

# MAT 125B FIRST MIDTERM EXAM

Last Name (PRINT): \_\_\_\_\_

First Name (PRINT): \_\_\_\_\_

Student ID #: \_\_\_\_\_

## Instructions:

1. Do not open your test until you are told to begin.
2. Use a pen to print your name in the spaces above.
3. No notes, books, calculators, or any other electronic devices allowed.
4. Show all your work. Unsupported answers will receive NO CREDIT.
5. You are expected to do your own work.
6. Problem 4 is a bonus problem.

#1	#2	#3	#4	TOTAL

1. Give an example of a function  $f : [a, b] \rightarrow \mathcal{R}$  which is not Riemann integrable. Explain. Compute the upper Riemann integral and the lower Riemann integral of the function from your example.

2. If  $f : \mathcal{R} \rightarrow \mathcal{R}$  is continuous, find the derivative of  $F$  for

$$F(x) = \int_0^{x \sin x} t f(t) dt.$$

3. Let  $f(x) = \frac{\sin(x)}{x^\gamma}$ . For what values of  $\gamma > 0$  the function  $f$  is a) Riemann integrable on  $[0, 1]$  b) improperly integrable on  $[0, 1]$ .

4. Let  $f : [0, 1] \rightarrow \mathcal{R}$  be such that  $f$  is identically zero at irrational points and  $f(p/q) = 1/q$  for rational points  $x = p/q$ ,  $\gcd(p, q) = 1$ . Prove or disprove that  $f$  is integrable on  $[0, 1]$ .