INSTRUCTIONS

(1) DO NOT TURN THIS PAGE UNTIL INSTRUCTED TO DO SO.
(2) FILL IN THE INFORMATION ON THIS PAGE (your name and id).
(3) SHOW YOUR WORK on every problem.
    PROVE YOUR CLAIMS on every answer.
    Computations or answers with no support work will not receive full credit.
(4) NO EXTRA ASSISTANCE ALLOWED.
    No notes, books, phone, laptop, calculator...

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1. (7 points)

Find an LU-decomposition for

\[ A = \begin{pmatrix} 1 & 4 & 4 \\ 1 & 3 & 3 \\ 0 & 2 & 1 \end{pmatrix}. \]
2. (8 points) Consider the matrix

\[
A = \begin{pmatrix}
1 & 1 & -1 & 0 & 1 \\
1 & 0 & -1 & 1 & 1 \\
0 & 1 & 1 & 1 & 1
\end{pmatrix}.
\]

Find a basis for the

(a) column space \(C(A)\),
(b) row space \(R(A)\),
(c) nullspace \(N(A)\).
3. (9 points)

(a) Give the projection of vector \( b = \begin{bmatrix} 2 \\ 2 \\ -2 \end{bmatrix} \) onto the line (through 0) in direction \( a = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \).

(b) Give the projection of vector \( b = \begin{bmatrix} 2 \\ 2 \\ -2 \end{bmatrix} \) onto the column space of \( A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} \).
4. (6 points) Decide whether the following statements are true or false!
Give a **ONE-LINE REASON** for your decision. (*You only get a point for a correct reason.*)

(a) The set of 3\times 3-matrices with rank 2 is a subspace of the vector space of 3\times 3-matrices.
(b) The set of upper triangular \(n\times n\)-matrices is a subspace of the vector space of all matrices.
(c) If \(m > n\), then the dimension of the nullspace of an \(m\times n\)-matrix is greater than the dimension of the left nullspace of the matrix.
(d) \(n\times n\)-matrices with rank \(n\) are invertible.
(e) If the column space of a 2\times 2-matrix has dimension 1, then the row space has dimension 1.
(f) The nullspace and left nullspace are orthogonal to each other.