Show all work and label all answers. Also, don’t forget to check your answers.

For the problems 1 through 4, let \( f(x, y) = \sin(2x + y^3) - e^{x-y} \)

1. (3 points) Find \( f(1, 0) \).

2. (3 points) Find \( \frac{\partial f}{\partial y} \).

3. (4 points) Find \( f_x(-1, 2) \).

4. (3 points) Find \( f_{xy}(x, y) \).

5. (2 points) Let \( g(t, u, v, w) \) be a function. What is the largest set that could be the domain of \( g \)?
6. (4 points) Show that \( \lim_{(x,y) \to (2,-1)} (3x^2y + 5xy^2) \) exists, and evaluate it.

7. (3 points) If the limit of a function \( h(x, y) \) as \((x, y) \to (0,0)\) along the line \( x = 0 \) is 2, and the limit along the line \( y = 0 \) is also 2, then what (if anything) can we conclude about \( h \)?

8. (4 points) Compute
\[
\lim_{(x,y) \to (0,0)} \frac{6x^2 + 3y^2}{2x^2 + 3y^2}
\]
along the \( x \)-axis and \( y \)-axis. What can you conclude?

9. (4 points) Let \( w = f(x, y) = x^2e^{3y} \) with \( x(t) = 3e^t \) and \( y(t) = t^4 \). Find the derivative \( \frac{dw}{dt} \) when \( t = 0 \).
10. (3 points) Define continuity of a function \( f(x, y) \) at a point \((x_0, y_0)\).

11. (5 points) Find the linearization of \( h(x, y) = e^{x^2+2y} \) at \((0, 0)\) and use it to approximate \( h(0.1, 0.1) \)

12. (2 points) True or False? If a function \( f(x, y) \) is differentiable and has a horizontal (parallel to \( xy\)-plane) tangent plane at \((x_0, y_0)\), then it has a maximum or minimum value there.

13. (2 points) True or False? If a function \( f(x, y) \) is differentiable and has a maximum or minimum value at \((x_0, y_0)\), then it has a horizontal tangent plane there.

14. (2 points) True or False? Given a function \( f(x, y) \), if \( f_x(x_0, y_0) \) and \( f_y(x_0, y_0) \) exist, then \( f \) is differentiable at \((x_0, y_0)\).

15. (3 points) What information does the direction of the gradient give?
For the next three problems, use \( g(x, y) = 2x^2 - 2y^2 - x^4 \)

16. (6 points) Find all critical points of \( g \) and determine if each point is a relative maximum, relative minimum, or a saddle point.

17. (4 points) Find \( \nabla g(-2, 3) \), the gradient of \( g \) at \((-2, 3)\).

18. (4 points) Find the directional derivative of \( g \) at \((-2, 3)\) in the direction \( \begin{bmatrix} 3 \\ 4 \end{bmatrix} \).
For the next four questions, use the following system of differential equations

\[
\frac{dx_1}{dt} = -4x_1 + 3x_2
\]

\[
\frac{dx_2}{dt} = -5x_2
\]

19. (3 points) Write the system in matrix form.

20. (8 points) Solve the differential equations with the initial values \( x_1(0) = -2 \) and \( x_2(0) = 2 \)
21. (3 points) Analyze the stability of the equilibrium (0, 0)

22. (4 points) Draw a compartment model for the system.

23. (5 points) Suppose two species have a community matrix $\begin{bmatrix} -3 & -5 \\ 0 & -4 \end{bmatrix}$. Describe the relationship between species 1 and species 2. Is the equilibrium associated with this matrix stable?
24. (6 points) Following the Lotka-Volterra model, we have a system for two species:

\[
\frac{dN_1}{dt} = 3N_1 \left( 1 - \frac{N_1}{12} - \frac{2N_2}{12} \right)
\]

\[
\frac{dN_2}{dt} = 2N_2 \left( 1 - \frac{N_2}{15} - \frac{5N_1}{15} \right)
\]

To save you computation, one of the equilibria is at (10, 10). What happens to these two species over time?
**Bonus Questions**

25. What is interesting about the answer to problem 16.

26. Who played the main protagonist in the movie *Equilibrium*?

27. Describe a relationship between two specific species which might yield the community matrix from problem 23. Absurdity, fiction, and illustrations highly encouraged.