MAT 135A Homework 5

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Solution of one selected problem

Problem 3: We know that $A \sim U[0,5]$, therefore the pdf of A is given by

$$f_A(a) = \begin{cases} \frac{1}{5}, & \text{if } 0 \leq a \leq 5 \\ 0, & \text{otherwise.} \end{cases}$$

The quadratic equation $4x^2 + 4Ax + (A+2) = 0$ has two real roots if and only if the Discriminant is positive, i.e., $(4A)^2 - 4 \cdot 4 \cdot (A+2) > 0$. So the probability that the given quadratic equation has two real root is

$$\begin{aligned}
\mathbb{P}((4A)^2 - 4 \cdot 4 \cdot (A+2) > 0) &= \mathbb{P}(A^2 - A - 2 > 0) \\
&= \mathbb{P}((A-2)(A+1) > 0) \\
&= \mathbb{P}((A-2) > 0 \;\&\; (A+1) > 0) + \mathbb{P}((A-2) < 0 \;\&\; (A+1) < 0) \\
&= \mathbb{P}(A > 2) + \mathbb{P}(A < -1) \\
&= \int_2^5 \frac{1}{5}\, da + 0 \\
&= \frac{3}{5}.
\end{aligned}$$

Remark: If $A \sim U[-3,3]$, then the above probability would be

$$\mathbb{P}(A > 2) + \mathbb{P}(A < -1) = \int_2^3 \frac{1}{6}\, da + \int_{-3}^{-1} \frac{1}{6}\, da = \frac{1}{6} + \frac{2}{6} = \frac{1}{2}.$$

Moral: $A^2 - A - 2 > 0$ can happen in two possible ways, namely $A > 2$ or $A < -1$. You should consider both of the cases.