Note that if $z_1, z_2$ are complex numbers then $|z_1 + z_2| = |z_1| + |z_2|$ iff $z_1$ and $z_2$ have the same angle.

hence $P$ for any $m,n \ P_m e^{i \tan \alpha_m}$ and $P_n e^{i \tan \alpha_n}$ have the same angle. Hence $e^{i \tan \alpha_m} = e^{i \tan \alpha_n}$ and $\tan(\alpha_m - \alpha_n) = 2\pi N_{m,n}$ for some $N_{m,n} \in \mathbb{Z}$.

i.e. $\alpha_m - \alpha_n \in \frac{2\pi}{\alpha_m} \mathbb{Z}$.

Hence $X$ must have a lattice distribution.