Show all work and label all answers. Also, don’t forget to check your answers.

For the problems 1 through 4, let \( f(x, y) = x \ln(x^2 + 3y) - e^{x-y} \)

1. Find \( f(1, 0) \)
2. Find \( \frac{\partial f}{\partial y} \)
3. Find \( f_x(-1, 2) \)
4. Find \( f_{xy}(x, y), f_{yx}(x, y), f_{xx}(x, y) \) and \( f_{yy}(x, y) \)
5. Let \( g(x, y, z) \) be a function. What is the largest set that could be the domain of \( g \)?
6. Show that \( \lim_{(x,y) \to (1,1)} (2xy + 3x^2) \) exists, and evaluate it.
7. Compute
\[
\lim_{(x,y) \to (0,0)} \frac{4xy}{2x^2 + 3y^2}
\]
along the \( x \)-axis, \( y \)-axis, and the line \( y = x \). What can you conclude?
8. Find the linearization of \( h(x, y) = e^{2x-4y} \) at \((0,0)\) and use it to approximate \( h(0.1,0.1) \)

For the next three problems, use \( g(x, y) = x^2 y - 3xy + 2 \)

9. Find all critical points of \( g \) and determine if that point is a relative maximum, relative minimum, or a saddle point.
10. Find \( \nabla g(2,-1) \) the gradient of \( g \) at \((2,-1)\)
11. Find the directional derivative of \( g \) at \((2,-1)\) in the direction \( \begin{bmatrix} 3 \\ 4 \end{bmatrix} \)
12. Let \( w = f(x, y) = xe^y \) with \( x(t) = e^t \) and \( y(t) = t^2 \). Find the derivative \( \frac{dw}{dt} \) when \( t = 0 \).

For the next four questions, use the following system of differential equations
\[
\begin{align*}
\frac{dx_1}{dt} &= -x_1 \\
\frac{dx_2}{dt} &= x_1 - 2x_2 
\end{align*}
\]
13. Write the system in matrix form
14. Solve the differential equations with the initial values \( x_1(0) = -1 \) and \( x_2(0) = -2 \)
15. Analyze the stability of the equilibrium \((0,0)\)
16. Draw a compartment model for the system.

17. Find the equilibria of the following system of differential equations, and analyze the stability of each equilibrium. (ignore any complex eigenvalues that may pop up. I haven’t checked this problem yet, but I want to get this practice exam out asap)

\[
\begin{align*}
\frac{dx_1}{dt} &= 2x_1(3 - x_2 - 5x_2) \\
\frac{dx_2}{dt} &= 3x_2(1 - 8x_1 - 3x_2)
\end{align*}
\]

18. Following the Lotka-Volterra model, we have a system for two species:

\[
\begin{align*}
\frac{dN_1}{dt} &= N_1(1 - \frac{N_1}{40} - \frac{.2 N_2}{40}) \\
\frac{dN_2}{dt} &= N_2(1 - \frac{N_2}{50} - \frac{.2 N_1}{50})
\end{align*}
\]

To save you computation, one of the equilibria is at \((31.25, 43.75)\). What happens to these two species over time? (coexistence, one species dominates, they both die off, or it depends on initial conditions)

19. Suppose two species have a community matrix \[
\begin{bmatrix}
-3 & 2 \\
-2 & -4
\end{bmatrix}
\]. Describe the relationship between species 1 and species 2. Is the equilibrium associated with this matrix stable?