MAT 125B Final Exam

Last Name (PRINT): ____________________________________

First Name (PRINT): ____________________________________

Student ID #: ____________________________________

Instructions:

1. Do not open your test until you are told to begin.
2. Use a pen to print your name in the spaces above.
3. No notes, books, calculators, or any other electronic devices allowed.
4. Show all your work. Unsupported answers will receive NO CREDIT.
5. You are expected to do your own work.

#1	#2	#3	#4	#5	#6	TOTAL

1. Let $f : [0, 1] \to R$ be nondecreasing on the set $[0, 1]$. Show that f is integrable on $[0, 1]$.

2. Compute
$$\lim_{n\to\infty}(\frac{1}{n+1}+\frac{1}{n+2}+\ldots+\frac{1}{2n}).$$

3. Let $f : R^2 \to R$ be defined as $f(x, y) := \frac{xy}{|x|+|y|}$ for $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$. Show that f is continuous everywhere. Where it is differentiable?

4. Show that if f is a continuously differentiable real-valued function in R^2 and $\frac{\partial^2 f}{\partial x \partial y} = 0$ everywhere, then there are continuous differentiable real-valued functions g and h on R such that

$$f(x, y) = g(x) + h(y).$$

5. Let $F : R^2 \to R$ be defined as

$$\begin{cases} 0, & \text{if } |y| \geq x^2 \\ x, & \text{if } y = 0 \\ -\frac{1}{x}(y - x^2) & \text{if } 0 < y < x^2 \\ \frac{1}{x}(y + x^2) & \text{if } -x^2 < y < 0 \end{cases}$$

(a) Verify analitically that f is continuous and has directional derivative $D_u f(0,0) = 0$, for all directions with the exception of $(1,0)$ and $(-1,0)$.

(b) Calculate the first order partial derivatives f_1 and f_2 and show that they are discontinuous at $(0,0)$.

6. Let (X_1, d_1) and (X_2, d_2) be metric spaces. The set

$$X_1 \times X_2 = \{(x_1, x_2) : x_1 \in X_1, x_2 \in X_2\}$$

is called the Cartesian product of X_1 and X_2. For

$$u = (x_1, x_2) \in X_1 \times X_2,\ \ v = (y_1, y_2) \in X_1 \times X_2,$$

define $d(u, v) = d_1(x_1, y_1) + d_2(x_2, y_2)$.
Prove that d is a metric on $X_1 \times X_2$.