MAT 22B Summer Session 1, 2016 - Homework 1*†

1. Find \( \frac{dy}{dx} \) of following functions:

(a) \( y = x^3 - x - 3\sqrt{x} + 2 - \frac{1}{x^2}; \)
(b) \( y = x^{\ln x}; \)
(c) \( y = e^x - \sin^2 x + \frac{x + \cos x}{x^2 + 1}. \)

2. Find following indefinite integrals:

(a) \( \int \frac{1}{x} \, dx; \)
(b) \( \int x \ln x \, dx; \)
(c) \( \int x \cos x^2 \sin x^2 \, dx. \)

3. Problem 1-6 of Section 1.3.

4. Show that following functions are all solutions to the differential equation

\[ \frac{d^2y}{dx^2} + \omega^2 y = 0 \]

where \( \omega > 0 \) is a constant. (\( c_1, c_2, A \) and \( B \) below are arbitrary constants.)

(a) \( y = \cos \omega x; \)
(b) \( y = c_1 \cos \omega x; \)
(c) \( y = \sin \omega x; \)
(d) \( y = c_1 \cos \omega x + c_2 \sin \omega x; \)
(e) \( y = A \sin(\omega x + B). \)

5. Verify that each given function is a solution of the differential equation.

(a) DE: \( y'' - y = 0, \)  Function: \( y = \cosh t; \)

*Due Date: Wed. 6/29.
†Problem 3, 7, 8 are based on *Elementary Differential Equations and Boundary Value Problems*, 9th/10th Edition, by Boyce and DiPrima.
(b) DE: \( t^2 y'' + 5ty' + 4y = 0 \) with \( t > 0 \), Function: \( y = t^{-2} \ln t; \)

(c) DE: \( xy' + y = \cos x \), Function: \( y = \frac{\sin x}{x}; \)

(d) DE: \( y' - 2ty = 1 \), Function: \( y = e^{t^2} \int_0^t e^{-s^2} ds + e^{t^2}; \)

(e) DE: \( y' = \frac{f'(x)}{g(x)} y^2 - \frac{g'(x)}{f(x)} \) with \( f(x) \neq 0 \) and \( g(x) \neq 0 \) for all \( x \in \mathbb{R} \) some differentiable functions, Function: \( y = -\frac{g(x)}{f(x)}. \)

6. Determine the values of \( r \) for which the given differential equation has solutions of the form \( y = e^{rt}. \)

(a) \( y' + 2y = 0; \)

(b) \( y'' - 3y' + 2y = 0. \)

7. Problem 14, 15, 16, 19, 20 of Section 2.1.

8. Problem 1-8 of Section 2.2.