MAT 125B Second Midterm Exam

Last Name (PRINT): ____________________________________

First Name (PRINT): ____________________________________

Student ID #: ____________________________________

Instructions:

1. Do not open your test until you are told to begin.
2. Use a pen to print your name in the spaces above.
3. No notes, books, calculators, or any other electronic devices allowed.
4. Show all your work. Unsupported answers will receive NO CREDIT.
5. You are expected to do your own work.

#1	#2	#3	#4	TOTAL

1. Transform the Laplacian $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}$ into polar coordinates (i.e. consider the substitution $x = r\cos\theta,\ y = r\sin\theta$).

2. Let $f(x,y) = \frac{x^3 - xy^2}{x^2+y^2}$ if $(x,y) \neq (0,0)$ and $f(0,0) = 0$. Show that f is continuous and has first-order partial derivatives on R^2, but is not differentiable at $(0,0)$.

3. Suppose $f, g : R^n \to R^m$ and $a \in R$. Prove that if f and g are differentiable at $x \in R^n$, then so too are $f + g$ and af.

4. Prove or disprove the following statement:

   Let the Jacobian determinant of a mapping $T : R^2 \to R^2$, $T(u, v) = (x, y)$, is never zero (i.e. it does not vanish for all $(u, v) \in R^2$). Then T must be one-to-one on all of R^2.