Solution of one selected problem

Problem 3: We know that $A \sim U[0, 5]$, therefore the pdf of $A$ is given by

$$f_A(a) = \begin{cases} \frac{1}{5}, & \text{if } 0 \leq a \leq 5 \\ 0, & \text{otherwise}. \end{cases}$$

The quadratic equation $4x^2 + 4Ax + (A + 2) = 0$ has two real roots if and only if the Discriminant is positive, i.e., $(4A)^2 - 4 \cdot 4 \cdot (A + 2) > 0$. So the probability that the given quadratic equation has two real root is

$$P((4A)^2 - 4 \cdot 4 \cdot (A + 2) > 0) = P(A^2 - A - 2 > 0)$$

$$= P((A - 2)(A + 1) > 0)$$

$$= P((A - 2) > 0 \& (A + 1) > 0) + P((A - 2) < 0 \& (A + 1) < 0)$$

$$= P(A > 2) + P(A < -1)$$

$$= \int_2^5 \frac{1}{5} \, da + 0$$

$$= \frac{3}{5}.$$

Remark: If $A \sim U[-3, 3]$, then the above probability would be

$$P(A > 2) + P(A < -1) = \int_2^3 \frac{1}{6} \, da + \int_{-3}^{-1} \frac{1}{6} \, da = \frac{1}{6} + \frac{2}{6} = \frac{1}{2}.$$

Moral: $A^2 - A - 2 > 0$ can happen in two possible ways, namely $A > 2$ or $A < -1$. You should consider both of the cases.