Math 22A  
Homework 2 (MATLAB Part)  
January 21, 2014  
Submit before Jan. 27, 11:59pm to SmartSite-Assignment

1. (3 points)

(1). Input vectors \( u = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \) and \( v = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \), then calculate the vector \( w = 3 \cdot u + 2 \cdot v \).

(Note: \( u \) and \( v \) are column vectors!)

\[
\begin{align*}
>> & u = [1 1 1]' \\
& u = \\
& \quad 1 \\
& \quad 1 \\
& \quad 1 \\
>> & v = [2 3 4]' \\
& v = \\
& \quad 2 \\
& \quad 3 \\
& \quad 4 \\
>> & w = 3*u + 2*v \\
& w = \\
& \quad 7 \\
& \quad 9 \\
& \quad 11 \\
\end{align*}
\]

(2). Calculate the vector \( w = 5 \cdot u - 7 \cdot v \).

\[
\begin{align*}
>> & w = 5*u - 7*v \\
\end{align*}
\]
\[ w = \]
\[-9 -16 -23 \]

(3). Use the MATLAB `==` command to confirm that addition of \( u \) and \( v \) is commutative (remember, it is for all vectors!).

\[ \gg u + v \]
\[ \text{ans} = \]
\[ 3 4 5 \]

\[ \gg v + u \]
\[ \text{ans} = \]
\[ 3 4 5 \]

\[ \gg v + u == u + v \]
\[ \text{ans} = \]
\[ 1 1 1 \]

(4). Calculate the lengths of vectors \( u \) and \( v \).
(Note: “length” means “norm” here, but no deduction point if misunderstood as dimension of vector; use `norm()` will be fine.)
\[ \text{len}_u = \sqrt{u'^*u} \]
\[
\begin{align*}
\text{len}_u &= 1.7321 \\
\text{len}_v &= \sqrt{v'^*v} \\
\text{len}_v &= 5.3852
\end{align*}
\]

(5). Calculate the inner product of \( u \) and \( v \).

\[ u'^*v \]
\[
\begin{align*}
\text{ans} &= 9
\end{align*}
\]

(6). Calculate the angle between vectors \( u \) and \( v \). You only need to show the value of \( \cos(u, v) \).

\[ \frac{(u'^*v)}{\text{len}_u \times \text{len}_v} \]
\[
\begin{align*}
\text{ans} &= 0.9649
\end{align*}
\]

2. (5 points)
You are given the system of linear equations below:
\[
\begin{align*}
x + y + z &= 4 \\
2x + y &= 3 \\
y + z &= 3
\end{align*}
\]

(1). Find the associated matrix (denoted as \( A \)) and the right-hand side vector (denoted as \( b \)).

\[ A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \]
A =

\[
\begin{pmatrix}
1 & 1 & 1 \\
2 & 1 & 0 \\
0 & 1 & 1
\end{pmatrix}
\]

>> b = [4 3 3]'

b =

\[
\begin{pmatrix}
4 \\
3 \\
3
\end{pmatrix}
\]

(2). Calculate the rank of A.

>> rank(A)

ans =

\[
3
\]

(3). Calculate the inverse of A.

>> inv_A = inv(A)

inv_A =

\[
\begin{pmatrix}
1 & 0 & -1 \\
-2 & 1 & 2 \\
2 & -1 & -1
\end{pmatrix}
\]

(4). Calculate the transpose of A.

>> tra_A = A'

tra_A =

\[
\begin{pmatrix}
1 & 2 & 0 \\
1 & 1 & 1
\end{pmatrix}
\]
1  0  1

(5). Calculate the solution of the system of linear equations.  
(Note: use solve() or other related functions will be fine.)

>> inv_A*b

ans =

1
1
2

(6). Input matrix $B = \begin{bmatrix} 1 & 3 & 5 \\ 7 & 9 & 11 \\ 13 & 15 & 17 \end{bmatrix}$. Then calculate $C = A + 2 \cdot B$, $D = A \cdot B$.

>> B = [1 3 5; 7 9 11; 13 15 17]

B =

1   3   5
7   9  11
13  15  17

>> C = A + 2*B

C =

3   7  11
16  19  22
26  31  35

>> D = A*B

D =

21  27  33
9  15  21
20  24  28
If the first equation $x + y + z = 4$ is replaced by $x + y + 0.5z = 4$, MATLAB cannot find a solution to the new system of equations. Try this for yourself in MATLAB, then use what you have learned to explain why.

```matlab
>> E = [1 1 0.5; 2 1 0; 0 1 1]

E =

1.0000    1.0000    0.5000
2.0000    1.0000         0
 0    1.0000    1.0000

>> inv(E)*b
Warning: Matrix is singular to working precision.

ans =

Inf
Inf
Inf

>> A_b = [E b]

A_b =

1.0000    1.0000    0.5000    4.0000
2.0000    1.0000         0    3.0000
 0    1.0000    1.0000    3.0000

>> rref(A_b)

ans =

1.0000         0
-0.5000         0
0    1.0000    1.0000         0
0         0         0    1.0000

Because by change the first equation, the system will have no solution at all as it met the problem that the row of zeros end with non-zero constant.