambda} \right)$$

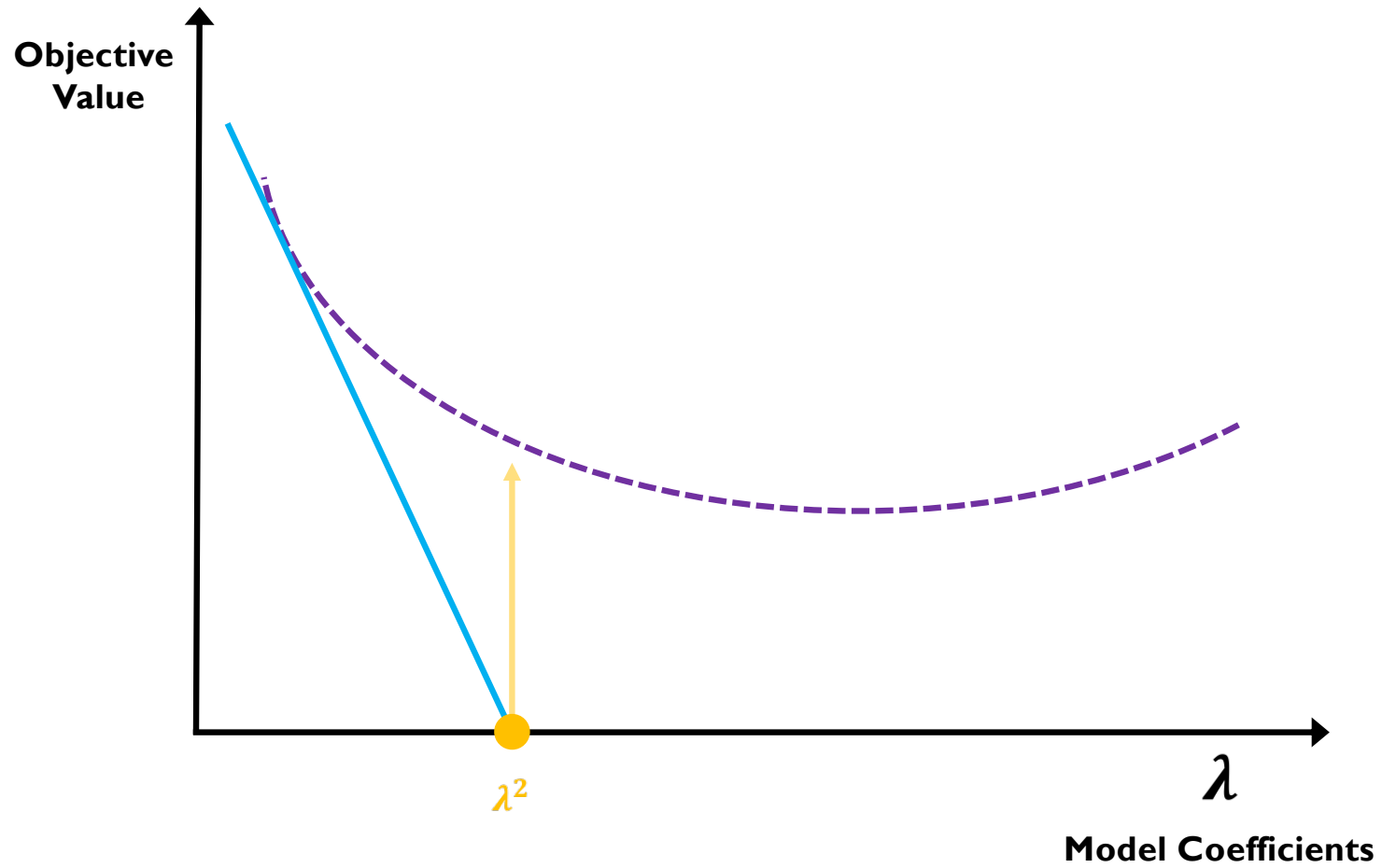
# Traditional cutting planes



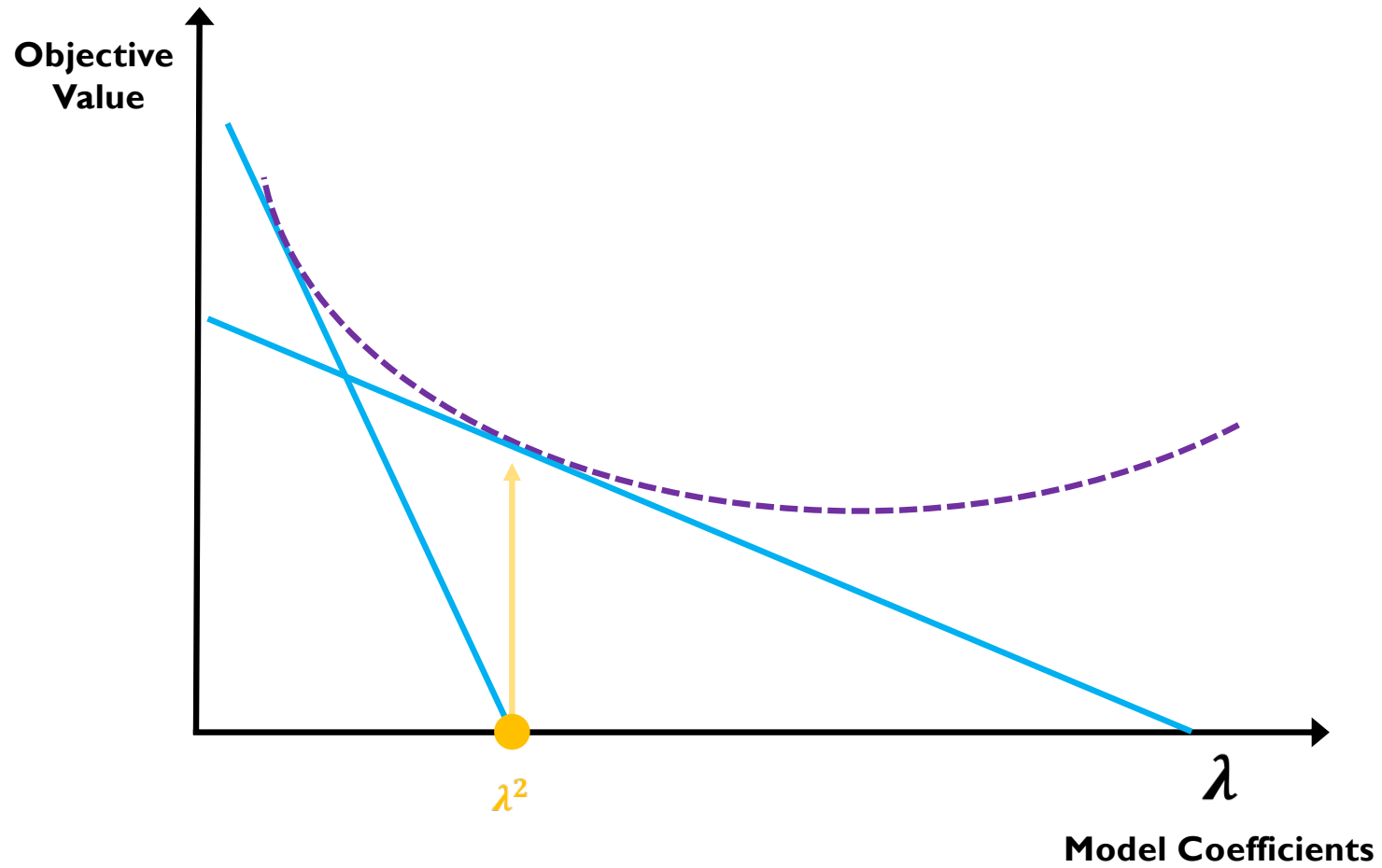
# Traditional cutting planes



# Traditional cutting planes

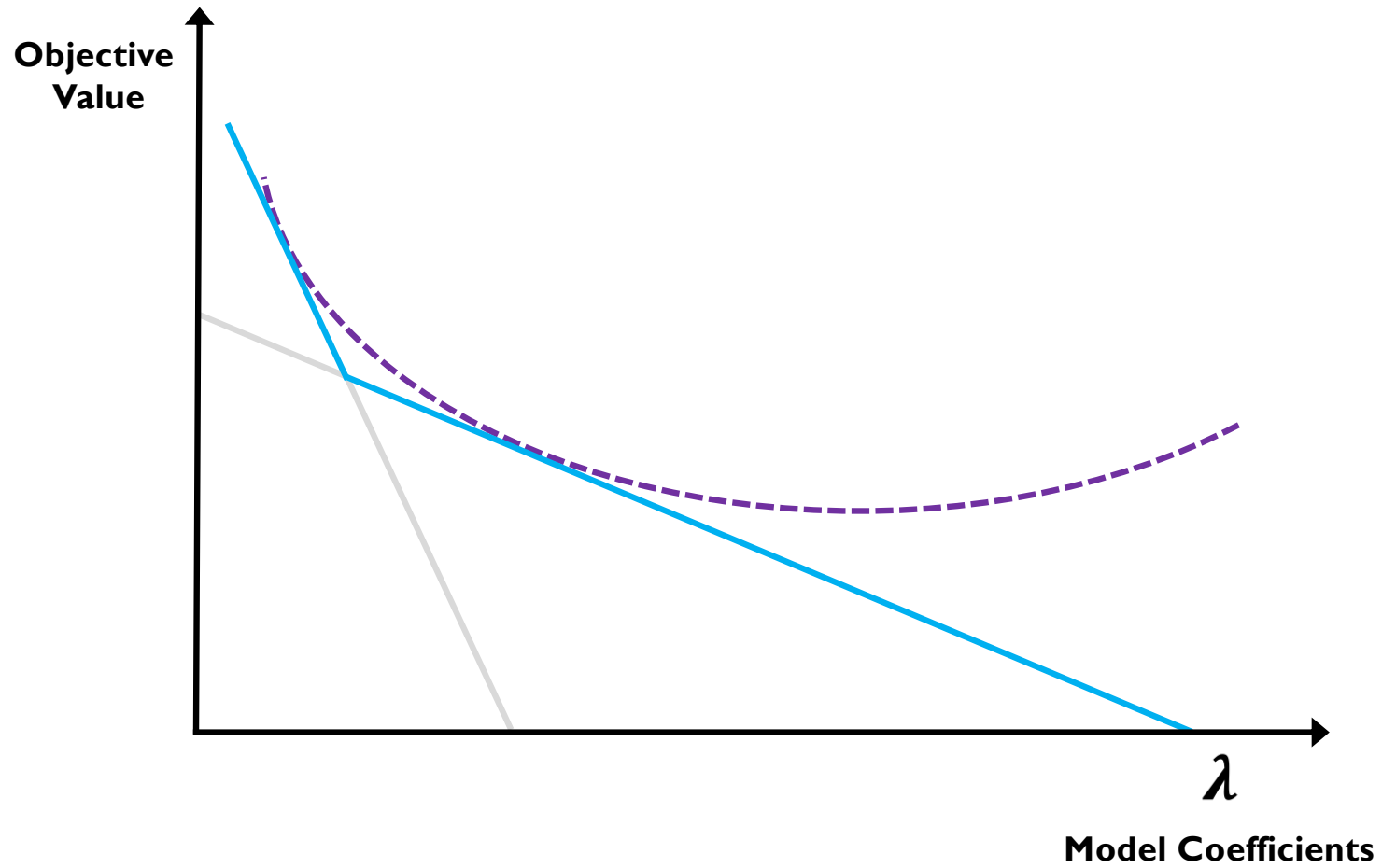


# Traditional cutting planes

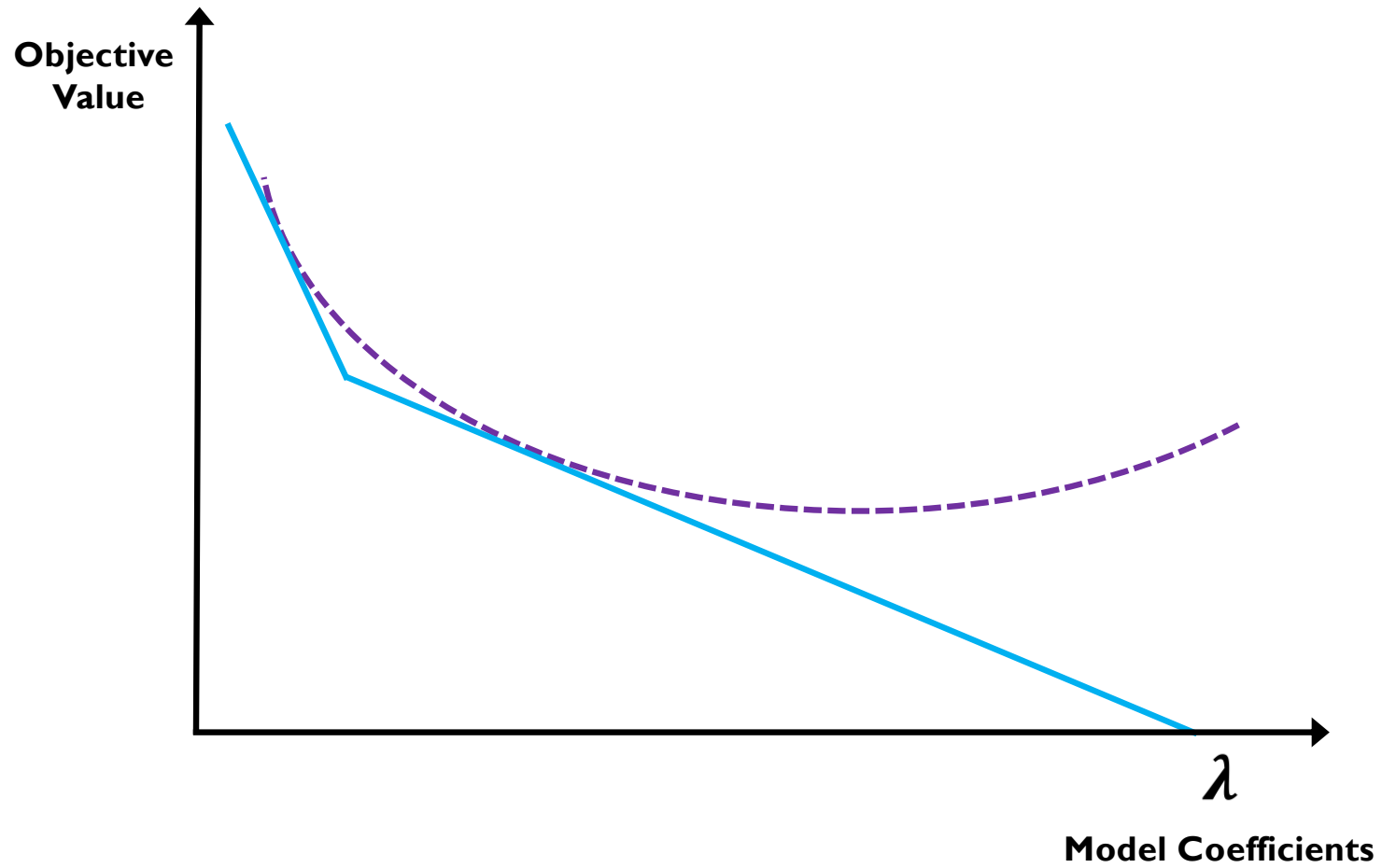




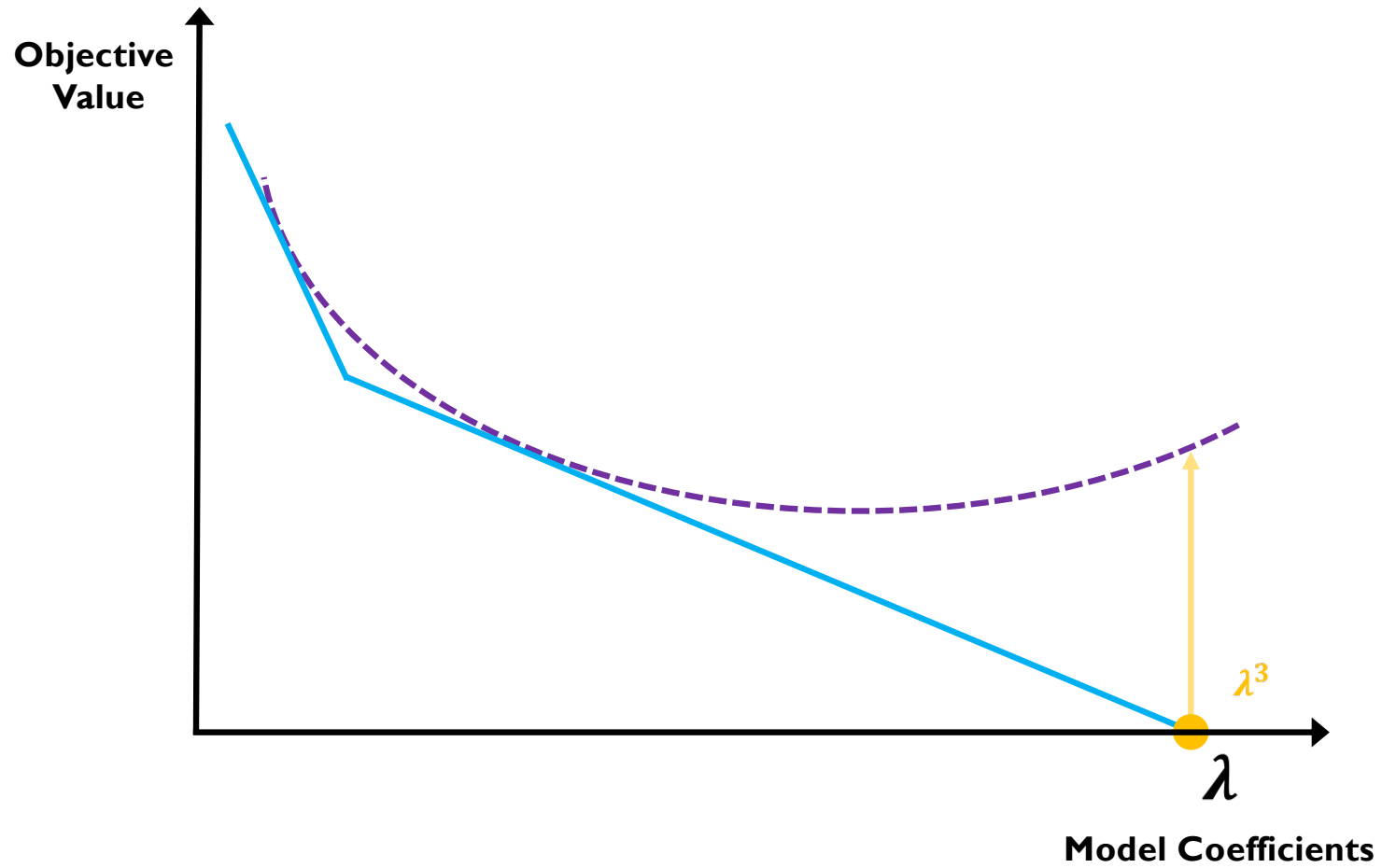
# Traditional cutting planes



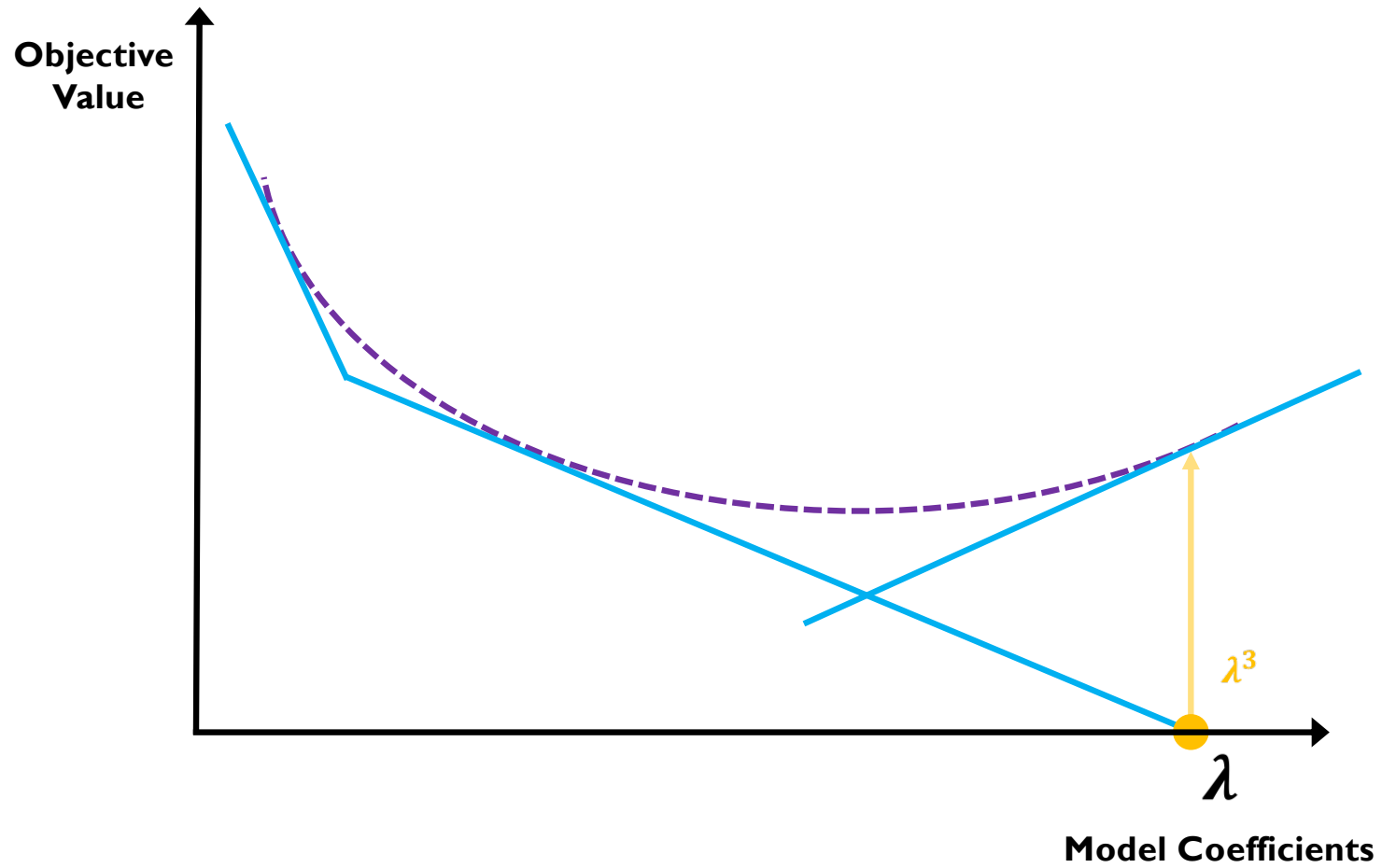
# Traditional cutting planes



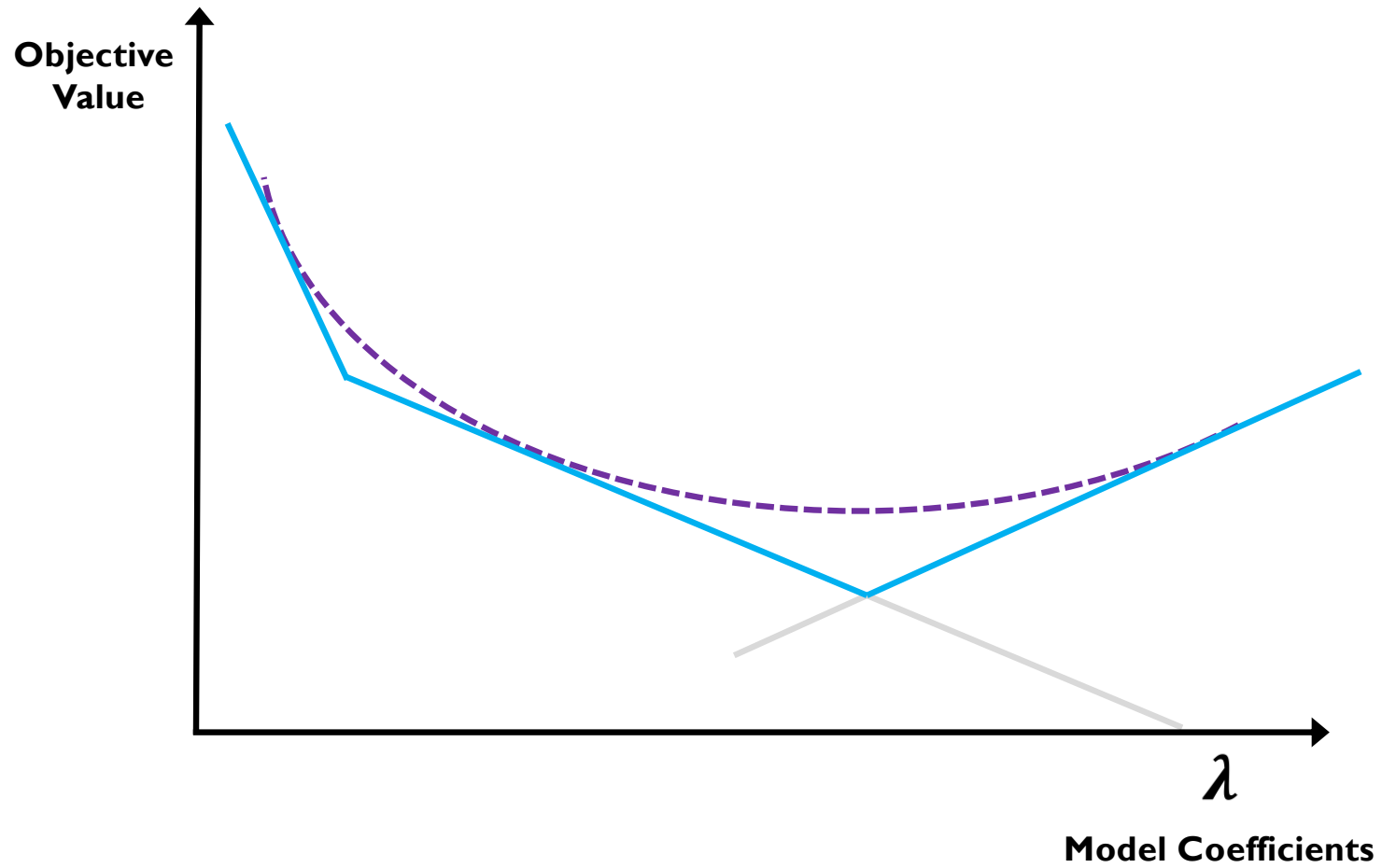
# Traditional cutting planes



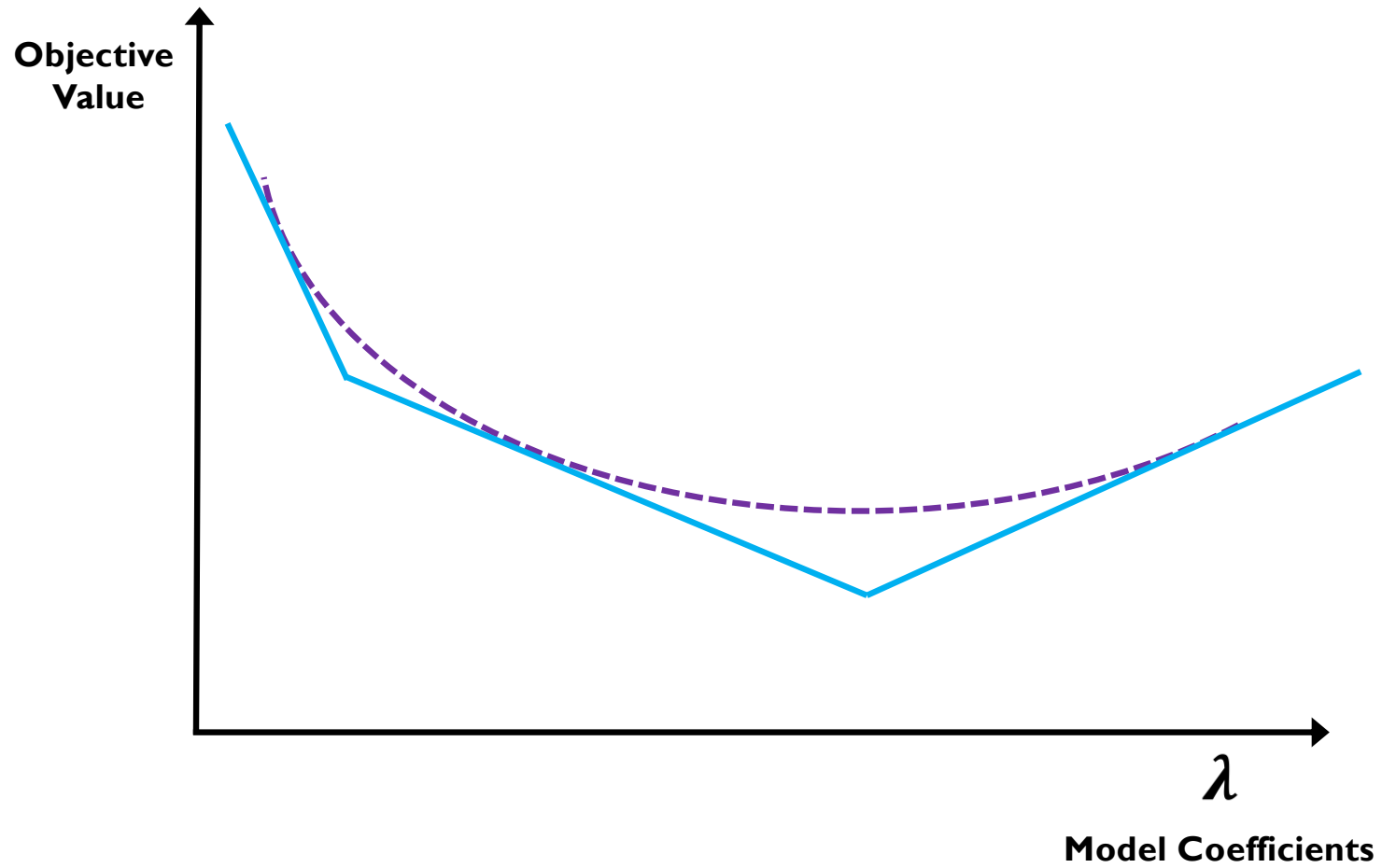
# Traditional cutting planes



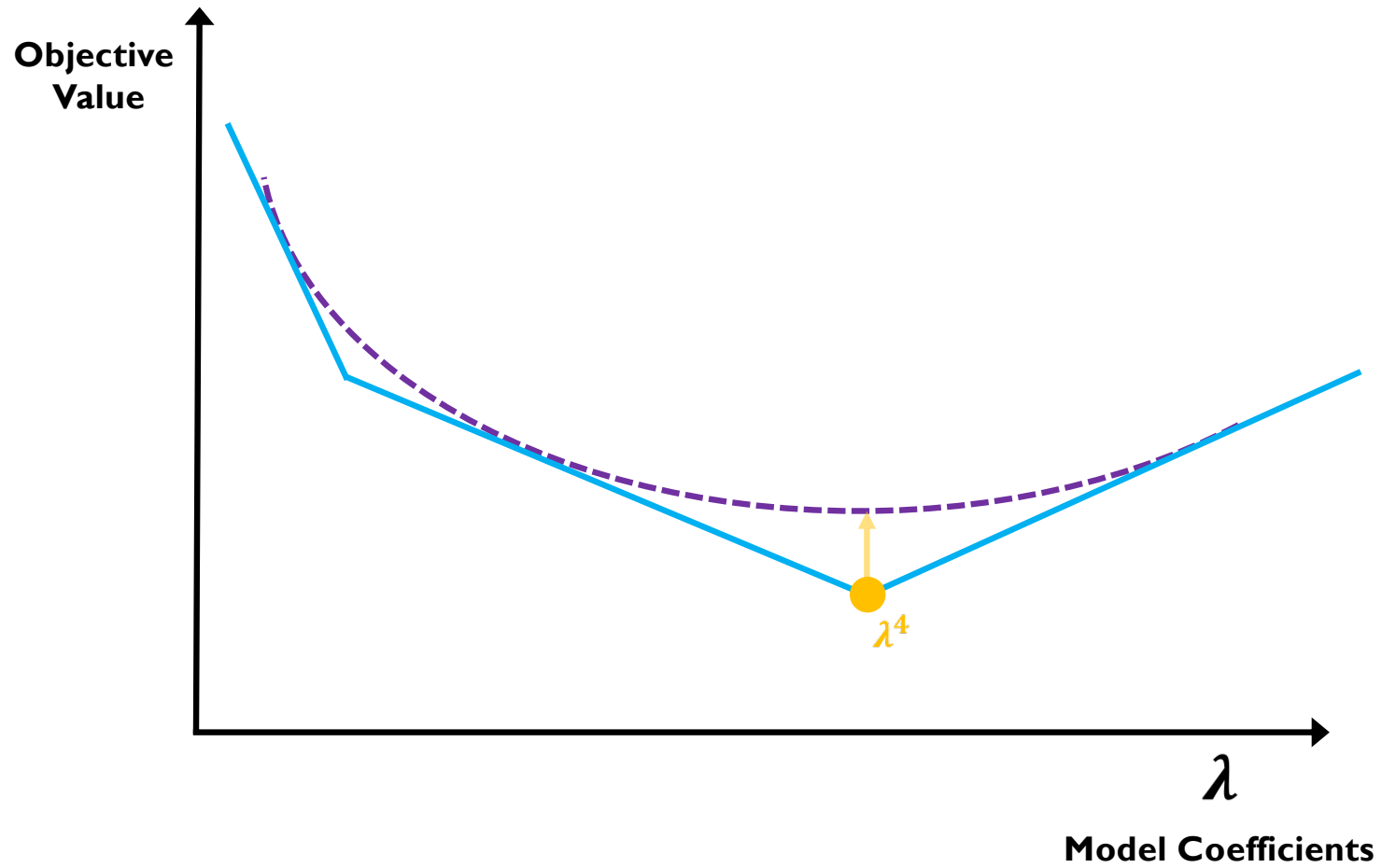
# Traditional cutting planes



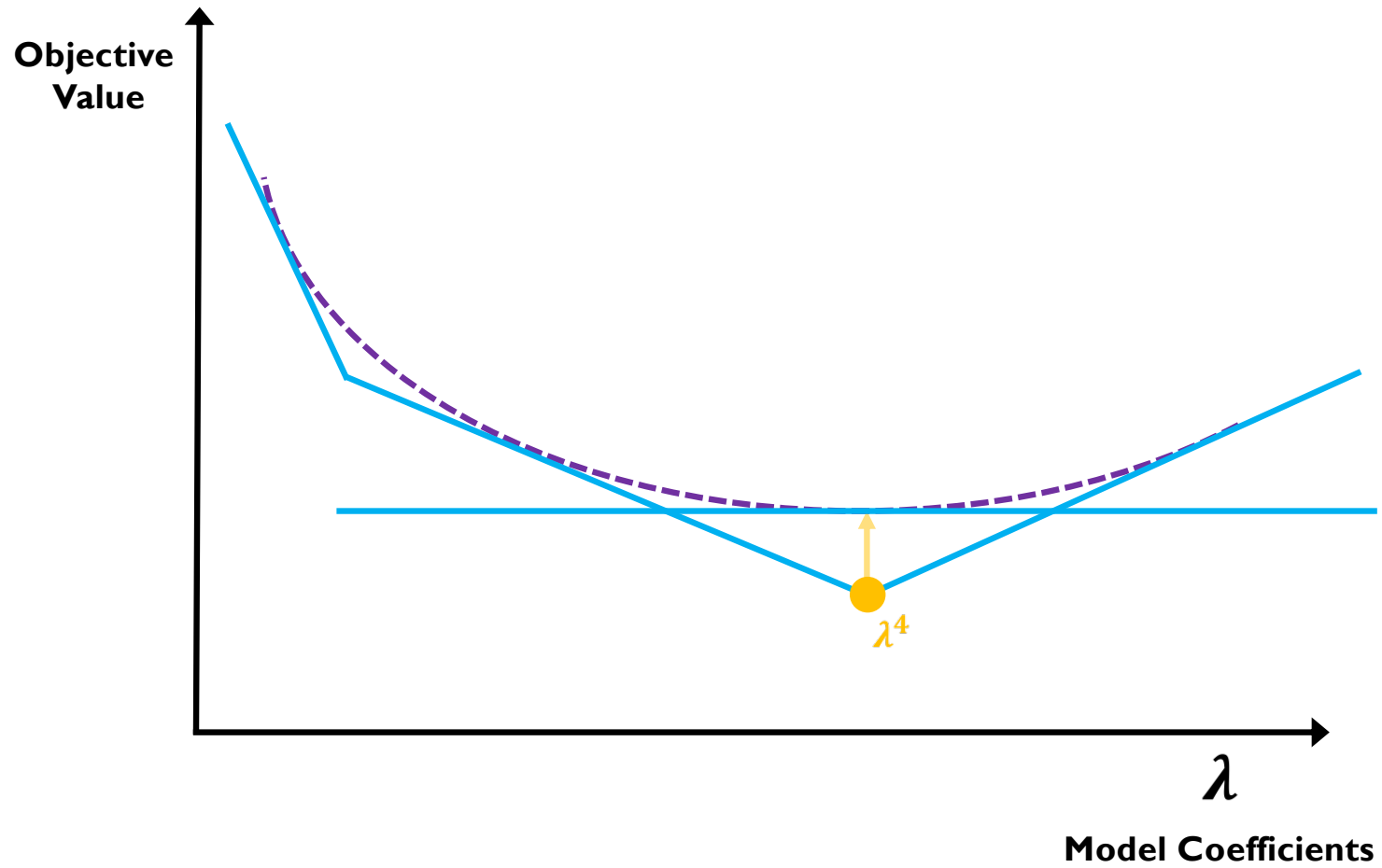
# Traditional cutting planes



# Traditional cutting planes

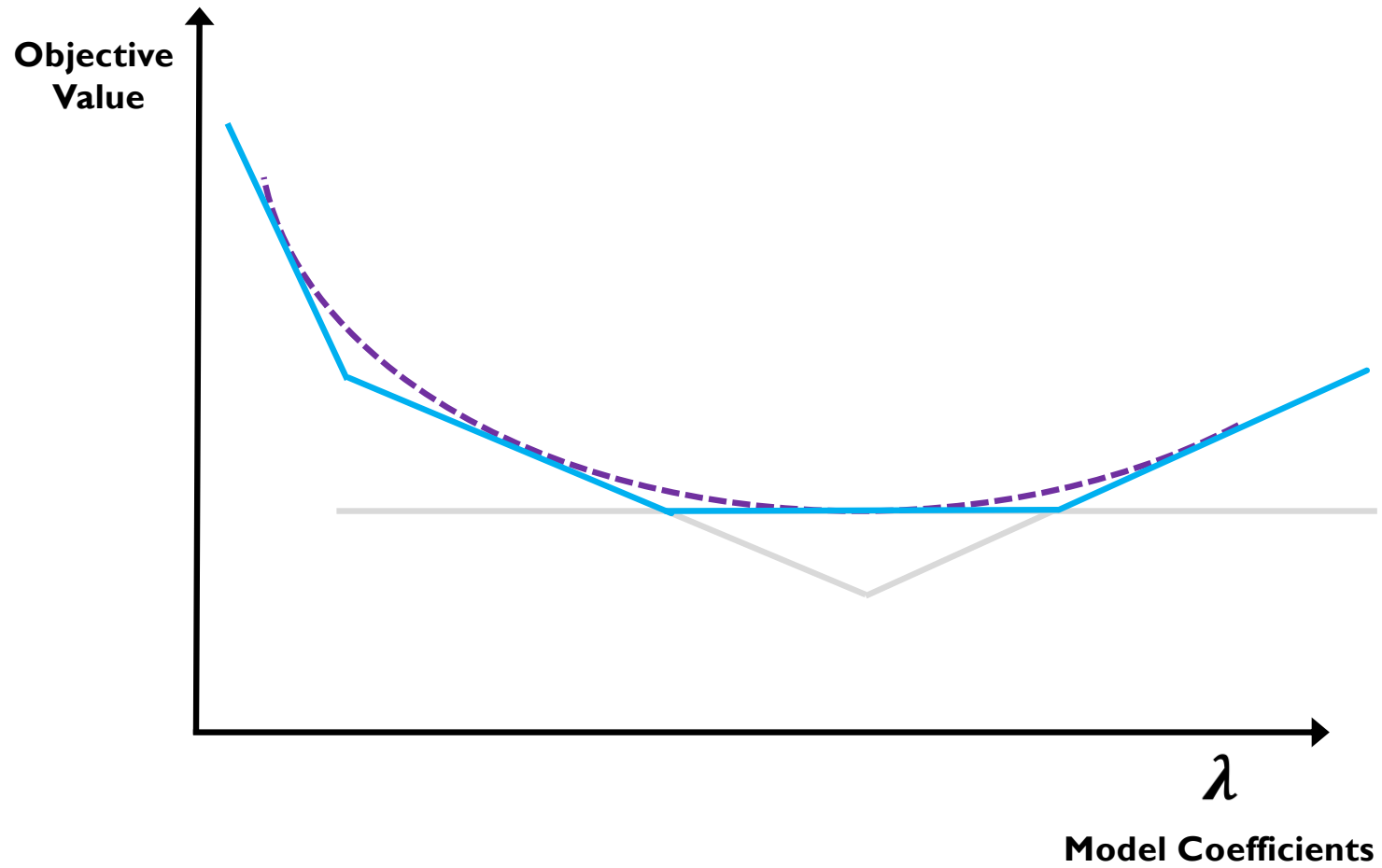


# Traditional cutting planes

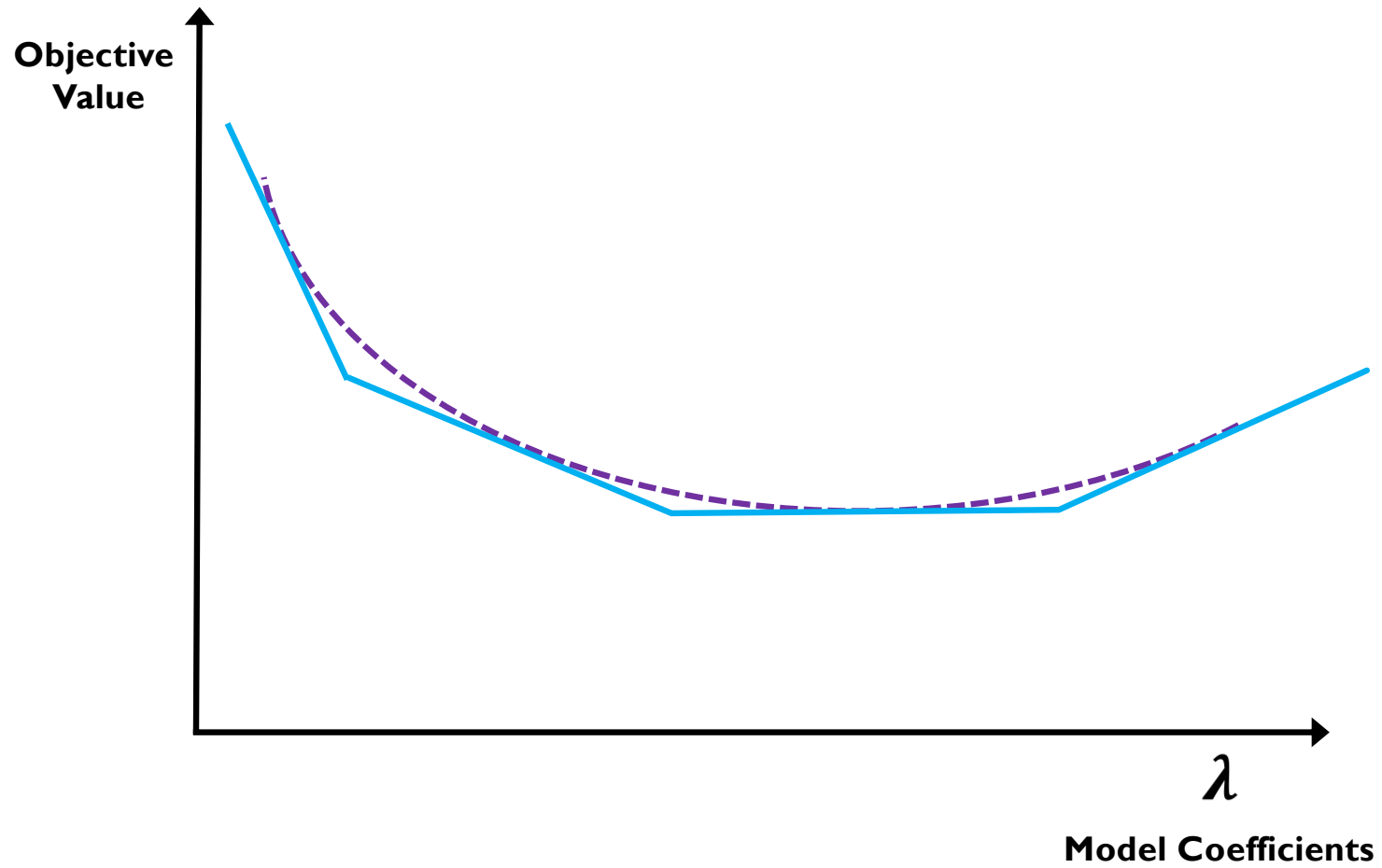




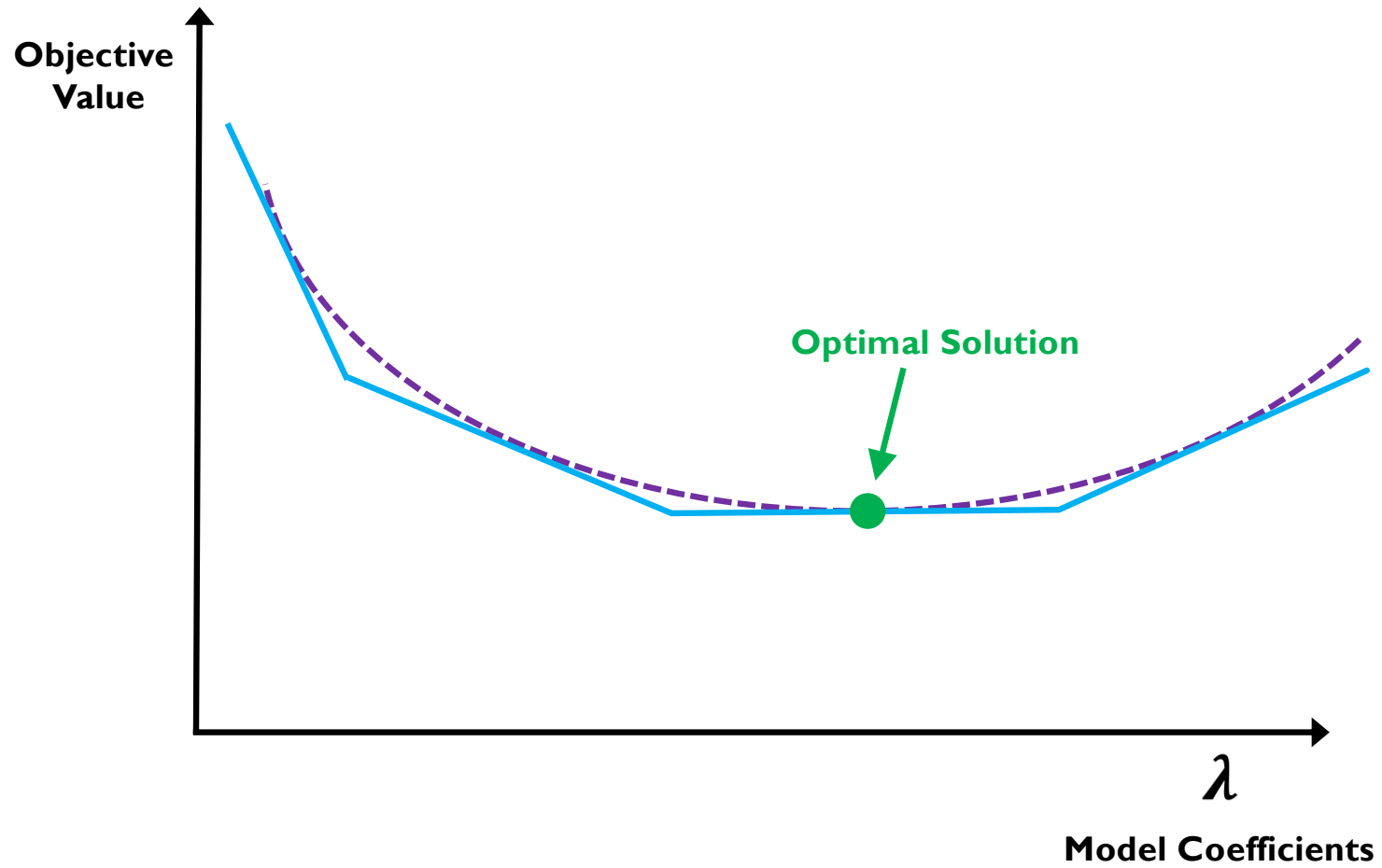
# Traditional cutting planes



# Traditional cutting planes

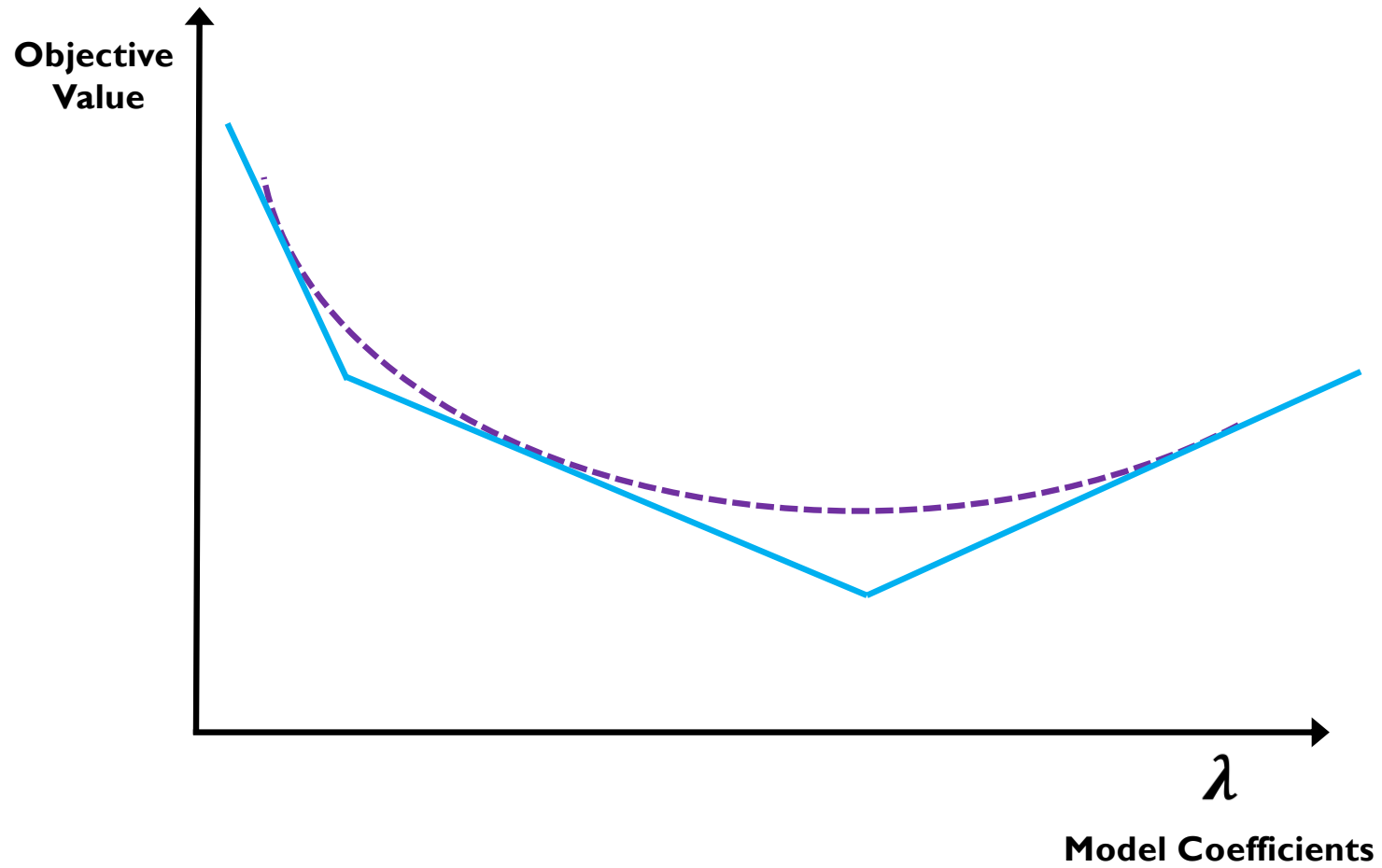


# Traditional cutting planes

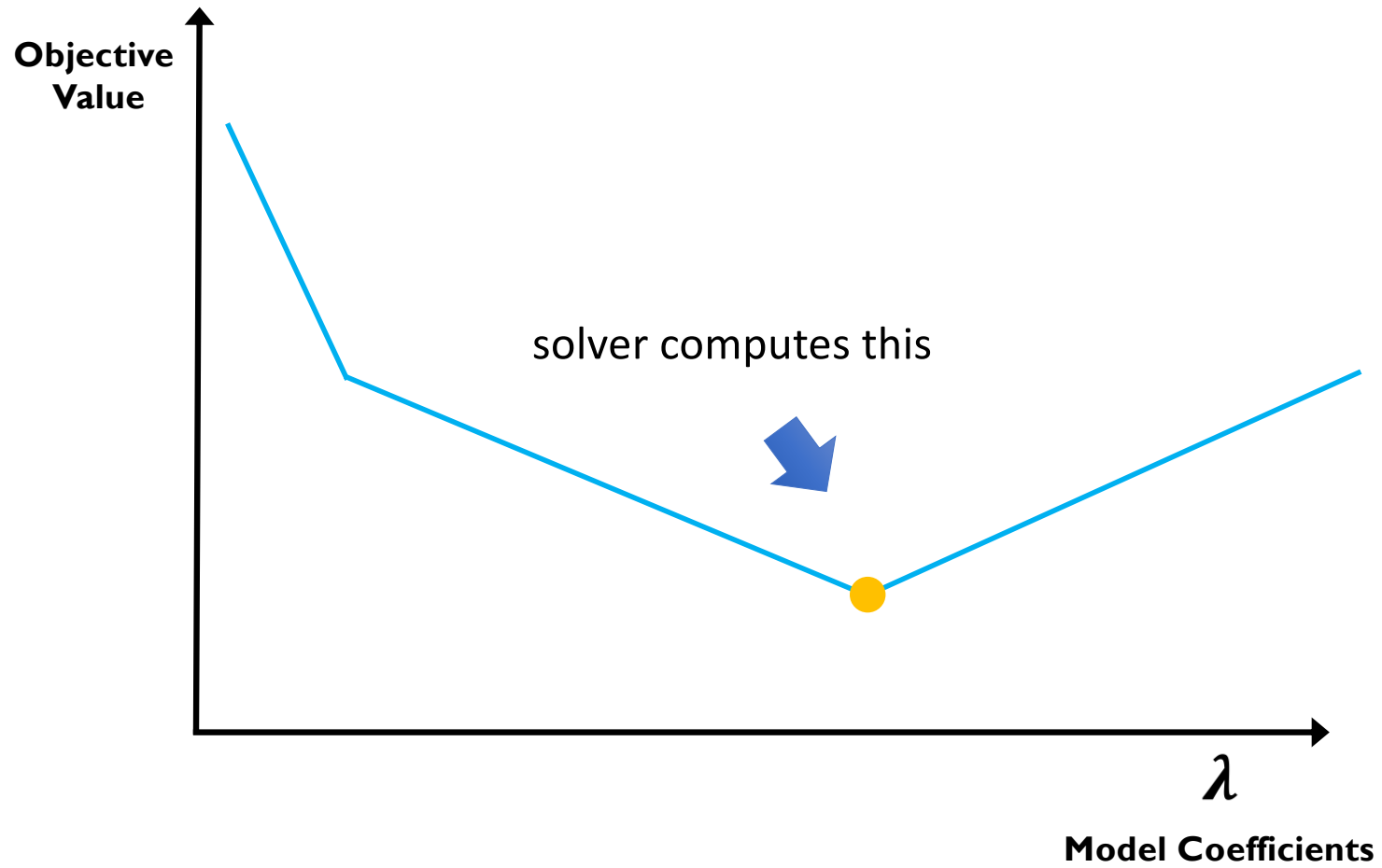


- Something goes wrong when creating models with integer coefficients.

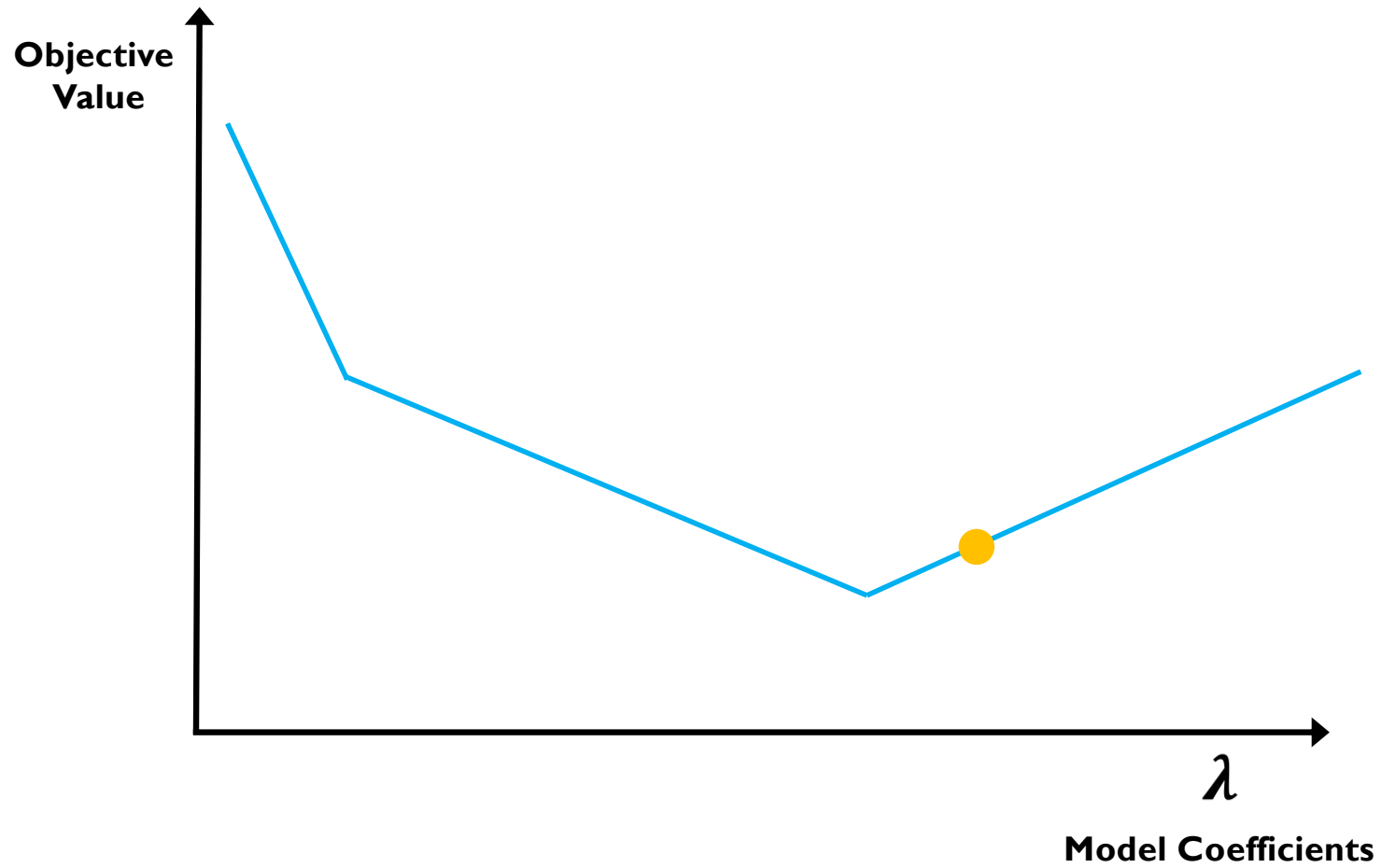
# Traditional cutting planes



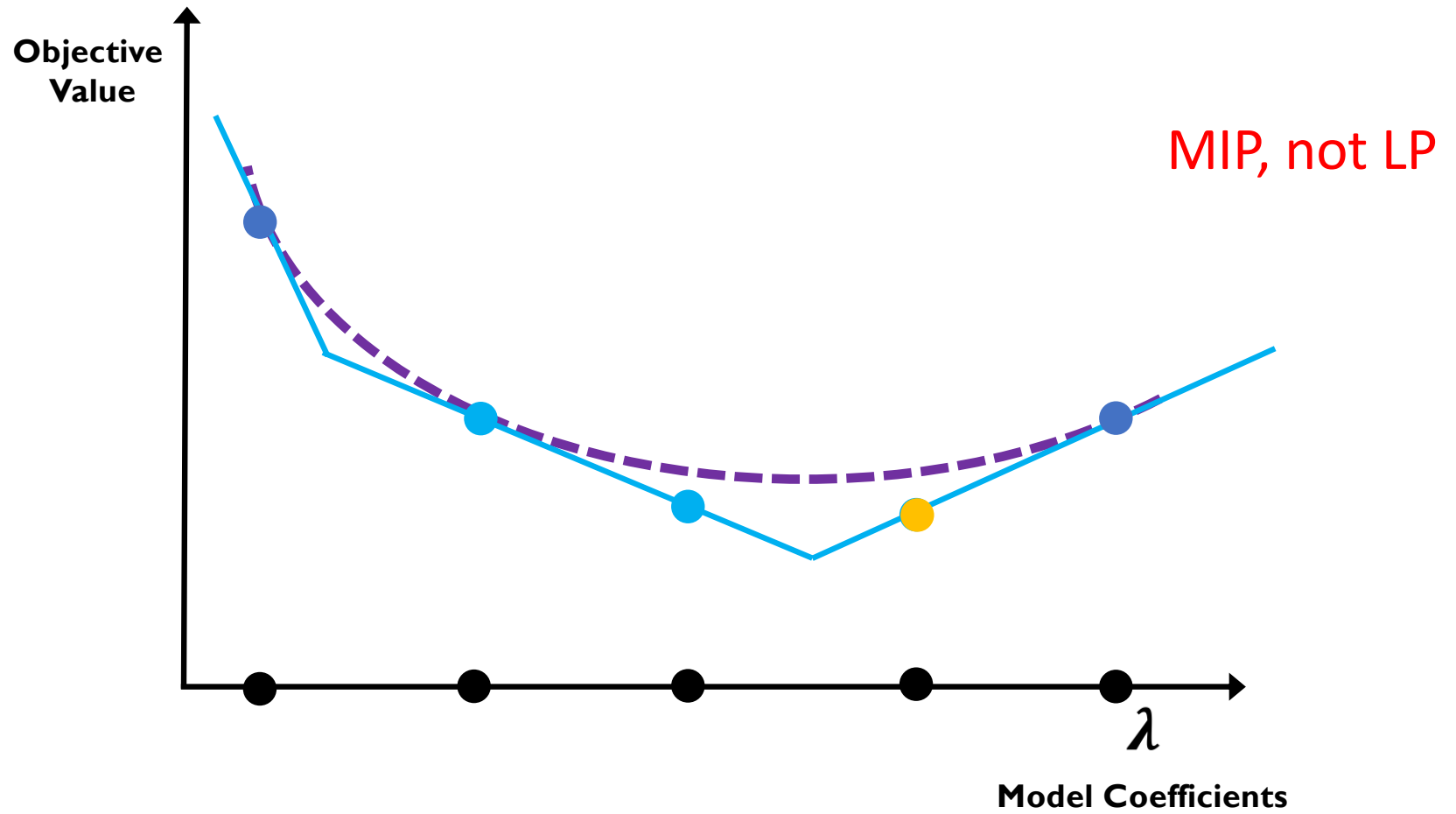
# Traditional cutting planes



# Traditional cutting planes



# Traditional cutting planes

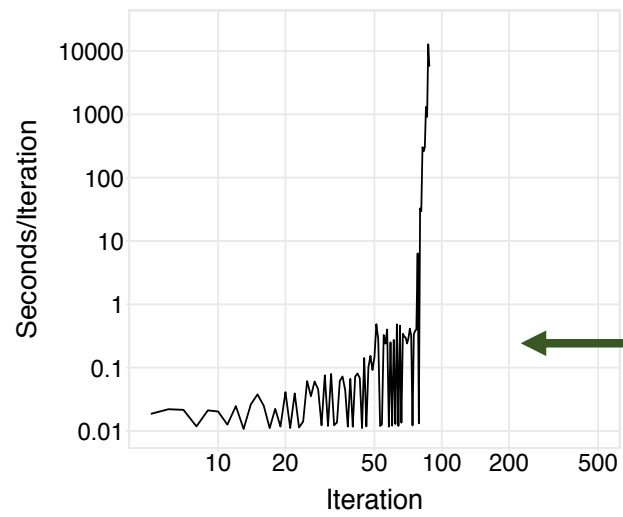




## Stalling

$$d = 20$$

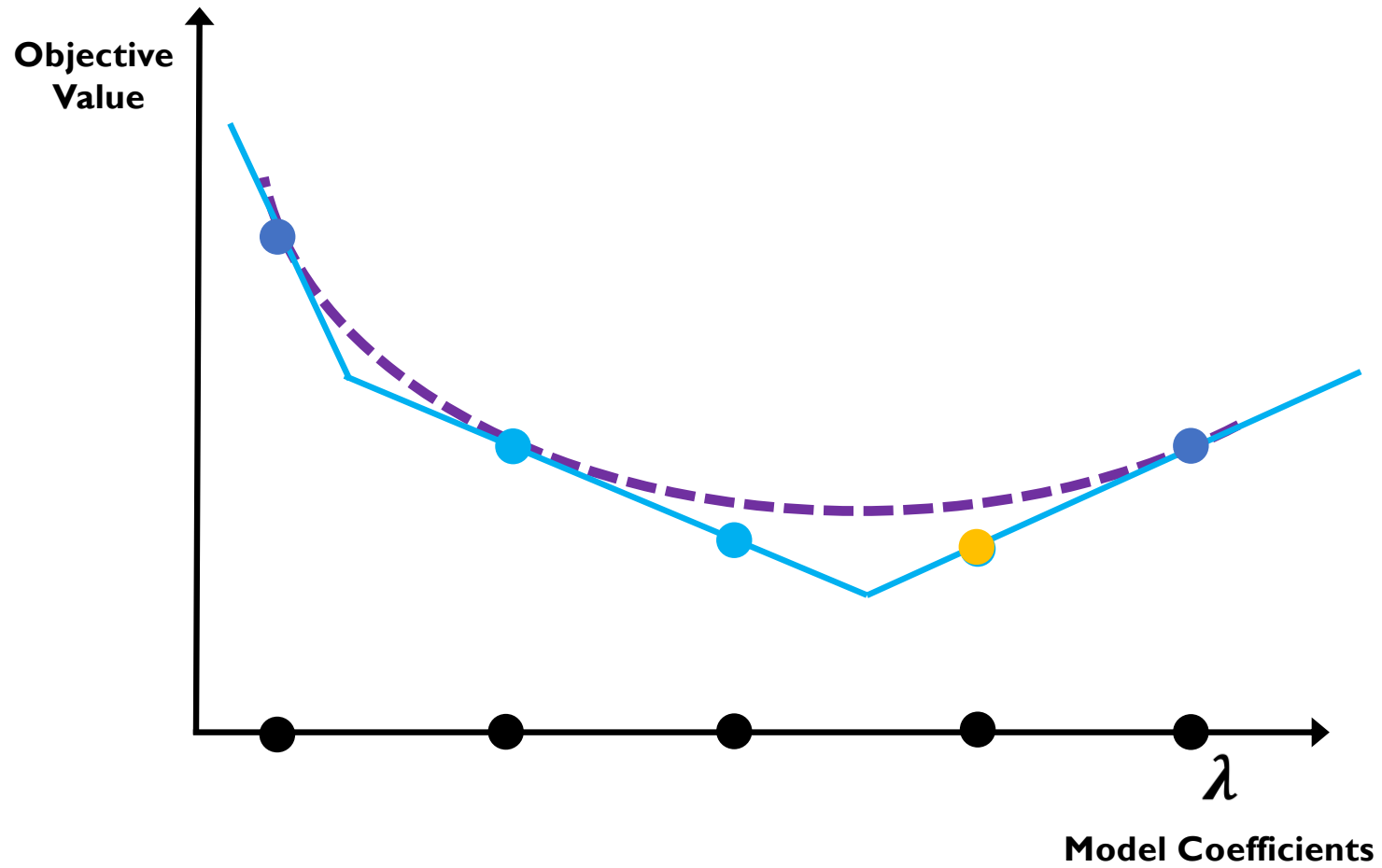
Seconds per iteration



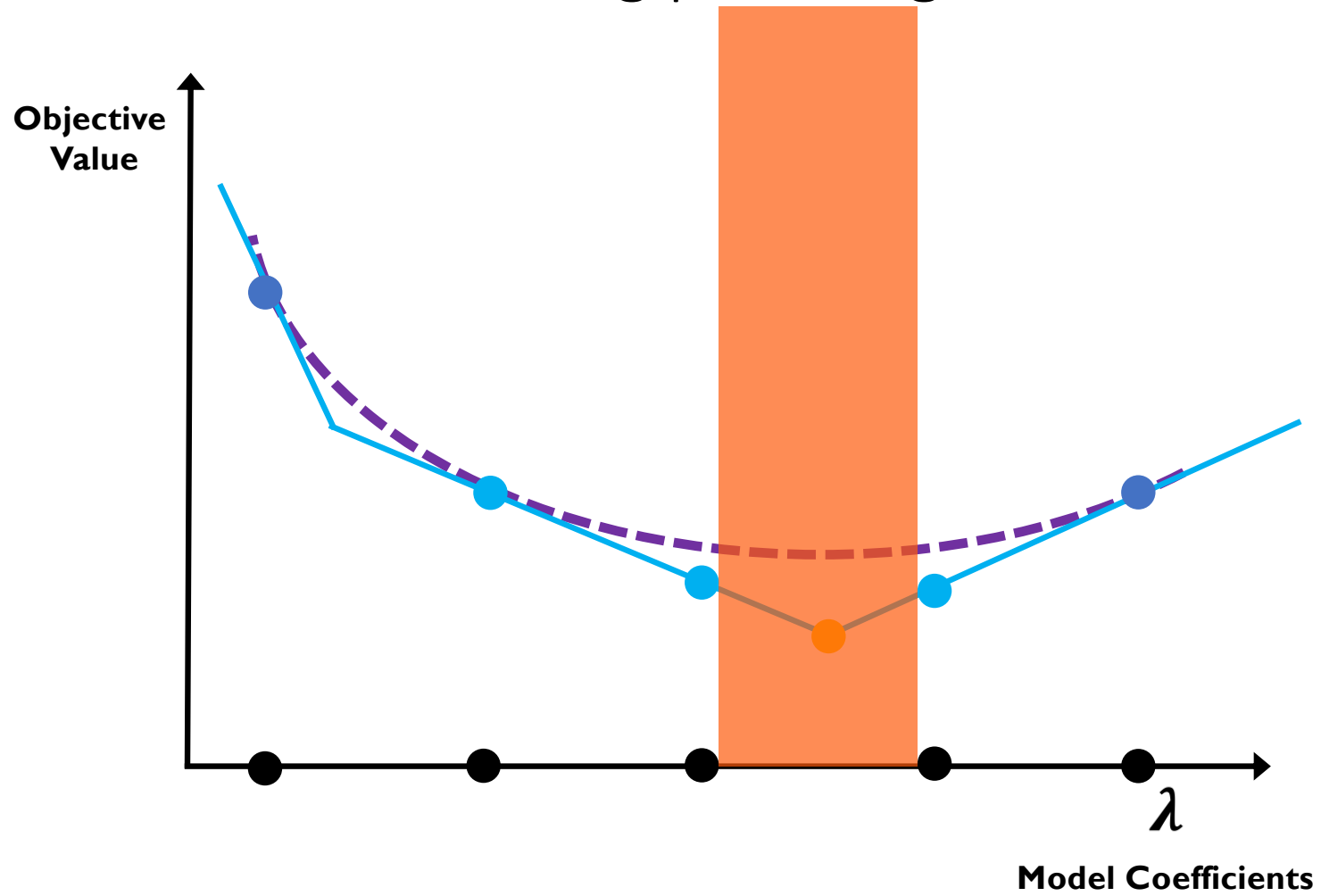
Stalling in traditional cutting planes

**RiskSLIM's *Lattice Cutting Plane Algorithm***  
**(Ustun & Rudin, KDD 17)**

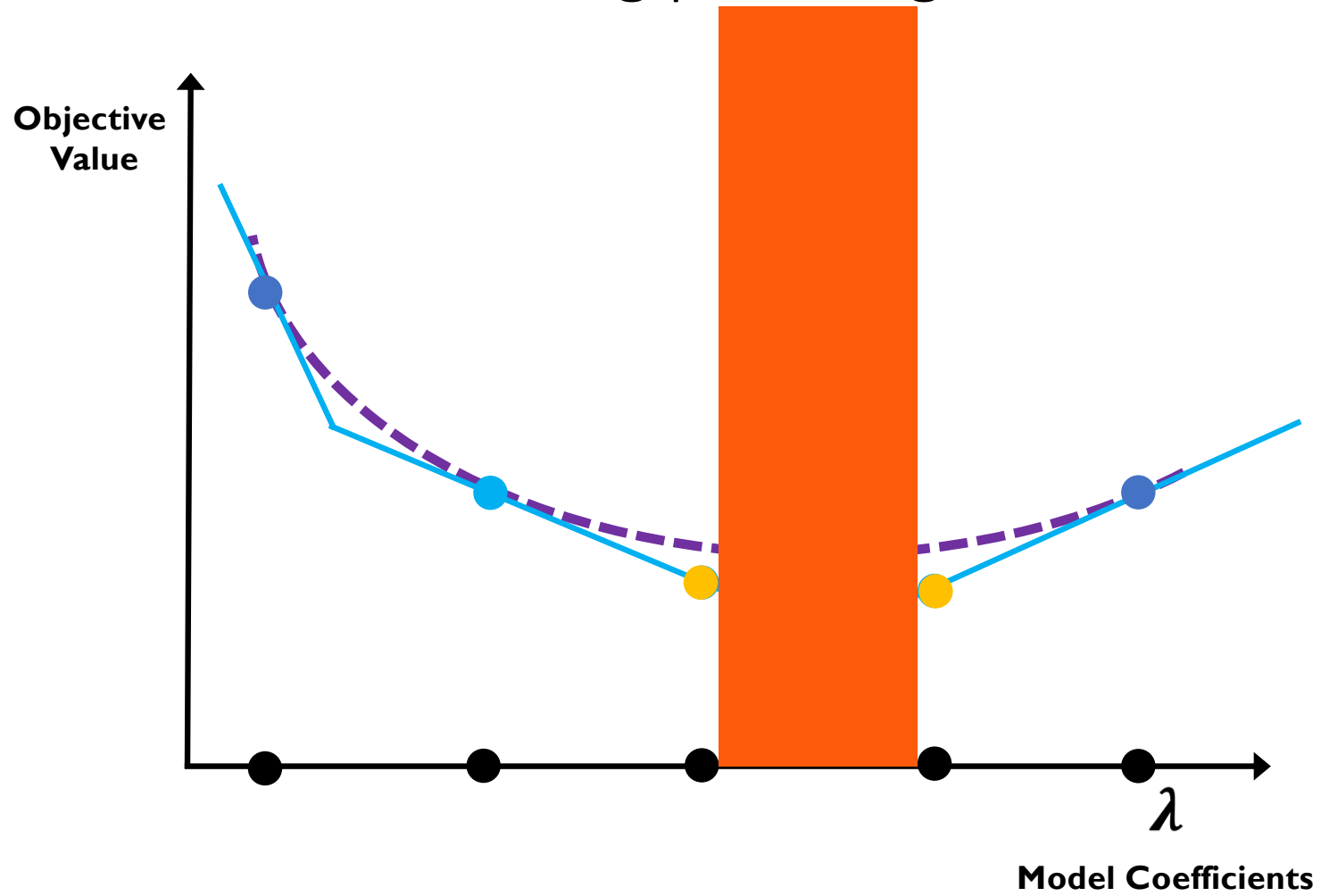
# Lattice cutting plane algorithm



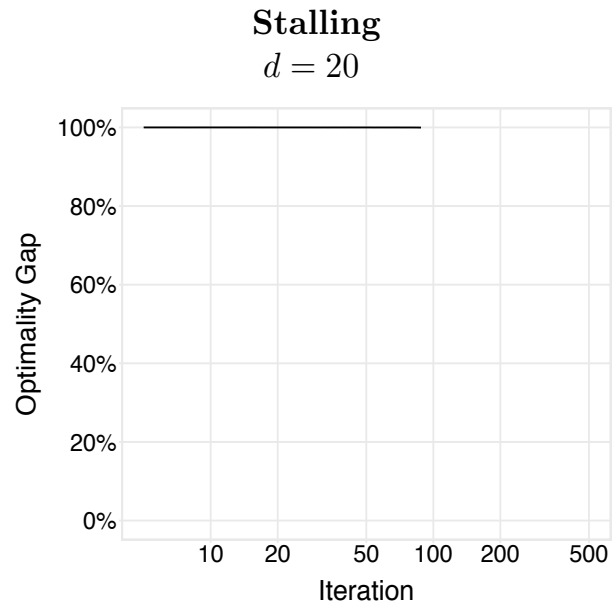
# Lattice cutting plane algorithm



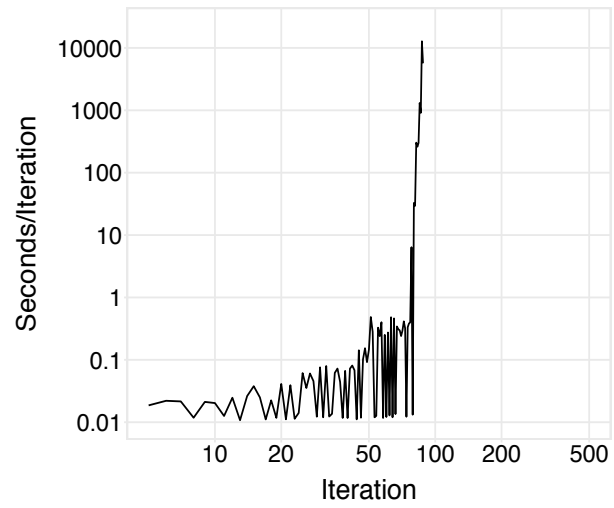
# Lattice cutting plane algorithm



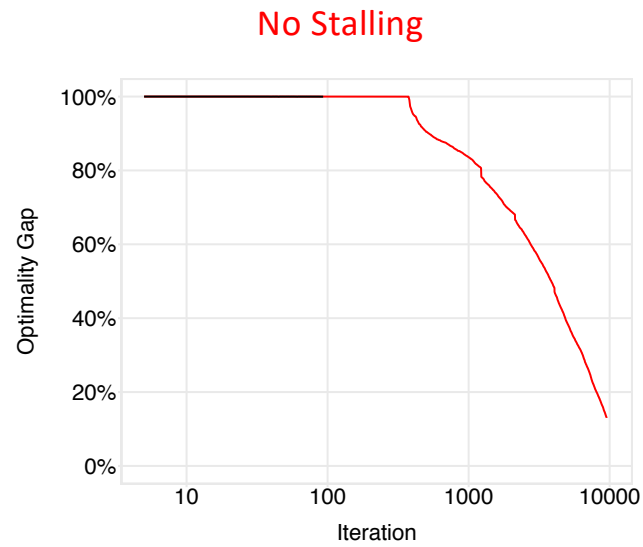
Optimality Gap



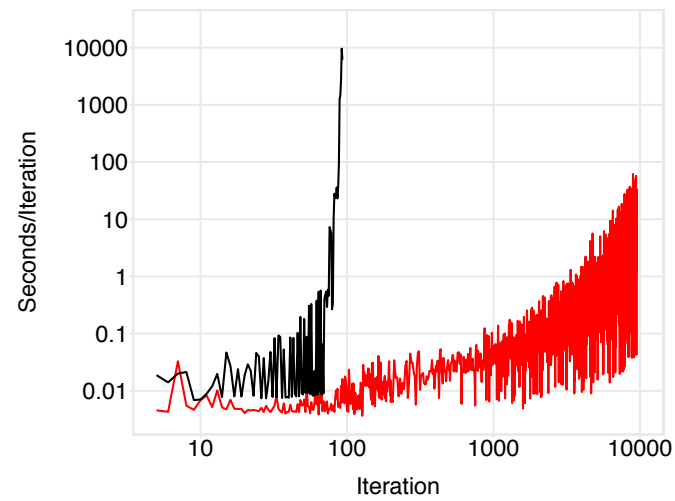
Seconds per iteration



Optimality Gap



Seconds per iteration

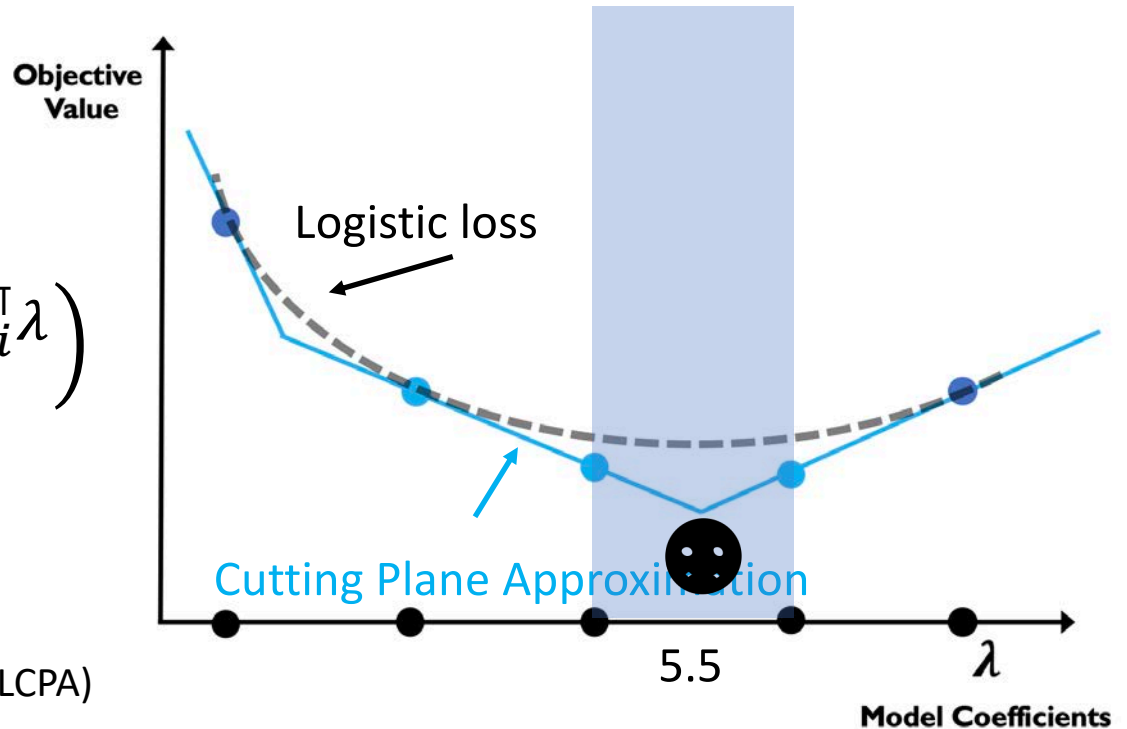


No Stalling for Lattice  
Cutting Plane Algorithm

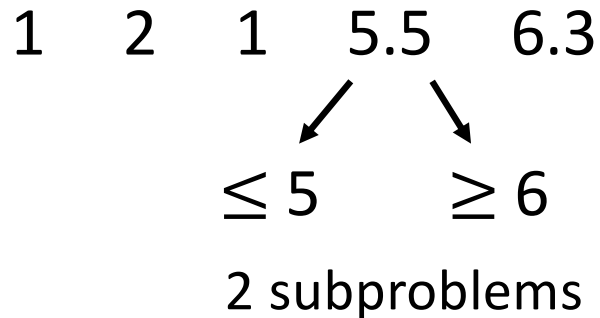
# Risk-SLIM

(Ustun, R, JMLR 2019)

$$\min_{\lambda \in L} \sum_{i=1}^n \log \left( 1 + e^{-y_i x_i^\top \lambda} \right)$$



RiskSLIM's Lattice Cutting Plane Algorithm (LCPA)



3.8    1    0    9    7

If a subproblem leads to a feasible integer solution, add a cutting plane.

Otherwise split into 2 subproblems (linear programs).  
If min cutting planes = objective, solved!



## Risk-SLIM

(Ustun, R, JMLR 2019)

- LCPA is the only method that generates solutions within a reasonable time.
  - MINLP solvers don't work
  - standard cutting planes require solving larger and larger MIPs.

## Polishing with SequentialRounding and Discrete Coordinate Descent (DCD)

(Ustun, R, 2019)

1	2	1	5.5	6.3	3.8	1	0	9.8	7	SequentialRounding
1	2	1	5.5	6.3	4	1	0	9.8	7	
1	2	1	5	6.3	4	1	0	9.8	7	
1	2	1	5	7	4	1	0	9.8	7	
1	2	1	5	7	4	1	0	10	7	DCD
1	2	1	5	7	4	2	0	10	7	
1	2	4	5	7	4	1	0	10	7	
1	1	4	5	7	4	1	0	10	7	“1-opt solution”

# Preventing Brain Damage in Critically Ill Patients



CT-angiography, Anterior Communicating Saccular Aneurysm



Head CT without contrast showing Subarachnoid Hemorrhage

- Seizure are common (20%)
- Seizure → Brain Damage
- Need EEG to detect seizures

Need to use EEG data to predict seizures, determine EEG duration

EEG is expensive and limited: 24hrs of monitoring is \$1600-\$4000

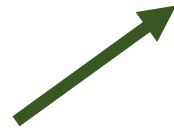
- 2HELPS2B was not created by doctors
- It is a ML model
- It is just as accurate as black box models.
- Doctors can decide themselves whether to trust it
- Doctors can calibrate the score with information not in the database
- Score can be explained to non-physicians

## 2HELPS2B

1.	Any cEEG Pattern with Frequency <b>2 Hz</b>	1 point	...
2.	<b>Epileptiform Discharges</b>	1 point	+ ...
3.	Patterns include [LPD, LRDA, BIPD]	1 point	+ ...
4.	<b>Patterns Superimposed with Fast or Sharp Activity</b>	1 point	+ ...
5.	Prior <b>Seizure</b>	1 point	+ ...
6.	<b>Brief Rhythmic Discharges</b>	2 points	+ ...
<b>SCORE</b>			= ...

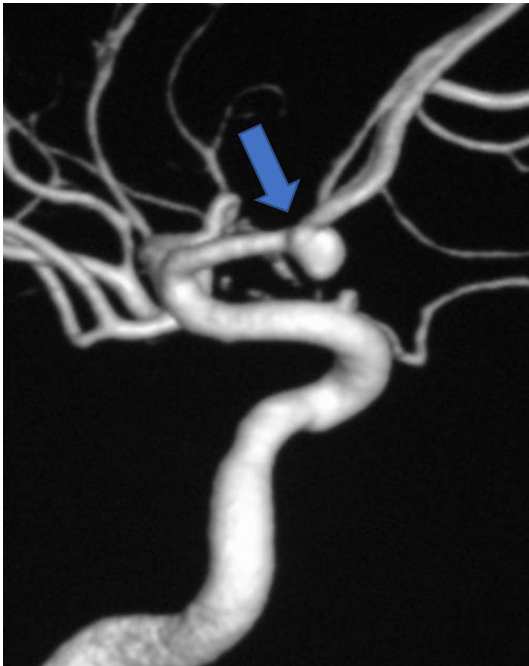
<b>SCORE</b>	0	1	2	3	4	5	6+
<b>RISK</b>	<5%	11.9%	26.9%	50.0%	73.1%	88.1%	95.3%

There are many variables to choose from.



Variable
PDR
BRDs
Unreactive background
Prior Sz
GRDA
LRDA
GPDs
LPDs
BIPDs
Infection
Inflammation
Neoplasm
ICH
Metabolic encephalopathy
Stroke
SAH
SDH
TBI
Hypoxic/ischemic
IVH
Hydrocephalus
Discharges
Frequency (>2Hz) <sup>c</sup>

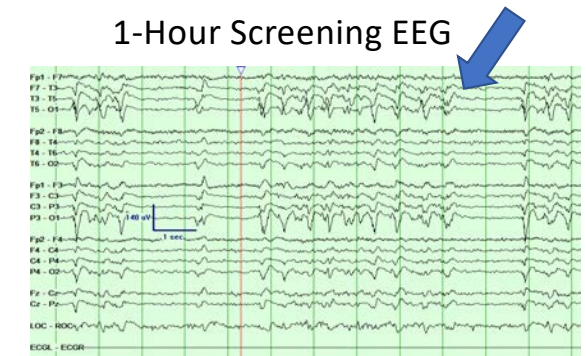
# Preventing Brain Damage in Critically Ill Patients



CT-angiography, Anterior Communicating Saccular Aneurysm



Head CT without contrast showing Subarachnoid Hemorrhage



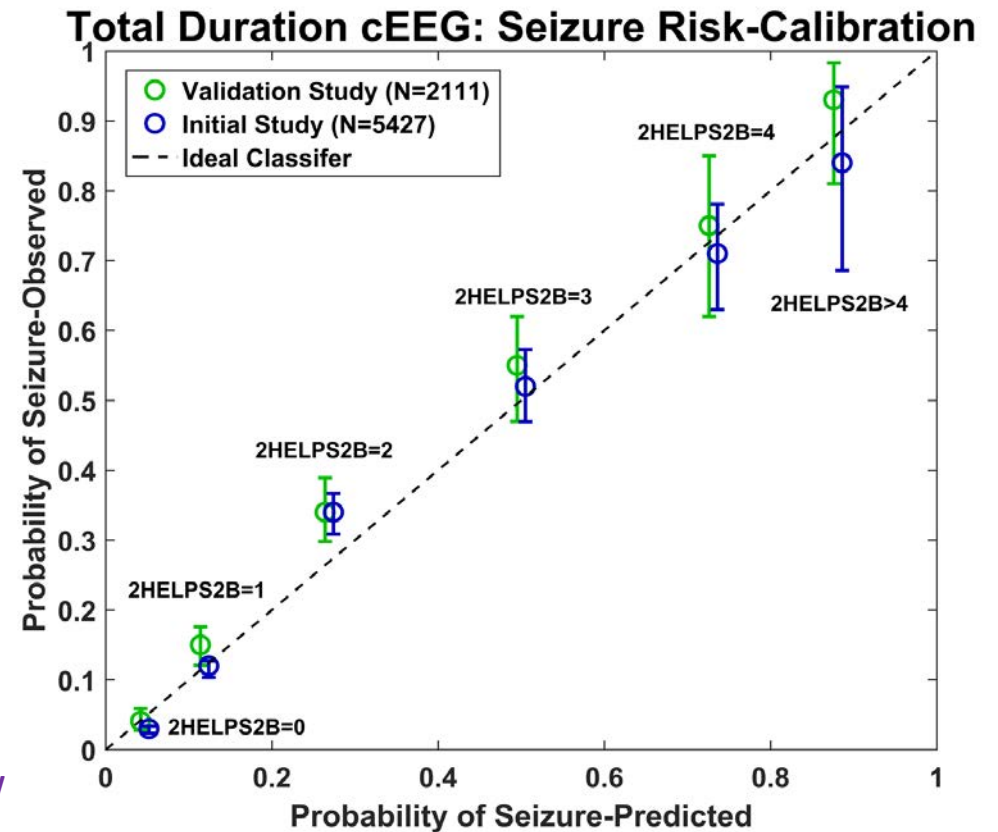
2HELPS2B=3 (high-risk)



- Placed on Continuous EEG for >72H
- Start on preventative medications

# So far...

- 2HELPS2B validated on independent multicenter cohort (N=2111)
- Implemented: University of Wisconsin, Massachusetts General Hospital/Harvard Medical School
- Ongoing implementation: Emory University, Duke University, Medical University of South Carolina, Free University of Brussels (Belgium)
- Resulted in **63.6%** reduction in duration of EEG monitoring per patient
  - \$1,134.831 saving per patient<sup>1</sup>
- **2.82 X** More Patients Monitored
- **\$6.1M** estimated savings in FY 2018 at MGH,UW



<sup>1</sup>2016 Medicare Reimbursement Most Common Professional Code

## Problem spectrum

Very sparse models (trees, scoring systems)

Neural networks

With minor pre-processing, all methods have similar performance

**Tabular**: All features are interpretable

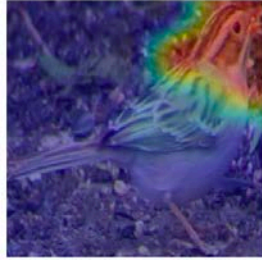
- many problems in criminal justice, healthcare, social sciences, equipment reliability & maintenance, etc.
- features include counts, categorical data

**Raw**: Features are individually uninterpretable

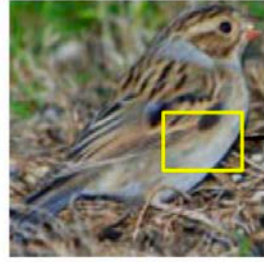
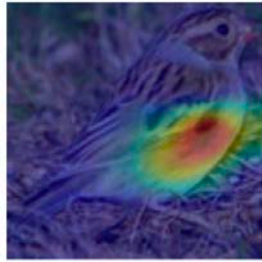
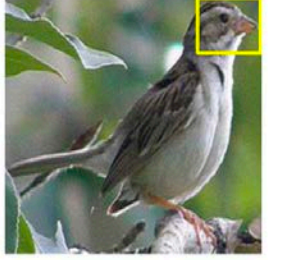
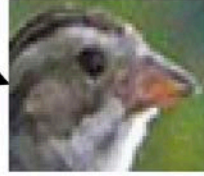
- pixels/voxels, words, a bit of a sound wave



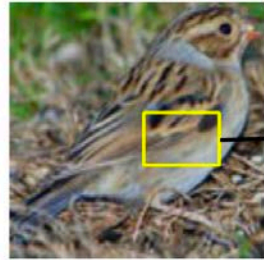
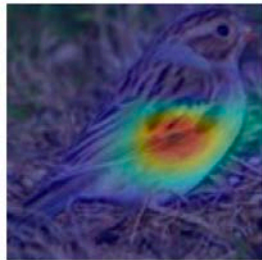
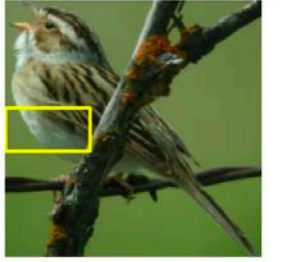
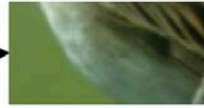
Interpretable neural networks?



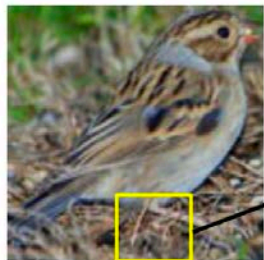
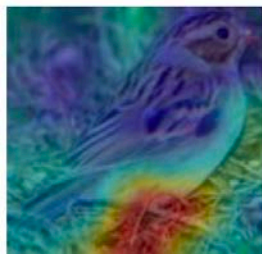
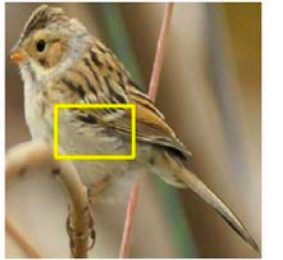
looks like



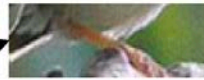
looks like



looks like



looks like



NeurIPS 2019 (spotlight)

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## This looks like that: deep learning for interpretable image recognition

Chaofan Chen, Oscar Li, Alina Barnett, Jonathan Su, Cynthia Rudin

(Submitted on 27 Jun 2018)

When we are faced with challenging image classification tasks, we often explain our reasoning by dissecting the image, and pointing out prototypical aspects of one class or another. The mounting evidence for each of the classes helps us make our final decision. In this work, we introduce a deep network architecture that reasons in a similar way: the network dissects the image by finding prototypical parts, and combines evidence from the prototypes to make a final classification. The algorithm thus reasons in a way that is qualitatively similar to the way ornithologists, physicians, geologists, architects, and others would explain to people on how to solve challenging image classification tasks. The network uses only image-level labels for training, meaning that there are no labels for parts of images. We demonstrate the method on the CIFAR-10 dataset and 10 classes from the CUB-200-2011 dataset.



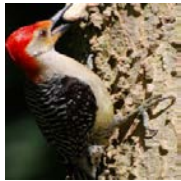
Oscar



Chaofan

Accuracy  $\approx$  black box baselines

Why is this bird classified as a red-bellied woodpecker?



Evidence for this bird being a red-bellied woodpecker:

Original image (box showing part that looks like prototype)	Prototype	Training image where prototype comes from	Activation map	Similarity score	Class connection	Points contributed
				6.499	1.180	$6.499 \times 1.180 = 7.669$
				4.392	1.127	$4.392 \times 1.127 = 4.950$
				3.890	1.108	$3.890 \times 1.108 = 4.310$
⋮	⋮	⋮	⋮	⋮	⋮	⋮

Total points to red-bellied woodpecker: 32.736

Why is this bird classified as a Wilson's warbler?



Evidence for this bird being a Wilson's warbler:

Original image (box showing part that looks like prototype)	Prototype	Training image where prototype comes from	Activation map	Similarity score	Class connection	Points contributed
				3.341	1.443	$3.341 \times 1.443 = 4.821$
				3.302	1.450	$3.302 \times 1.450 = 4.788$
				2.159	1.442	$2.159 \times 1.442 = 3.113$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
Total points to Wilson's warbler:						19.473

Base model: VGG-16

Why is this bird incorrectly classified as a prothonotary warbler, instead of a Wilson's warbler?

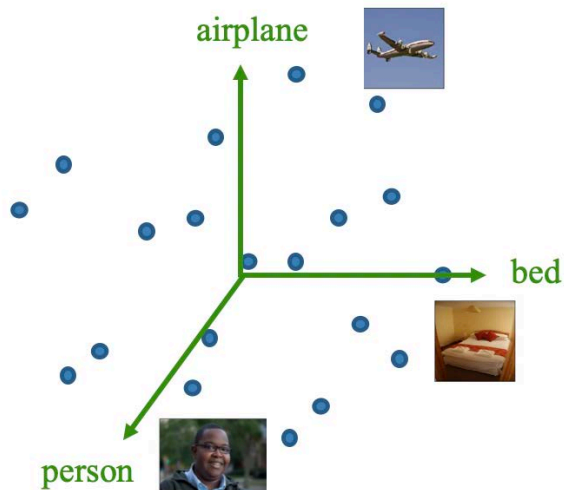


Evidence for this bird being a Wilson's warbler:

Original image (box showing part that looks like prototype)	Prototype	Training image where prototype comes from	Activation map	Similarity score	Class connection	Points contributed
				1.342	Wilson's warbler	$1.342 \times 1.357 = 1.821$
				1.189	Wilson's warbler	$1.189 \times 1.247 = 1.483$
				1.189	Wilson's warbler	$1.189 \times 1.247 = 1.483$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
Total points to Wilson's warbler:						<u>9.744</u>

Base model: DenseNet161

- Even for computer vision, we can still have an interpretable model of the same accuracy as a black box.



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## Concept Whitening for Interpretable Image Recognition

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Zhi Chen<sup>1</sup> Yijie Bei<sup>2</sup> Cynthia Rudin<sup>1,2</sup>

### Abstract

What does a neural network encode about a concept as we traverse through the layers? Interpretability in machine learning is undoubtedly

The questions listed above are important, but it is not clear that they would naturally have satisfactory answers when performing posthoc analysis on a pretrained neural network. In fact, there are several reasons why various types of posthoc analyses would not answer these questions.

Nature Machine Intelligence, accepted, Oct 2020

## The Idea

- Create a latent space that tells us *how* it is disentangling concepts
- Form the latent space so that its axes represent known concepts
- It's easy to do: Just replace a batch normalization step with a "Concept Whitening" step.
- Instead of normalizing, whiten and rotate.



# Summary

- Trees: Modern decision tree methods are not your old CART.
- Scoring systems: Rounding linear model coefficients can go against the performance gradient. LCPA helps.
- Interpretable neural networks for computer vision: yes, they exist.

Jimmy Lin, Chudi Zhong, Diane Hu, Cynthia Rudin, Margo Seltzer  
[Generalized and Scalable Optimal Sparse Decision Trees](#). ICML, 2020.

Berk Ustun and Cynthia Rudin  
[Learning Optimized Risk Scores](#). JMLR, 2019. Shorter version at KDD 2017.

Aaron F. Struck, Berk Ustun, ....., Cynthia Rudin, M Brandon Westover.  
[Association of an Electroencephalography-Based Risk Score With Seizure Probability in Hospitalized Patients](#). JAMA Neurology, 2017

Chaofan Chen, Oscar Li, Chaofan Tao, Alina Barnett, Jonathan Su, Cynthia Rudin  
[This Looks Like That: Deep Learning for Interpretable Image Recognition](#). NeurIPS, 2019.

Zhi Chen, Yijie Bei, Cynthia Rudin  
[Concept Whitening for Interpretable Image Recognition](#). Nature Machine Intelligence, accepted 2020.



